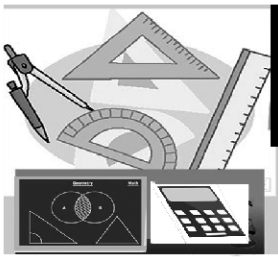
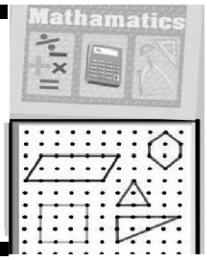


Unit 5

TRIGONOMETRY



Points to Remember

- For triangles with the same set of angles, the ratio of the lengths of the sides is the same.
- The angles of a triangle determines the ratio of the sides. The perpendicular sides of a right triangle with angles 45°, 45°, 90° are equal. To find the length of the hypotenuse, multiply the perpendicular side by $\sqrt{2}$.

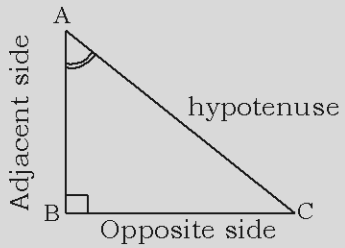
  (Ratio of the sides of this triangle is $1 : 1 : \sqrt{2}$)

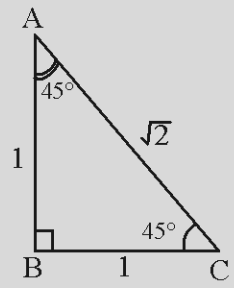
  In a right angled triangle with angles 30°, 60°, 90°, hypotenuse will be two times the length of the side opposite to 30° angle. Also the length of the side opposite to 60° angle will be $\sqrt{3}$ times the length of the side opposite to 30° angle.

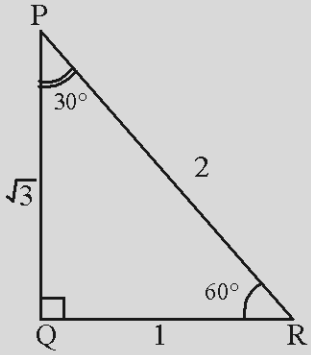
  (Ratio of the sides of this triangle is $1 : \sqrt{3} : 2$)
- In all right triangles with the same angles, the number got by dividing the opposite side of an acute angle by the hypotenuse is the same. It is called the sine of the angle, written as 'sin'.
- The number got by dividing the adjacent side of an acute angle (Shorter of the two sides containing the angle) by the hypotenuse is also the same. It is called the cosine of the angle. It is shortened as 'cos'.
- The number got by dividing the opposite side of an angle by the adjacent side will be same number. It is called the tangent of the angle. It is shortened as 'tan'.

$$\sin A = \frac{BC}{AC} \qquad \cos A = \frac{AB}{AC}$$

$$\tan A = \frac{BC}{AB}$$



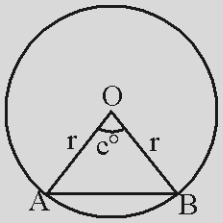




	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

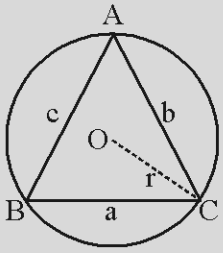
- In a circle, the length of a chord is double the product of the sine of half the central angle and the radius.

  In a circle of radius 'r', the length of the chord with central angle c is 2r Sin $\left(\frac{c}{2}\right)$

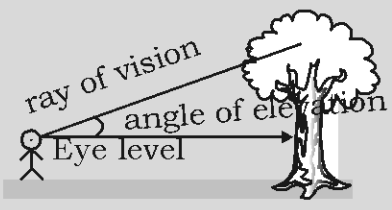


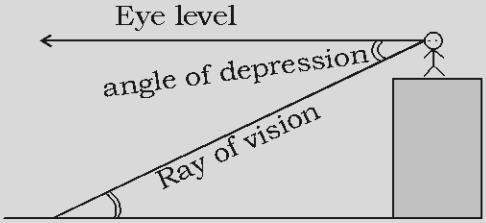
- In any triangle, the ratio of the sides is equal to the ratio of the sines of the angles opposite them.
- The length of the sides of a triangle are the sines of its angles opposite to that side, multiplied by the diameter of its circumcircle. If any angle is greater than the right angle the sine of its supplementary angle should be taken. If the angle is 90° the opposite side is equal to the circum diameter.
- To find the circum diameter, divide the length of one side of a triangle by its Sine of angle opposite to that side.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$$

