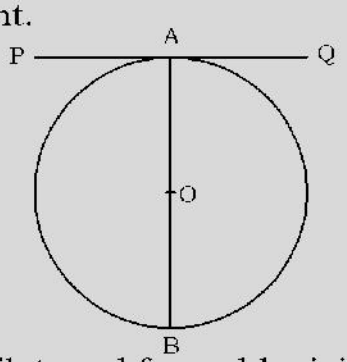
Unit 7

TANGENTS

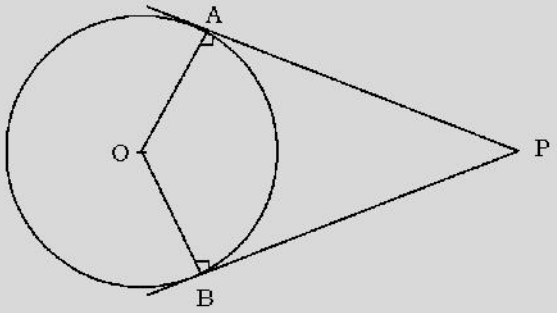
Points To Remember

- A tangent at a point to a cricle is perpendicular to the diameter through that point.



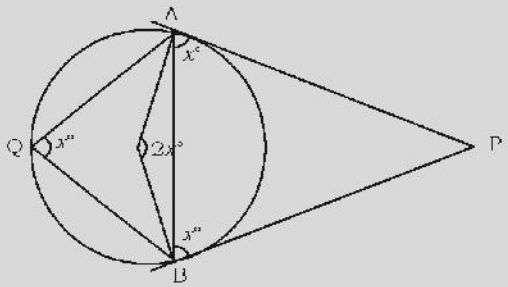
$AB \perp PQ$

- A quadrilaternal formed by joining the centre of the circle, two points on that circle, and the point of inersection of two tangents at these points is cyclic.

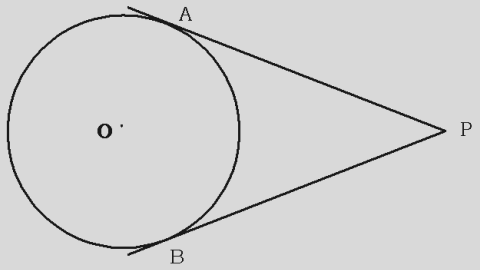


The quadrilateral PAOB is a cyclic quadrilateral $\angle AOB + \angle P = 180°$

- An angle formed by two radi of a circle through two points and the angle formed by two tangents through these points are supplimentary.
- The angle made by chord and the tangents at it's end point is half the central angle of the chord. Besides, the angles made by a chord and a tangent at it's one end point is same as the angle made in the alternate segment.

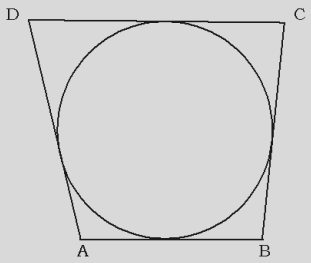


- Two tangents can be drawn from an exterior point of a circle. These two tangents will be equal in length.

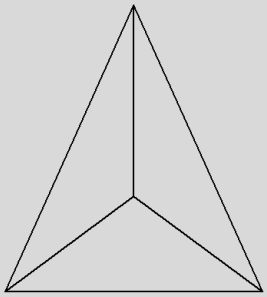


PA = PB

- The sum of the opposite sides of quadrilateral formed by the tangents at four points of a circle are equal. Conversely, if the sum of the opposite sides of a quadrilateral are equal then we can draw a circle with the four sides as tangents to that circle.



- In any triangle, the angle bisectors meet at a point. We can draw a circle with this point as centre and touching the there sides of the triangle. This cricle is called incircle of the triangle. The perpendicular distance from this centre to a side is the radius of the circle.



- The radius of the incircle of a triangle is the quotient obtained when the area of the triangle is divided by it's semiperimeter.

$$r = \frac{A}{S}$$

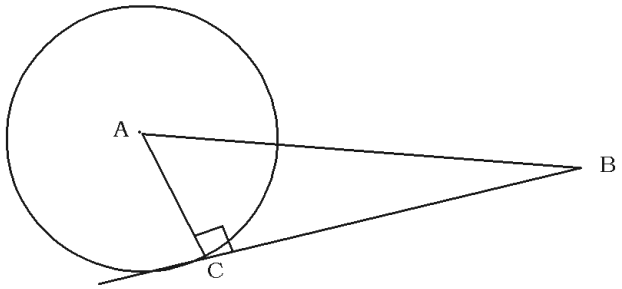
A = Area of the triangle

S = Semi perimeter of the triangle

Worksheet - 1

In the figure, BC is a tangent to the circle. The radius of the circle is 5cm and AB= 13cm. Find the length of the tangent.

ABC is a right angled triangle



AC = 5cm

AB = 13cm

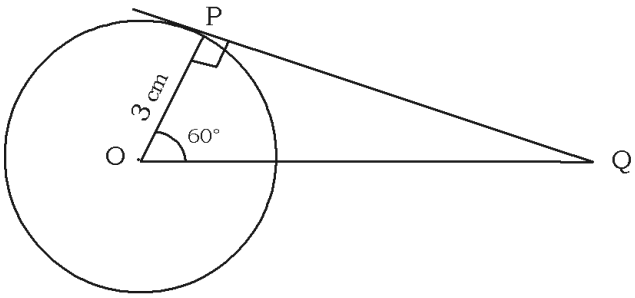
BC = $\sqrt{\square^2 - \square^2}$

= $\sqrt{\square - \square}$ = $\sqrt{\square}$ = $\square$cm

Worksheet - 2

In the figure, 'O' is the centre of the circle. Find the length of the tangent PQ?

OPQ is a right angled triangle



OP = 3cm

$\angle O = 60°$

$\angle P = 90°$

$\angle Q = 30°$

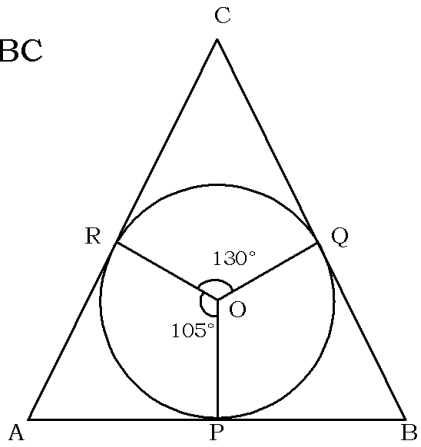
The ratio of the sides of the triangle is _____

Then OQ = $\square$ cm

PQ = $\square$ cm

Work Sheet - 3

Find all the angles of ΔABC



$\angle ROQ = 130°$

$\angle ROP = 105°$

$\angle ROQ + \angle C = 180°$

$\angle C = \square°$

$\angle ROP + \angle A = 180°$

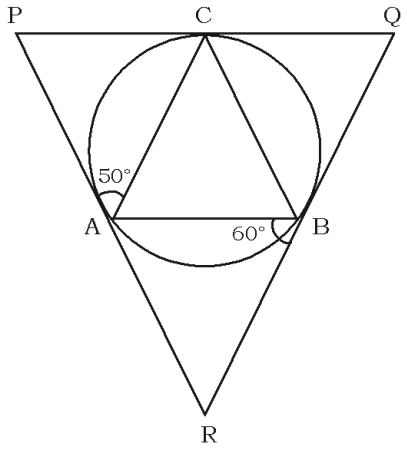
$\therefore \angle A = \square$

$\angle POQ = \square$

$\angle B = \square$

Worksheet - 4

Find the measure of all angles of ΔABC and ΔPQR in the figure.



