MATHEMATICS

(Maximum Marks: 100)

(Time allowed: Three hours)

(*Candidates are allowed additional 15 minutes for* ***only*** *reading the paper*.

They must NOT start writing during this time.)

This Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions* ***EITHER*** *from* ***Section B OR Section C***

Section A: *Internal choice has been provided in three questions of four marks each and two questions of six marks each.*

Section B: *Internal choice has been provided in two questions of four marks each.*

Section C: *Internal choice has been provided in two questions of four marks each.*

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A (80 Marks)

Question 1 **[10×2]**

(i) Determine whether the binary operation $*$ on R defined by $a*b = |a-b|$ is commutative. Also, find the value of $(-3)*2$.

(ii) Prove that:

$\tan^2(\sec^{-1} 2) + \cot^2(\operatorname{cosec}^{-1} 3) = 11$.

(iii) Without expanding at any stage, find the value of the determinant:

$$\Delta = \begin{vmatrix} 20 & a & b+c \\ 20 & b & a+c \\ 20 & c & a+b \end{vmatrix}$$

(iv) If $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$, find x.

(v) Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3axy$

This Paper consists of 6 printed pages.

(vi) The edge of a variable cube is increasing at the rate of 10 cm/sec. How fast is the volume of the cube increasing when the edge is 5 cm long?

(vii) Evaluate: $\int_4^5 |x-5|\,dx$

(viii) Form a differential equation of the family of the curves $y^2 = 4ax$.

(ix) A bag contains 5 white, 7 red and 4 black balls. If four balls are drawn one by one with replacement, what is the probability that none is white?

(x) Let A and B be two events such that

$$P(A) = \frac{1}{2}, P(B) = p \text{ and } P(A \cup B) = \frac{3}{5}$$

find 'p' if A and B are independent events.

Question 2 **[4]**

If the function f: R→R be defined as $f(x) = \frac{3x+4}{5x-7}, \left(x \neq 7/5\right)$ and

g: R→R be defined as $g(x) = \frac{7x+4}{5x-3}, \left(x \neq 3/5\right)$

show that $(g\,o\,f)(x) = (f\,o\,g)(x)$.

Question 3 **[4]**

(a) If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$, then prove that

$$9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$$

OR

(b) Evaluate: $\cos\left(2\cos^{-1}x + \sin^{-1}x\right)$ at $x = \frac{1}{5}$.

Question 4 **[4]**

Using properties of determinants, show that

$$\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} = (x-p)(x^2 + px - 2q^2)$$

Question 5 **[4]**

Verify Rolle's theorem for the function, $f(x) = -1 + \cos x$ in the interval $[0, 2\pi]$

Question 6 **[4]**

If $y = e^{m\sin^{-1}x}$, prove that

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2 y$$

Question 7 **[4]**

(a) The equation of tangent at (2, 3) on the curve $y^2 = px^3 + q$ is $y = 4x - 7$. Find the values of 'p' and 'q'.

OR

(b) Using L'Hospital's rule, evaluate:

$$\lim_{x \to 0} \frac{xe^x - \log(1 + x)}{x^2}$$

[4]

Question 8

(a) Evaluate: $\int \frac{dx}{\sqrt{5x - 4x^2}}$

OR

(b) Evaluate: $\int \sin^3 x \ \cos^4 x \, dx$

Question 9 **[4]**

Solve the differential equation

$$\left(1 + x^2\right)\frac{dy}{dx} = 4x^2 - 2xy$$

[4]

Question 10

Three persons A, B and C shoot to hit a target. Their probabilities of hitting the target are $\frac{5}{6}$, $\frac{4}{5}$ and $\frac{3}{4}$ respectively. Find the probability that:

(i) Exactly two persons hit the target.

(ii) At least one person hits the target.

Question 11 **[6]**

Solve the following system of linear equations using matrices:

$$x-2y=10, \quad 2x-y-z=8, \quad -2y+z=7$$

Question 12 **[6]**

(a) Show that the radius of a closed right circular cylinder of given surface area and maximum volume is equal to half of its height.