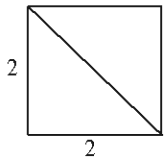
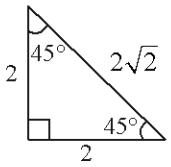
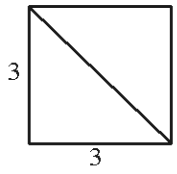
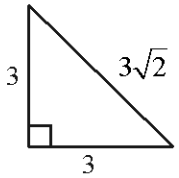
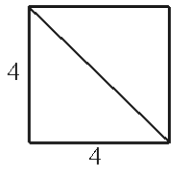
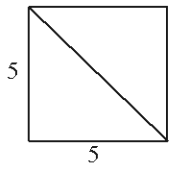
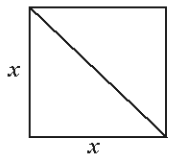
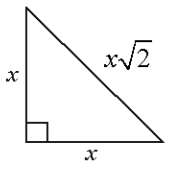


- Heights and distances which cannot be directly measured can be computed using trigonometric ratios.



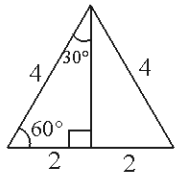
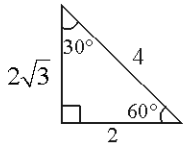
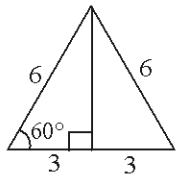
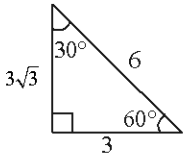
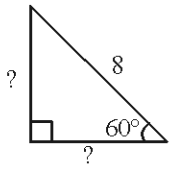
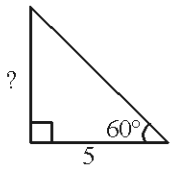
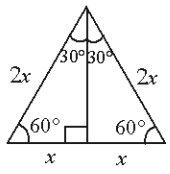
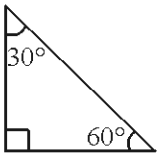


Worksheet 1

Square	Length of diagonal $d = a\sqrt{2}$	Isosceles triangle	Length of sides			Ratio of Sides
			Opposite to 45°	Opposite to 45°	Opposite to 90°	
	$d = 2\sqrt{2}$		2	2	2	$2:2:\sqrt{2} = 1:1:\sqrt{2}$
	$d = 3\sqrt{2}$		—	—	$3\sqrt{2}$	— : — : — = — : — : —
	d = —	———	———	———	———	— : — : — = — : — : —
	d = —	———	———	———	———	— : — : — = — : — : —
	$d = x\sqrt{2}$		———	———	———	$x : x : \sqrt{2}$

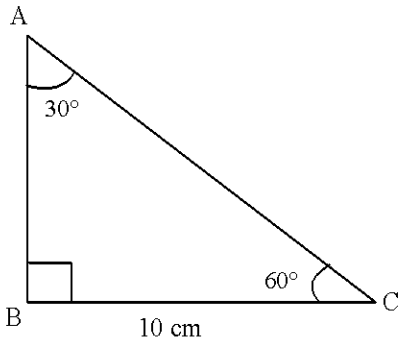
Worksheet 2

Complete the following table.

Equilateral triangle	altitude $h=\frac{a\sqrt{3}}{2}$	Triangle	Length of sides			Ratio of Sides
			Opposite to 30°	Opposite to 60°	Opposite to 90°	
	$h=2\sqrt{3}$		2	$2\sqrt{3}$	4	$2:2\sqrt{3}:4$ $=1:\sqrt{3}:1$
	$h=3\sqrt{3}$		———	———		—— : —— : —— = —— : —— : ——
———	h = ——		———	———	———	—— : —— : —— = —— : —— : ——
———	h = ——		———	———	———	—— : —— : —— = —— : —— : ——
	h = ——		———	———	———	$x:x:\sqrt{2}$

Worksheet 3

Angles of ΔABC are 30°, 60°, 90°. Calculate the perimeter of the triangle.