$\angle PAC = 50°$

$\angle PCA = \square°$

$\therefore \angle P = \square°$

$\angle ABR = 60°$

$\angle BAR = \square°$

$\angle R = \square°$

$\angle Q = \square°$

$\angle PAC = 50°$

$\angle RAB = 60^0$

$\angle ACB = \angle ABR = \square$

$\angle ABC = \angle PAC = \square$

$\therefore \angle BAC = 180 - (\square + \square)$

$= \square$

Worksheet - 5

Circumcircle of ΔPQR is the incircle of ΔXYZ. Find all the angles of ΔXYZ and ΔPQR.

$\angle X = 60^0$

$\angle Y = 70$

$\angle Z = \square°$

$\angle X + \angle XPR + \angle XRP = 180°$

$\angle XPR = \angle XRP$

$60 + \angle XPR + \angle XRP = 180°$

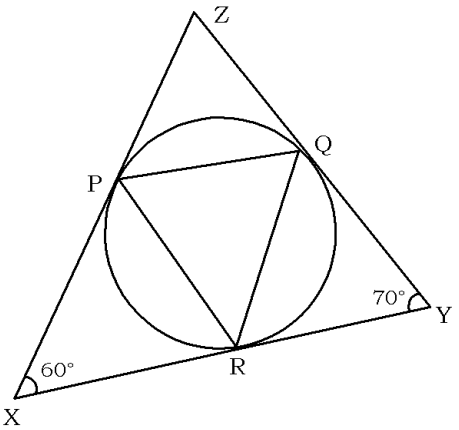
$\angle XPR + \angle XRP = 180 - \square = \square$

$\angle XPR = \square°$

$\angle XRP = \square°$

$\angle Y + \angle YRQ + \angle YQR = 180°$

$\angle YRQ = \angle YQR$



$70 + \angle YRQ + \angle YQR = 180°$

$\angle YRQ + \angle YQR = 180 - \square - \square$

$\angle YRQ = \square$

$\angle YQR = \square$

$\angle RPQ = \angle YRQ = \square$

$\angle PQR = \angle XRP = \square$

$\angle PRQ = 180 - [\square + \square] = \square$

$\angle Z = 180 - [\square + \square] = \square$

Worksheet - 6

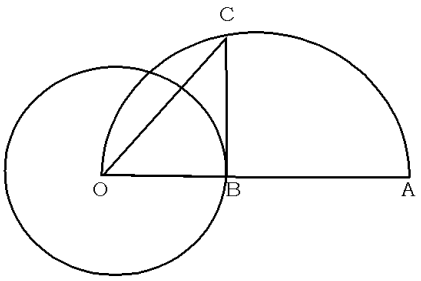
In the figure, 'O' is the centre of the circle, 'C' is a point on th semicircle having OA as diameter and BC is a tangent through BC. OB = 1cm, AB = 3cm. Find BC. What are the measures of the angles of ΔOBC?

$OB \times AB = BC^2$

$\square \times \square = BC^2$

$BC^2 = \square$

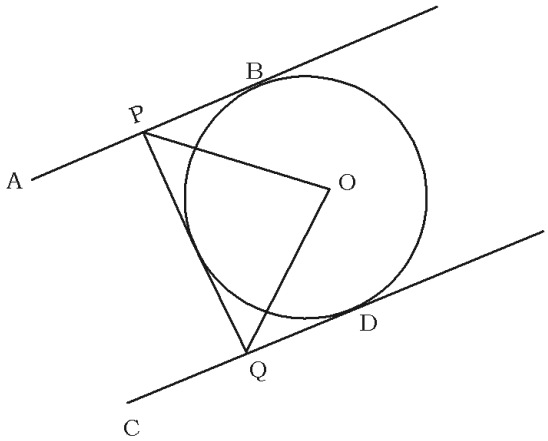
$BC = \square$



OB = 1cm, BC = cm, AB = 3cm. OC = $\square$cm. The sides of the triangle OBC are in the ratio _____. There for $\angle O = \square°$, $\angle B = \square°$, $\angle C = \square°$

Worksheet - 7

AB and CD are two tangents which are parallel to each other of a circle centered at 'O' PQ is another tangent of the circle. Prove that OPQ is a right angled triangle.



AB, CD and PQ are the tangents of the circle. PO and QO are the bisectors of $\angle BPQ$ and $\angle DQP$ respectively,

If we take $\angle BPQ = 2x$, $\angle DPQ = 2y$, since AB and CD are parallel,

$2x + 2y = \square$

$2(x+y) = \square$

$x+y = \square$

Therefore, $\angle POQ = \square^\circ$

This is , _____

Worksheet - 8

In the figure, AP = 4cm, CQ = 2.5cm, BR = 7cm. Find all the sides of the triangle.

AQ = AP = $\square$cm

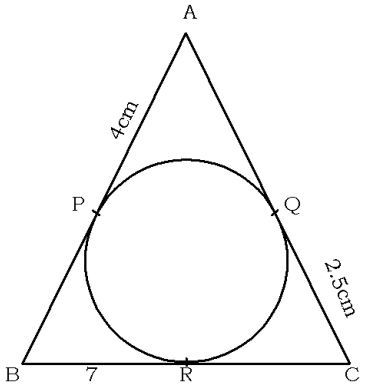
CR = $\square$ = 2.5cm

$\square$ = BR = $\square$ cm

AB = BP + AP = $\square$cm

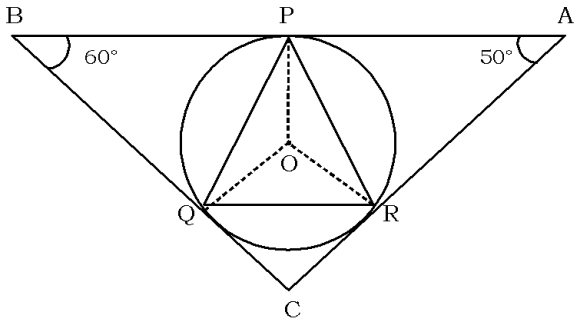
AC = AQ + QC = $\square$cm

BC = $\square$ + $\square$ = $\square$ cm



Worksheet - 9

In the figure O is the centre of the circle. The sides of ΔABC touches the circle at the points P.Q. R. IF $\angle Q = 60^\circ$ then find all angles of ΔPQR.