OR

(b) Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

Question 13 **[6]**

(a) Evaluate:

$$\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$$

OR

(b) Evaluate: $\int \frac{2x+7}{x^2-x-2}\,dx$

Question 14 **[6]**

The probability that a bulb produced in a factory will fuse after 150 days of use is 0·05. Find the probability that out of 5 such bulbs:

(i) None will fuse after 150 days of use.

(ii) Not more than one will fuse after 150 days of use.

(iii) More than one will fuse after 150 days of use.

(iv) At least one will fuse after 150 days of use.

SECTION B (20 Marks)

Question 15 **[3×2]**

(a) Write a vector of magnitude of 18 units in the direction of the vector $\hat{\imath}-2\hat{\jmath}-2\hat{k}$.

(b) Find the angle between the two lines:

$$\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4} \quad \text{and} \quad \frac{x-1}{5}=\frac{y+2}{2}=\frac{z-1}{-5}$$

(c) Find the equation of the plane passing through the point $(2,-3,1)$ and perpendicular to the line joining the points (4, 5, 0) and (1,$-$2, 4).

Question 16 **[4]**

(a) Prove that $\vec{a}.\left[\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+3\vec{b}+4\vec{c}\right)\right] = [\,\vec{a}\quad\vec{b}\quad\vec{c}\,]$

OR

(b) Using vectors, find the area of the triangle whose vertices are:
A (3,$-$1, 2), B (1,$-$1,$-$3) and C (4,$-$3, 1)

Question 17 **[4]**

(a) Find the image of the point (3,$-$2, 1) in the plane $3x - y + 4z = 2$

OR

(b) Determine the equation of the line passing through the point ($-$1, 3,$-$2) and perpendicular to the lines:

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{3} \quad \text{and} \quad \frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$$

Question 18 **[6]**

Draw a rough sketch of the curves $y^2 = x$ and $y^2 = 4 - 3x$ and find the area enclosed between them.

SECTION C (20 Marks)

Question 19 **[3×2]**

(a) The selling price of a commodity is fixed at ₹ 60 and its cost function is

C (x) = 35 x + 250

(i) Determine the profit function.

(ii) Find the break even points.

(b) The revenue function is given by R(x) = $100x - x^2 - x^3$. Find

(i) The demand function.

(ii) Marginal revenue function.

(c) For the lines of regression $4x - 2y = 4$ and $2x - 3y + 6 = 0$,
find the mean of 'x' and the mean of 'y'.

Question 20 **[4]**

(a) The correlation coefficient between x and y is 0.6. If the variance of x is 225 , the variance of y is 400 , mean of x is 10 and mean of y is 20 , find

(i) the equations of two regression lines.

(ii) the expected value of y when $x = 2$

OR

(b) Find the regression coefficients b_{yx} , b_{xy} and correlation coefficient 'r' for the following data : (2,8), (6,8), (4,5), (7, 6), (5 ,2)

Question 21 **[4]**

(a) The marginal cost of the production of the commodity is $30 + 2x$, it is known that fixed costs are ₹ 200, find

(i) The total cost.

(ii) The cost of increasing output from 100 to 200 units.

OR

(b) The total cost function of a firm is given by $C(x) = \frac{1}{3}x^3 - 5x^2 + 30x - 15$ where the selling price per unit is given as ₹ 6 . Find for what value of x will the profit be maximum.

Question 22 **[6]**

A company uses three machines to manufacture two types of shirts , half sleeves and full sleeves. The number of hours required per week on machine M_1 , M_2 and M_3 for one shirt of each type is given in the following table :

	M_1	**M_2**	**M_3**
Half sleeves	1	2	8/5
Full sleeves	2	1	8/5

None of the machines can be in operation for more than 40 hours per week . The profit on each half sleeve shirt is ₹ 1 and the profit on each full sleeve shirt is ₹1·50. How many of each type of shirts should be made per week to maximixe the company's profit?