Ratio of sides of ΔABC ☐ : ☐ : ☐

Length of the side BC opposite to 30° = ☐

Length of the side AB opposite to 60° = ☐

Length of the side AC opposite to 90° = ☐

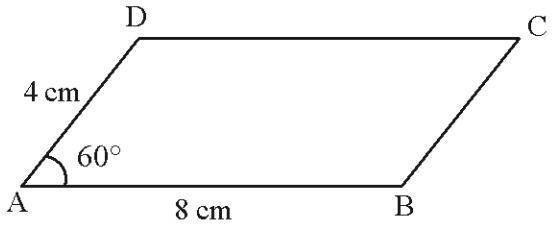
Perimeter of ΔABC = ☐ + ☐ + ☐

= ☐ + ☐ cm

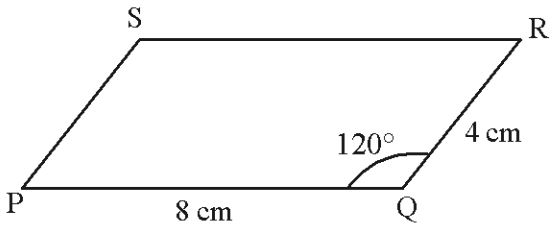
Worksheet 4

Calculate the area of the parallelogram in the figure.

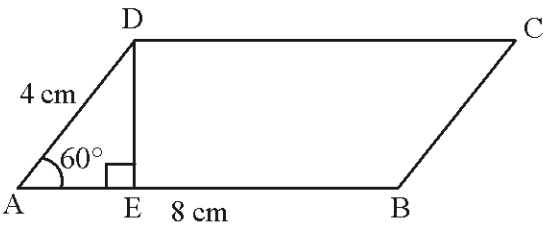
a.



b.



a.



Draw perpendicular from D to AB.

Then $\angle AED =$ ☐

$\angle ADE =$ ☐

Length of the side AD opposite to 90° = ☐

Length of the side AE opposite to 30° = ☐

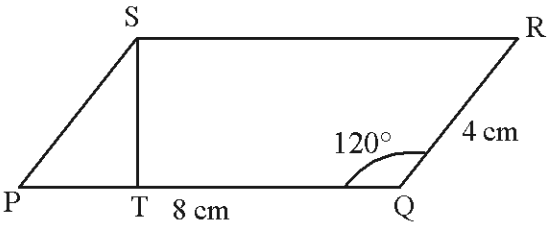
Length of the side DE opposite to 60° = ☐

Length of side AB = ☐ cm

Area of Parallelogram ABCD = AB × DE

= ☐ × ☐ sq. cm

b.



Draw ST perpendicular to PQ.

$\angle P = 180^0 -$ ☐ = ☐

$\angle PTS =$ ☐

$\angle PST =$ ☐

PS = ☐ = 4cm

PQ = ☐ cm

Length of the side PS opposite to 90^0 = ☐ cm

Length of the side PT opposite to 30^0 = ☐ cm

Length of the side ST opposite to 60^0 = ☐ cm

Area of parallelogram PQRS

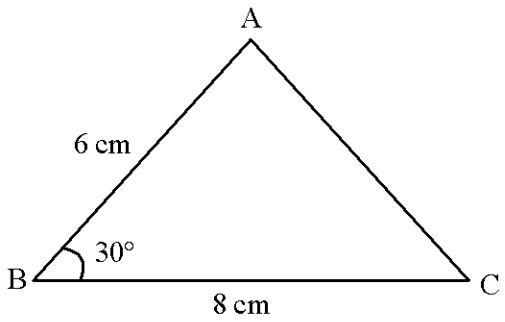
= PQ × ST

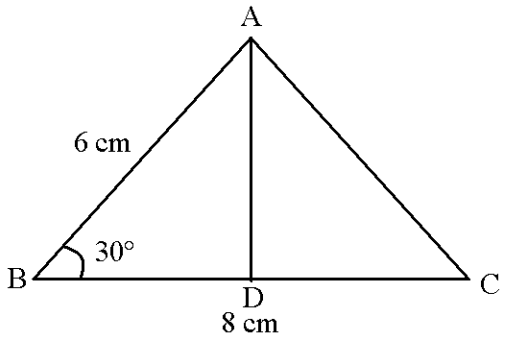
= ☐ × ☐

= ☐ sq.cm

Work Sheet - 5

In ΔABC, AB = 6cm, BC = 8cm, $\angle B = 30^0$. Calculate the area of the triangle.