Quadrilaterals APOR, BQOP, CROQ are cyclic quadrilaterals

$\angle POR = \square - \square = \square$

$\angle POQ = \square - \square = \square$

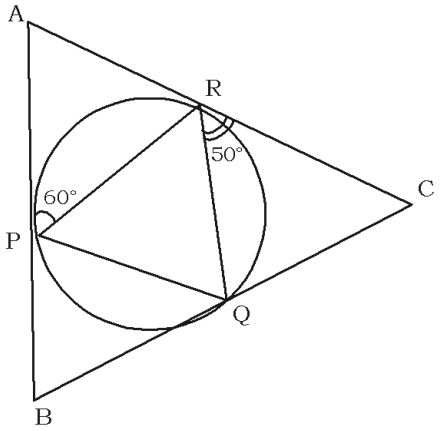
$\angle PQR = \frac{1}{2}\angle\square = \frac{1}{2}\times\square = \square$

$\angle PRQ = \frac{1}{2}\angle\square = \frac{1}{2}\times\square = \square$

$\angle QPR = \square - (\square + \square) = \square - \square = \square$

Worksheet - 10

In the figure the circumcirle of ΔPQR is drawn. ΔABC is a triangle formed by the tangents at P, Q and R. Find all angles of ΔABC and ΔPQR.



ΔAPR , ΔBPQ and ΔCRQ are isosceles triangles

In ΔAPR, AP = $\square$

$\angle ARP$ = $\square$

$\angle A$ = $180° - (\square + \square) = 180° - \square = \square$

In ΔCRQ, CR = $\square$

$\angle CQR$ = $\square$

$\angle C$ = $180° - (\square + \square) = 180° - \square = \square$

$\angle B = 180 - (\angle A + \angle C) = 180° - (\square + \square) = 180° - \square = \square$

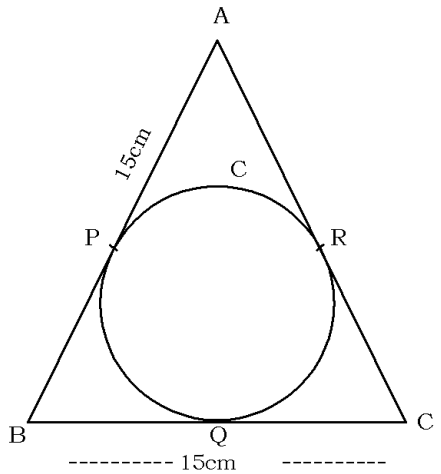
Since the angle made by chord and a tangent at it's one end point is same as the angle made in the alternate segment.

$\angle PQR = \angle\square = \square$ $\angle QPR = \angle\square = \square$

$\angle PRQ = 180° - (\angle\square + \angle\square) = 180° - (\square + \square) = 180° - \square = \square$

Worksheet - 11

In the figure, the incircle of ΔABC touches the sides AB, BC and AC at the points P, Q and R respectively. If AP = 5cm, BC= 15cm then find the perimeter of ΔABC.



Let BQ = x cm then CQ =$\square$cm Tangents drawn from an external point to a circle are equal in measure.

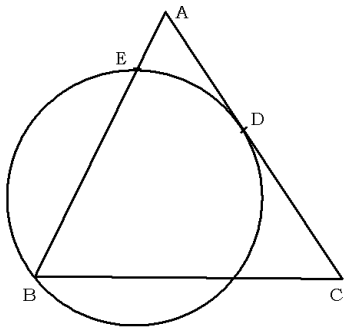
AR = $\square$ = $\square$, BP = $\square$= $\square$, CR = $\square$ = $\square$

Perimeter of ΔABC = $\square$ + $\square$ + $\square$

= $\square$ + $\square$ + $\square$ + $\square$ + $\square$ + $\square$

= $\square$ cm

Worksheet - 12



In the figure AB = AC. A circle passing through B intersect AB at E. The circle touches AC at it's midpoint D.
Prove that AB = 4AE.

$AE \times AB = \square^2$

$AD = \frac{\square}{2}$ Since D is the midpoint of AC

$AE \times AB = \left[\frac{\square}{2}\right]^2 = \left[\frac{\square}{2}\right]^2$ Since AB = AC

$= \frac{\square^2}{4}$

$AE = \frac{\square}{4}$

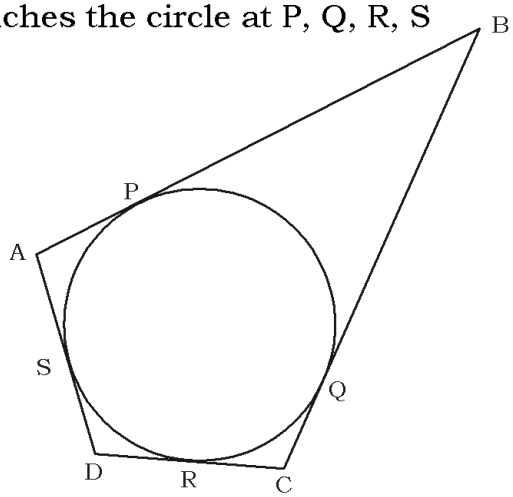
$\square = 4\square$

Worksheet - 13

The sides of the quadrilateral ABCD touches the circle at P, Q, R, S

a) Prove that AB + CD = AD + BC

b) If AB = 12cm, CD = 8cm AD = 14cm
Find BC



a) Tangents draw from an external point to the circle are equal in length

AP = $\square$, BP = $\square$, CR = $\square$, DR = $\square$

From the figure AB + CD = $\square$ + $\square$ + $\square$ + $\square$

= $\square$ + $\square$ + $\square$ + $\square$

= AD + BC

b) BC = □ + □ − □ = □

Worksheet - 14

Find the area of the triangle with sides 12cm, 16cm, 20cm. Also find the radius of its incircle.

We have 12^2+16^2 = □ + □ = □ = $□^2$

∴ the given triangle is a □ triangle

Area of the triangle = ½×□×□ = □

Perimeter of the triangle = □ + □ + □ = □

Semi perimeter of triangle = $\frac{□}{2}$ cm = □cm

$$\text{Radius of incircle} = \frac{\text{Area of triangle}}{\text{Semi perimeter of triangle}}$$

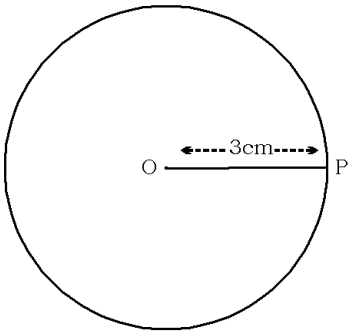
$$= \frac{□}{□} = □ \text{ cm}$$

Construction

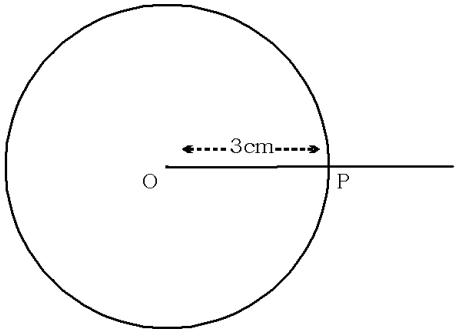
1. Draw a circle of radius 3cm and mark a point on it. Draw the tangent through that point

Ans :

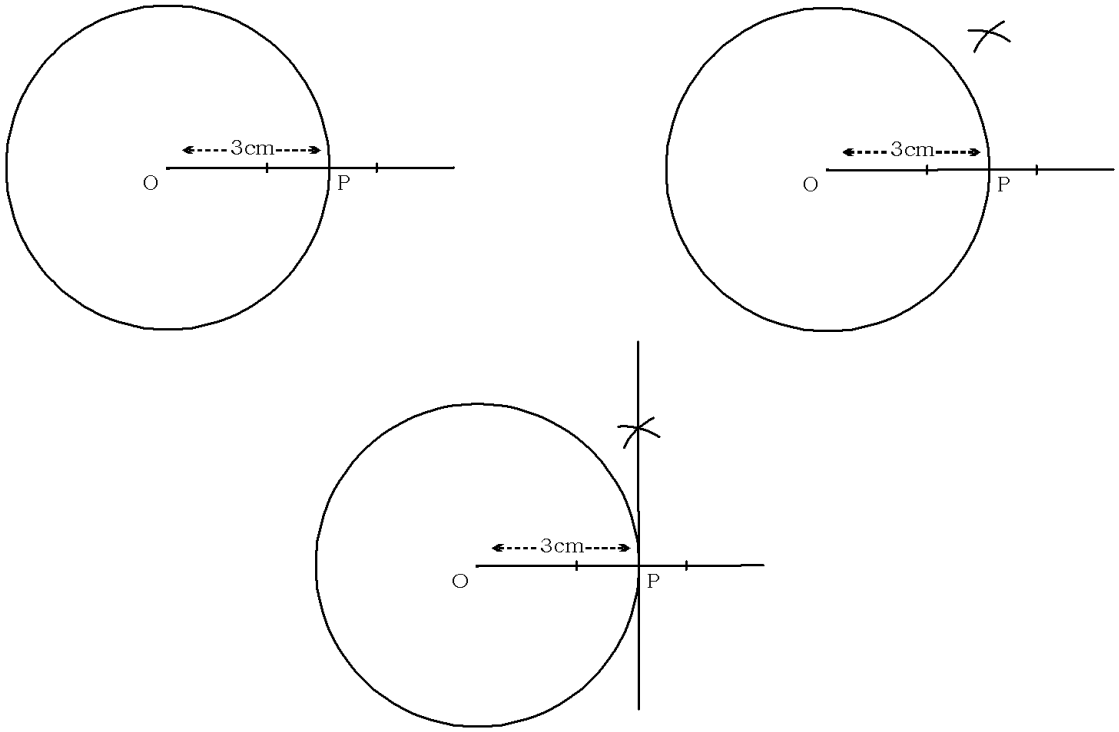
Step 1 : Draw a circle of radius 3cm



Step 2 : Extend the radius OP to outside



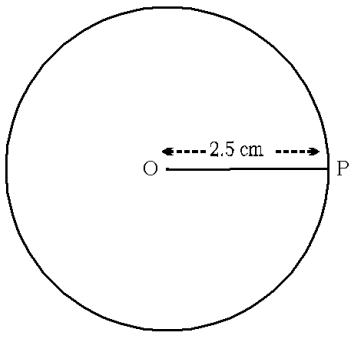
Step 3 : Draw a line perpendicular to OP through the point P.



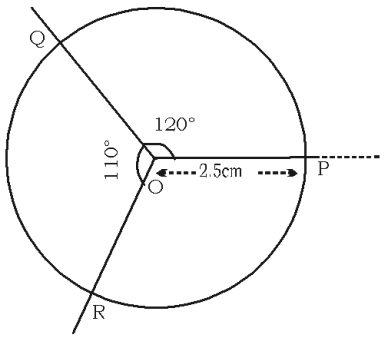
2. Draw a circle of radius 2.5cm. Draw a triangle with two angles 60°, 70° and the sides of the triangle touching the circle.

Ans.

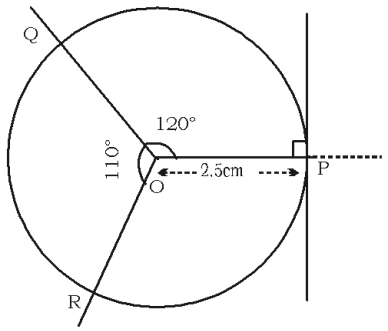
Step 1 : Draw a circle of radius 2.5cm



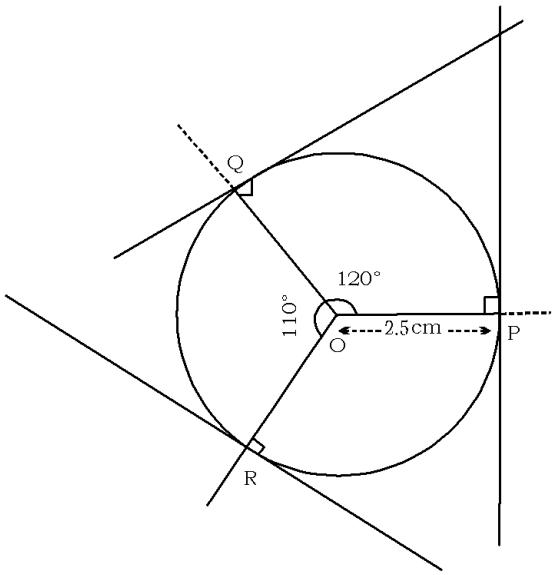
Step 2 : Draw angles of measures 120°, 110° among two consecutive radii.



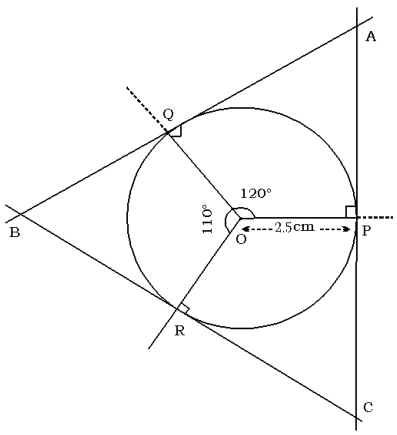
Step3 : Draw a line perpendicular to the radius OP through the point P.