Draw AD perpendicular to BC.

$\angle ADB =$ ☐

$\angle B =$ ☐

$\angle BAD =$ ☐

Length of side BC = ☐ cm

Length of the side AB opposite to 90^0 = ☐ cm

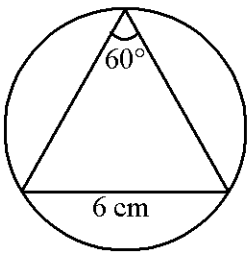
Length of the side AD opposite to 30^0 = ☐ cm

Area of $\Delta ABC = \frac{BC \times AD}{2}$

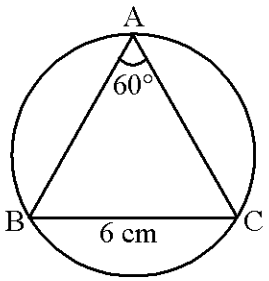
$= \frac{\square \times \square}{2}$

$=$ ☐ sq.cm

Work Sheet - 6



In the figure a triangle and its circum circle are given. Find the diameter of the circle.



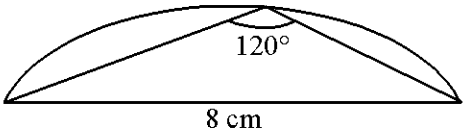
If the diameter is taken as 'd'

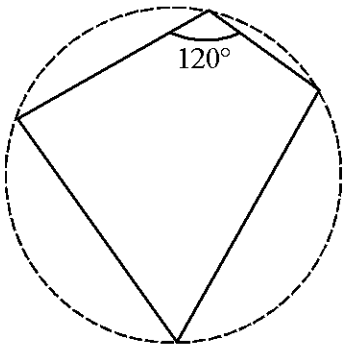
$$d = \frac{a}{\sin A}$$

Here, $d = \frac{6}{\text{Sin}\square}$

diameter = $\frac{\square}{\square} = \square \times \square = \square$

Work Sheet - 7





The figure shows a part of a circle. Find the radium of the circle.

Here the angle on the alternate are is $\square$

Then, to find out the diameter, we can consider that angle.

$\therefore$ diameter = $\frac{\square}{\text{Sin}\square} = \frac{\square}{\square}$

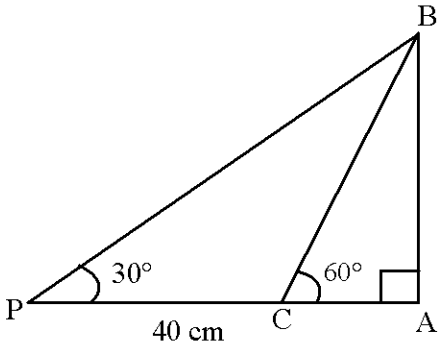
$= \square$

$\therefore$ Radius = $\square$

Work Sheet - 8

A boy standing at one bank of a river sees a tree opposite to the boy at an elevation of 60^0. Stepping 40 metres back, he sees the top at an elevation of 300.

Fill up the following step to find the height of the tree and width of the river about the rough figure given.



AB – Height of the tree

AC - Width of the river

In ΔPCB

$\angle$PCB = 180 – ☐ = ☐

$\therefore \angle$PBC = 180 – [☐+☐]

= ☐

$\therefore \Delta$PCB is an ——— triangle

$\therefore$ BC = ——— = 40cm

In right ΔACB

$\angle$ACB = 60^0, $\angle$A= 90^0, then $\angle$ABC = _____

Ratio of the sides of the $\angle$ACB = _____ : _____ : _____

Side opposite to 90^0, BC = _____

Side opposite to 30^0, AC = _____

Side opposite to 60^0, AB = _____

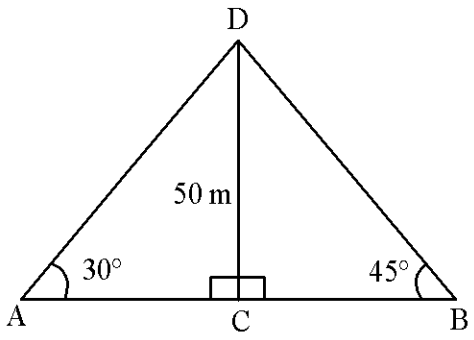
Width of the river = AC = _____

Height of the tree = AB = _____

Work Sheet - 9

Two persons standing either side of an eletric post of height 50m. One person see the top of the post at an anagle of elevation 30^0, and other person see the top of the post at an angle of 45^0.