Step 4 : Similarly draw perpendicular lines through the point Q, R.



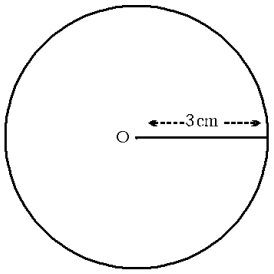
Step 5 : Name the points, intersecting these perpendicular lines as A,B,C



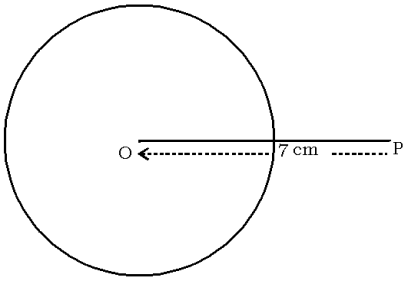
3. Draw a circle of radius 3cm and mark a point 7cm away from its centre. Draw the tangent to the circle from this point and measure it lengths.

Ans :

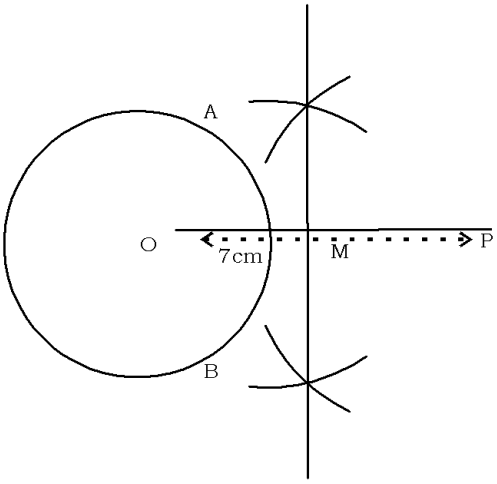
Step 1 : Draw a circle of radius 3cm



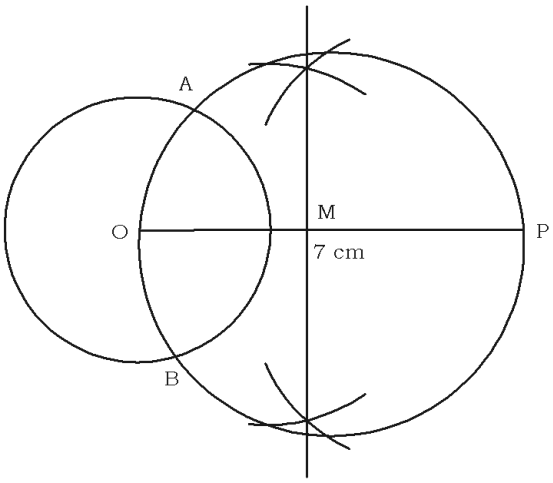
Step 2 : Mark a point 7cm away from the centre of the circle.



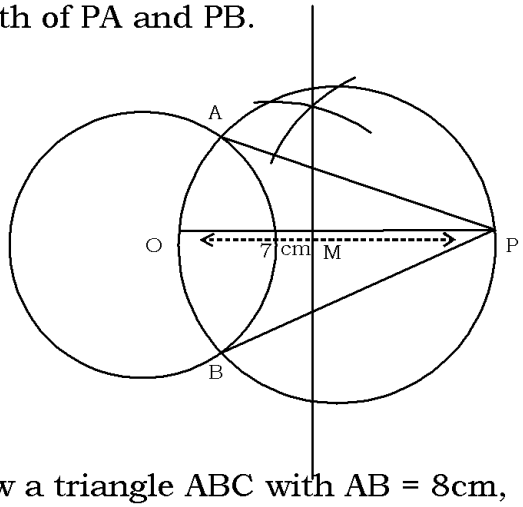
Step 3 : Draw the perpendicular bisector of the line OP.



Step 4 : M is the midpoint of OP. Draw a circle unit centre M and diameter OP. Mark the point of intersection of these two circles as A, B.



Step 5 : Draw the lines from the point P to A and B and measure the length of PA and PB.

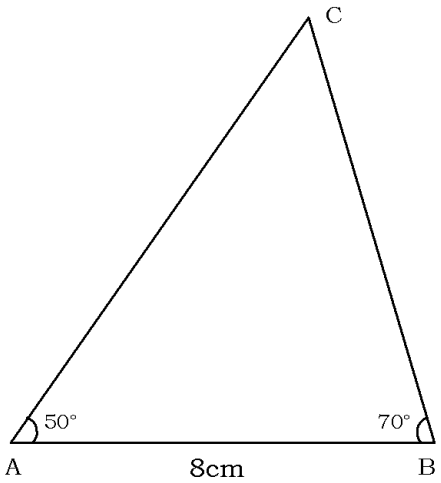


PA = 6.3cm, PB = 6.3cm

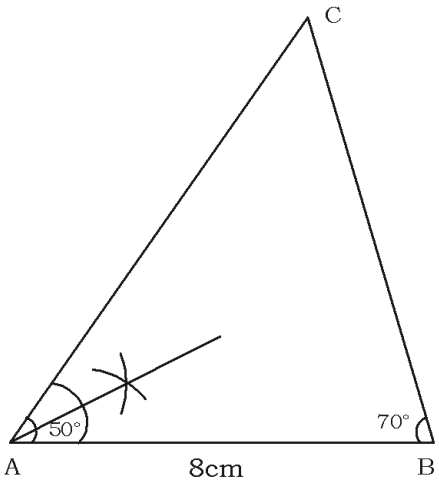
4. Draw a triangle ABC with AB = 8cm, $\angle A = 50°$, $\angle B = 70°$. Construct the incircle of ΔABC and write the measure of its radius.

Ans :

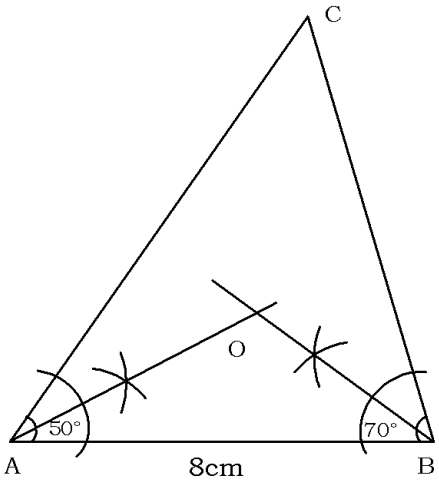
Step 1 : Draw the triangle ABC with given measures.



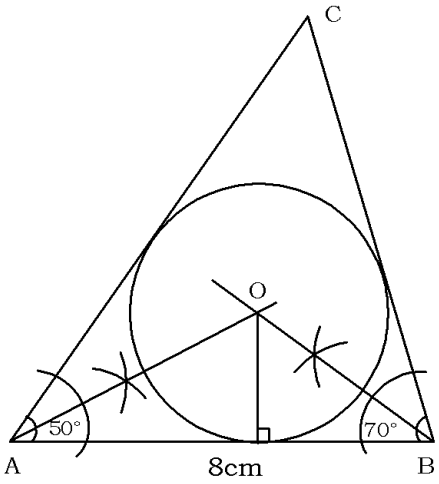
Step 2 : Draw the bisector of $\angle A$.



Step 3 : Draw the bisector of $\angle B$



Step 4 : The bisectors of $\angle A$ and $\angle B$ interset at O. Draw a circle with centre 'O' and the distance from 'O' to the side AB as radius. Measure the radius.



Radius = 2.2cm

More practice Problems

1. Draw a circle of radius 3.5cm and mark point on the circle. Draw a tangent through the point.
2. Draw a circle of radius 3cm and mark a point 6cm mark a point 6cm away from its centre. Draw tangents to the circle from this point and measure its lengths.
3. Draw a triangle of angle mesures 50°, 60° and radii of whose incircle is 2cm.
4. Draw a circle of radius 3cm. Mark a point P outside the circle, at a distance 8cm from the centre. Draw tangents from P to the circle and measure the length of tangents.
5. Draw a circle of radius 2.5cm. Draw a triangle with two angles 50°, 70° and the sides of the triangle touching the circle.

ANSWERS

Worksheet - 1

ABC is a right angled triangle

AC = 5cm

AB = 13cm

BC = $\sqrt{13^2 - 5^2}$

= $\sqrt{169 - 25}$

= $\sqrt{144}$

= 12cm

Worksheet - 2

OPB is a right angled triangle.

OP = 3cm

$\angle O = 60°$

$\angle P = 90°$

$\angle Q = 30°$

The ratio of the sides of the triangle is $1 : \sqrt{3} : 2$

Then, OQ = 6cm

PQ = $3\sqrt{3}$ cm

Worksheet - 3

$\angle ROQ = 130°$

$\angle ROP = 105°$

$\angle ROQ + \angle RCQ = 180°$

$\therefore \angle C = 50°$

$\angle ROP + \angle RAP = 180°$

$\therefore \angle A = 75°$

$\angle POQ = 125°$

$\angle B = 55°$

Worksheet - 4

$\angle PAC = 50°$

$\angle PCA = 50°$

$\angle P = 180 - (50 + 50)$

$= 180\ 100 = 80°$

$\angle ABR = 60°$

$\angle BAR = 60°$

$\angle R = 180 - (60 + 60)$

$= 180 - 120 = 60°$

$\therefore \angle Q = 40°$

$\angle PAC = 50°$

$\angle RAB = 60°$

$\angle ACB = \angle ABR = 60°$

$\angle ABC = \angle PAC = 50°$

$\angle BAC = 180 - [60+50] = 70°$

Worksheet - 5

$\angle X = 60°$

$\angle Z = 70°$

$\angle Z = 180 - (65 + 70)$

$= 180 - 135 = 50°$

$\angle X = \angle XPR + \angle XRP = 180°$

$\angle XPR = \angle XRP$

$60° + \angle XPR + \angle XRP = 180°$

$\angle XPR + \angle XRP = 180° - 60° = 120°$

$\angle XPR = 60°$

$\angle XRP = 60°$

$\angle RYQ + \angle YRQ + \angle YQR = 180°$

$\angle YRQ = \angle YQR$

$70 + \angle YRQ + \angle YQR = 180$

$\angle YRQ + \angle YQR = 180 - 70 = 110°$

$\angle YRQ = \frac{110}{2} = 55°$

$\angle YQR = \frac{110}{2} = 55°$

$\angle RPQ = \angle YRQ = 55°$

$\angle PQR = \angle XRP = 60°$

$\angle PRQ = 180 - [55+60] = 65°$

Worksheet - 6

$OB \times AB = BC^2$

$1 \times 3 = BC^2$

$BC^2 = 3$

$BC = \sqrt{3}$

OB = 1cm, BC = $\sqrt{3}$ cm, AB = 3cm

The sides of the triagle OBC are in the ratio 1: $\sqrt{3}$: 2

Therefore, $\angle O = 60°$

$\angle B = 90°$

$\angle C = 30°$

Worksheet - 7

AB, PQ and CD are the tangents of the circle PO and QO are the bisectors of $\angle BPQ$ and $\angle DQP$ respectively.

If we take $\angle BPQ = 2x$ and $\angle DPQ = 2y$, since AB and CD are parallel,

$2x + 2y = 180°$

$2(x+y) = 180°$

$x + y = 90°$

$\angle POQ = 90°$

That is , OPQ is a right angled triangle.

Worksheet - 8

AQ = AP = 4cm

CR = CQ = 2.5cm

BP = BR = 7cm

AB = BP + AP = 11cm

AC = AQ+QC = 6.5cm

BC = BR + RC = 9.5cm

Worksheet - 9

Quadrilaterals APOR, BQOP, CROQ are cyelic quarilaterals.

$\angle POR = 180° - 50° = 130°$

$\angle POQ = 180° - 60° = 120°$

$\angle PQR = ½ \angle POR = ½ \times 130° = 65°$

$\angle PRQ = ½ \angle POQ = ½ \times 120° = 60°$

$\angle QPR = 180° - (65° + 60°) = 130° - 125° = 55°$

Worksheet - 10

ΔAPR, ΔBPQ and ΔCRQ are isosceles triangles

In ΔAPR,

AP = AR

$\angle ARP = 60°$

$\angle A = 180° - (60° + 60°) = 180° - 120° = 60°$

In ΔCRQ, CR = CQ

$\angle CQR = 50°$

$\angle C = 180° - (50°+50°) = 180° - 100° = 80°$

$\angle B = 180° - (\angle A+\angle C) = 180° - (60° + 80°)$ $180° - 140° = 40°$

$\angle PQR = \angle APR = 60°$ [Since the angle made by a chord and a tangent

$\angle QPR = \angle CRQ = 50°$ at it's one end point is same as the angle made in alternate segment]

$\angle PRQ = 180° - (\angle PQR + \angle QPR) = 180° - (60° + 50°) = 180° - 110° = 70°$

Worksheet - 11

Let BQ = x then CQ = 15– x

AP = AR = 5 [Tangents drawn from an external point to a circle

BP = BQ = x are equal in measure]

CR = CQ = 15 – x

Perimeter of ABC = AB + BC + AC

= AP + BP + BQ + CQ + CR + AR

= 5 + x + x + 15 – x + 15 – x + 5

= 40cm

Worksheet - 12

$AE \times AB = AD^2$

$AD = \frac{AC}{2}$ Since D is the midpoint of AC

$AE \times AB = \left[\frac{AC}{2}\right]^2 = \left[\frac{AB}{2}\right]^2 = \frac{AB^2}{4}$ since AB = AC

$AE = \frac{AB}{4}$

AB = 4AE

Work Sheet - 13

Tangents drawn from an external point to the circle are equal in length.