Fill up the following step find the distance between the person about the rough figure given.



CD - Represent the height of the electric post. The points A and B where the person standing.

Consider the triangle BCD

$\angle B = 45^0$, $\angle BCD =$ _____ $\angle BDC$ _____

The ratio of the sides of BCD = _____

BCD = _____ = 50m

In right $\triangle ACD$, $A = 30^0$, $\angle ACD = \angle ADC$

Ratio of the sides of ACD = _____

Sides opposite to 30^0, CD = _____

Side opposite to 60^0, AC = _____

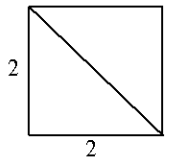
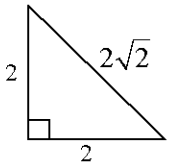
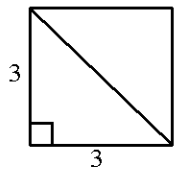
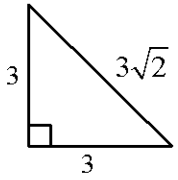
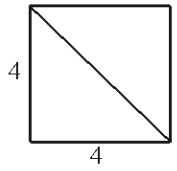
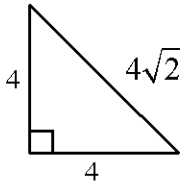
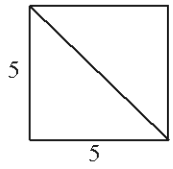
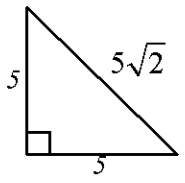
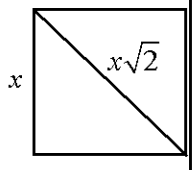
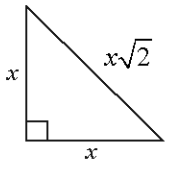
The AC + BC = _____ + _____

= _____

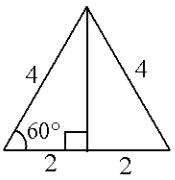
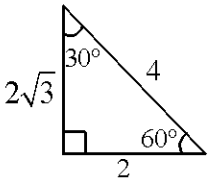
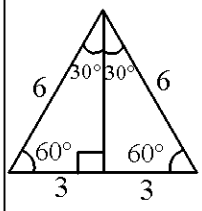
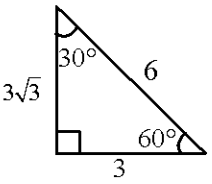
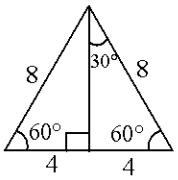
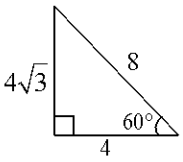
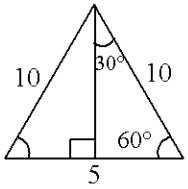
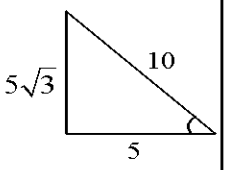
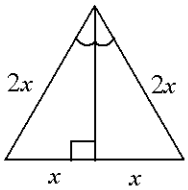
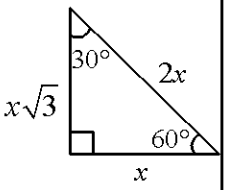
Distance between the two persons, AB = _____

ANSWERS

Worksheet 1

Square	Length of diagonal $d = a\sqrt{2}$	Isosceles triangle	Length of sides			Ratio of Sides
			Angle Opposite to 45°	Angle Opposite to 45°	Angle Opposite to 90°	
	$d = 2\sqrt{2}$		2	2	$2\sqrt{2}$	2:2:2 $\sqrt{3}$ 1 : 1: $\sqrt{3}$
	$d = 3\sqrt{2}$		3	3	$3\sqrt{2}$	3 :3 : $3\sqrt{2}$ = 1 : 1 : $\sqrt{2}$
	d = $4\sqrt{2}$		4	4	$4\sqrt{2}$	4 : 4 : $4\sqrt{2}$ = 1 : 1 : $\sqrt{2}$
	d = $5\sqrt{2}$		5	5	$5\sqrt{2}$	5 : 5 : $5\sqrt{2}$ = 1 : 1 : $\sqrt{2}$
	$d = x\sqrt{2}$		x	x	$x\sqrt{2}$	x : x : $x\sqrt{2}$ = 1 : 1 : $\sqrt{2}$