a) AP = AS

BP = BQ

CR = CQ

DR = DS

From the figure

AB + CD = AP + BP + CR + DR

= AS + BQ + CQ + DS

= AS + DS + BQ + CQ

= AD + BC

b) BC = AB + CD – AD = 12 + 8 – 14 = 20 – 14 = 6cm

Worksheet - 14

We have,

$12^2 + 16^2 = 144 + 256 = 400 = 20^2$

$\therefore$ the given triangle is a right triangle

Area of the triangle ½ ×12 ×16 = 6×16= 96sq.cm

Perimeter of the triangle = 12 + 16 + 20 = 48cm

Semiperimeter of the Δ = ½ ×48 = 24cm

$$\text{Radius of incircle} = \frac{\text{Area of triangle}}{\text{Semiperimeter of triangle}}$$

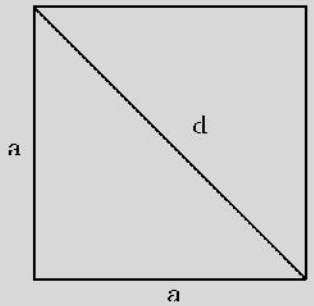
$$= \frac{96}{24} = 4\text{cm}$$

Unit 8

SOLIDS

Points to Remember

SQUARE



If the length of a side of a square is 'a' unit, then

Perimeter = 4a

Area = a^2

length of diagonal (d) = $\sqrt{2}\,a$

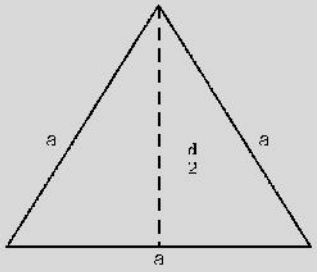
The length of diagonal of a squarew in 'd' unit then

Length of one side (a) = $\frac{d}{\sqrt{2}}$

Area (a^2) = $\frac{d^2}{2}$

EQUILATERAL TRIANGEL

Length of are side of a equilaternal triangle then

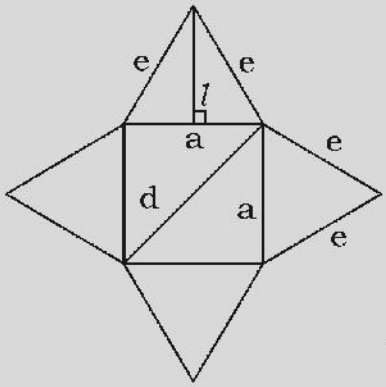


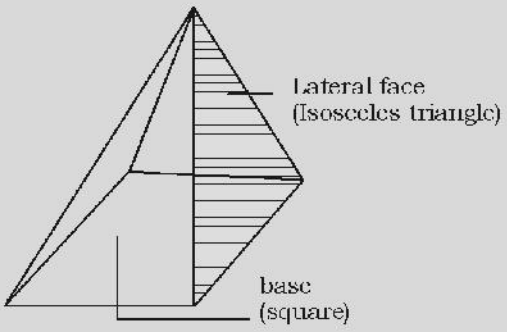
Perimeter = 3a

Area = $\frac{\sqrt{3}a^2}{4}$

Height (h) = $\frac{\sqrt{3}a}{2}$

SQUARE PYRAMID



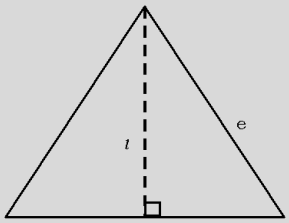


A square pyramid is a solid with base square and lateral face isosceles triangle.

Relation between base edge (e), slant height (l) and base edge(a)

$$e^2 = l^2 + \left(a/2\right)^2$$

$$\therefore l = \sqrt{e^2 - \left(a/2\right)^2}$$



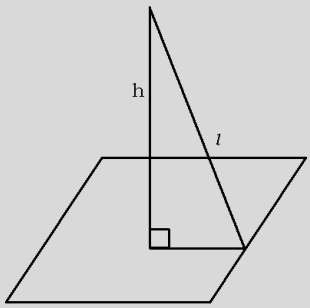
Relation betweeb base edge (a) , slant height (l) and height (h)

$$l^2 = h^2 + \left(a/2\right)^2$$

$$l = \sqrt{h^2 + \left(a/2\right)^2}$$

$$h = \sqrt{l^2 - \left(a/2\right)^2}$$

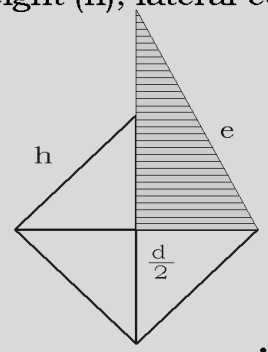
$$a/2 = \sqrt{l^2 - h^2}$$



Relation between height (h), lateral edge (e) base diagonal (d)

$$e^2 = h^2 + \left(d/2\right)^2$$

$$\therefore h = \sqrt{e^2 - \left(d/2\right)^2}$$



Area and volume of a square pyramid

Base area = a^2

Base perimeter = 4a

Area of one lateral face = $\frac{1}{2}al$

Total = $a^2 + 2al$

Volume = $\frac{1}{3} \times$ Base area $\times$ h

= $\frac{1}{3} \times a^2 \times h$

If base edge and leateral edge of a square pyramid are equal then lateral faces are equilaternal triangles.

If e = a then

$l = \frac{\sqrt{3}}{2} \times a$

$h = \frac{a}{\sqrt{2}}$

Then total surface area = $a^2 + \sqrt{3}a^2$

Worksheet 1

Complete the table

Length of Base edge (a)	**Half the length of base edge** $\left(a/2\right)$	**height (h)**	**Slant (*l*)**
12	____	8	____
____	3	____	5
____	12	____	13
____	____	15	17
____	15	20	____

Worksheet 2

Length of diagonal (d)	**Half the length of base edge** $\left(d/2\right)$	**Slant height (e)**	**Height (*h*)**
24	____	13	____
____	9	41	____
22	____	61	____
16	____	17	____
____	12	15	____

Worksheet 3

Length of one basic edge (a)	**Half the length of basic edge** $\left(a/2\right)$	**Slant height (*l*)**	**Lateral edge (*e*)**
8	____	____	5
____	6	____	10
____	____	____	13
24	____	____	25
40	16	____	20

Worksheet 4

Base edge and leateral edge of a square pyramid are equal and one edge length in 6cm. Find slant height and height?

a = ———— cm

$l = \frac{\sqrt{3}}{2} \times$ ____ $\qquad h = \frac{a}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2} \times$ ____ $\qquad =$ ____