Worksheet 2

Equilateral triangle	altitude $h=\frac{a\sqrt{3}}{2}$	Triangle	Length of sides			Ratio of Sides
			Angle opposite to 30°	Angle opposite to 60°	Angle opposite to 90°	
	$h=2\sqrt{3}$		2	$2\sqrt{3}$	4	2 : $2\sqrt{3}$: 4 =1 : $\sqrt{3}$: 2
	$h=3\sqrt{3}$		3	$3\sqrt{3}$	6	3 : $3\sqrt{3}$: 6 =1 : $\sqrt{3}$: 2
	$h=4\sqrt{3}$		4	$4\sqrt{2}$	8	4 : 4 : $\sqrt{3}$ = 8 = 1 : $\sqrt{3}$: 2
	h = $5\sqrt{3}$		5	$5\sqrt{3}$	10	5 : $5\sqrt{3}$: 10 = 1 : $\sqrt{3}$: 2
	h = $x\sqrt{3}$		x	$x\sqrt{3}$	$2x$	x : $x\sqrt{3}$:$2x$ = 1 : $\sqrt{3}$: 2

Worksheet 3

$1 : \sqrt{3} : 2$

BC = 10cm

AB = $10\sqrt{3}$ cm

AC = 20cm

Perimeter = $10 + 10\sqrt{3} + 20 = \left(30+10\sqrt{3}\right)$cm

Worksheet 4

$\angle ACD = 90^0$

$\angle ADE = 30^0$

AD = 4cm

AE = 2cm

DE = $2\sqrt{3}$ cm

AB = 8cm

Area of parallelogram ABCD = $8 \times 2\sqrt{3} = 16\sqrt{3}$ sq.cm

Worksheet 5

$\angle$ADB = 90^0

$\angle$B = 30^0

$\angle$BAD = 60^0

BC = 8cm

AB = 6cm

AD = 3cm

Area = $\frac{8 \times 2}{2}$ = 8sq.cm

Worksheet 6

$d = \frac{6}{\sin 60}$

diameter = $\frac{6}{\frac{\sqrt{3}}{2}} = 6 \times \frac{2}{\sqrt{3}} = \frac{3 \times 2 \times 2}{\sqrt{3}}$

$= \frac{\sqrt{3} \times \sqrt{3} \times 2 \times 2}{\sqrt{3}} = 4\sqrt{3}$

Worksheet 7

60^0

Diameter = $\frac{8}{\sin 60}$

$= \frac{8}{\frac{\sqrt{3}}{2}} = 8 \times \frac{2}{\sqrt{3}} = \frac{16}{\sqrt{3}}$

Worksheet 8

$\angle PCB = 180 - \boxed{60} = \boxed{120}$

$\angle PBC = 180 - \boxed{\boxed{30} + \boxed{120}}$

$= \boxed{30}$

PCB is an isosceles triangle

BC = PC = 40cm

$\angle ABC = 30^0$

Ratio of the sides of $\triangle$ ACB = $1 : \sqrt{3} : 2$

BC = 40m

AC = 20m

AB = $20\sqrt{3}$ m

Width of the River : AC = 20m

Height of the tree : AB = $20\sqrt{3}$ m

Worksheet 9

$\angle BCD = 50^0$ $\angle BDC = 45^0$

$\angle BCD$ is isosceles right triangle

BC = CD = 50m

Ratio of the sides of $\triangle$ BCD = $1 : 1 : \sqrt{2}$

$\angle ACD = 90^0$, $\angle ADC = 60^0$

Ration of the sides of $\triangle$ ACD = $1 : \sqrt{3} : 2$

CD = 50m

AC = $50\sqrt{3}$

AC + BC = $50\sqrt{3} + 50$

$= 50\left(\sqrt{3}+1\right)$

Distance between two persons AB = $50\left(\sqrt{3}+1\right)$