= ____ $\sqrt{3}$ $\qquad =$ ____ $\times \sqrt{2}$

Worksheet 5

The base edge length and slant height of a square pyramid are 12cm, 15cm. Find L.S.A and T.S.A

a = —— cm $\qquad$ l = cm

Basic area (a^2) = ——

Lateral surface area = (2al) = ——

Total surface area = $a^2 + 2al$

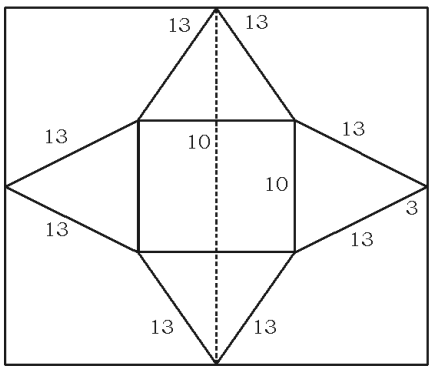
= —— + —— = —— cm^2

Worksheet 6

Length and beadth of a rectangler paper sheet are 40cm and 34cm given the figure. From the centre of the paper sheet a marked portion in cute and total it.

a) What is shape of the fold shape

b) Area of square (a^2) = ——— 2



Height of the triangle shape = $\sqrt{(\ )^2 - (\ )^2}$

= $\sqrt{\_\_\_ - \_\_\_}$

$= \sqrt{\_\_\_\_ - \_\_\_\_}$

$= \_\_\_\_\_$ cm

Area of one triangle $= \frac{1}{2} \times \_\_\_\_ \times \_\_\_\_$

$= \_\_\_\_$ cm^2

Area of the cutout portion = Area of square + 4× Area of triangle

$= \_\_\_\_\_ + 4 \times \_\_\_\_\_$

$= \_\_\_\_\_ + \_\_\_\_\_$

$= \_\_\_\_\_$ cm^2

Worksheet 7

A container in the shape of a square pyramid with base edge 10cm height 30cm. Find the capacity of the container (b) Find the capacity of a container in the shape of a square prism with same base edge and height of the give square pyramid.

a = _____ cm

h = _____ cm

$V = \frac{1}{3} a^2 \times h$

$= \frac{1}{3} \times \_\_\_\_^2 \times \_\_\_\_$

$= \_\_\_\_ \times \_\_\_\_ = \_\_\_\_ cm^3$

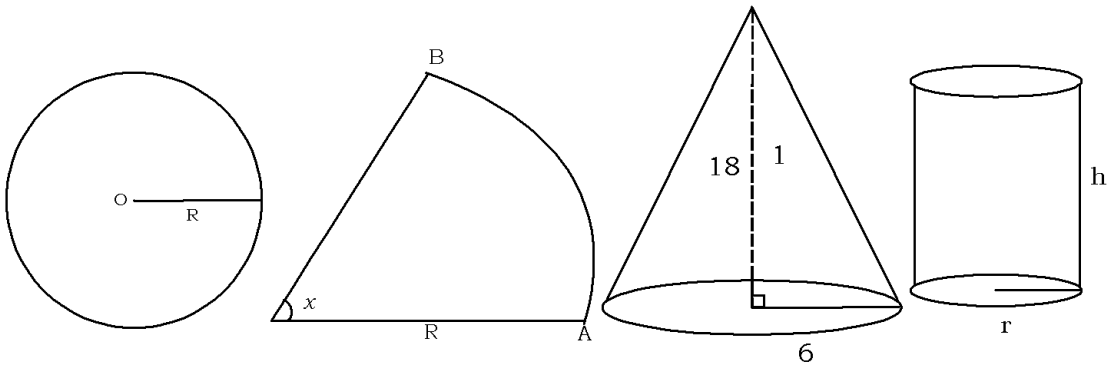
Capacity of the square pyramid container $= \frac{V}{1000} = \_\_\_\_$ liter

Volume of square pyramid = _____ × volume of square prism

∴ Capacity of square prism cantainer = _____ × volume of square prism

= _____ × _____ = _____ liter

Cone



Circle

Area of a circle with radius 'R' = πR^2

Circumfereace of the circle = $2\pi R$

Sector

Length the sector with radius (R) = $\frac{x}{360} \times 2\pi R$

Area of the sector = $\frac{x}{360} \times 2\pi R^2$

Cylineder

Volume = $\pi r^2 h$

When a sector is curved (bent(in to a cone,

The radius of the sector = Slant height of the cone

Area of the sector = C.S.A of the cone

$\frac{x}{360} = \frac{r}{R}$ OR $\frac{x}{360} = \frac{r}{l}$

Retation between height (h)

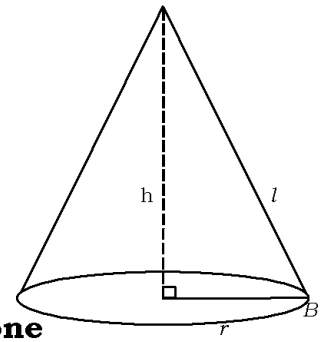
Slant height (l), base radius (r) of a cone

$l^2 = h^2 + r^2$

$l = \sqrt{h^2 + r^2}$

$h = \sqrt{l^2 - r^2}$

$r = \sqrt{l^2 - h^2}$



Area and volume of a cone

Basic perimeter = $2\pi r$

Curved surface area = πrl

Total surface area = $\pi r^2 + \pi rl$

Volume = $\frac{1}{3}\ \pi r^2 h$

Worksheet 8

Complete the table.

Base diameter (d)	**Base radius (r)**	**height (h)**	**slant leight (l)**
-----	3	4	-----
-----	6	8	10
40	-----	15	-----
24	12	5	-----
12	8	-----	17
60	-----	40	-----

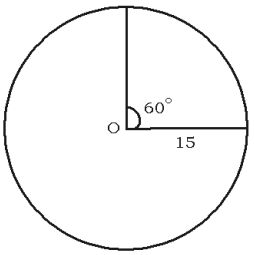
Worksheet 9

A sector is cut out from a circle of radius 24cm with central angle 60°.

The sector is bent cut a cone the a find the sector.

a) Slant height of the cone ———

b) Radius the cone (r)———



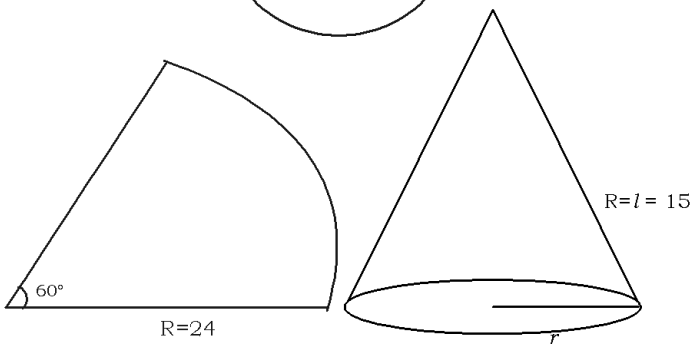
l = R =——— cm

$$\frac{x}{360} = \frac{r}{R}$$

$$\frac{\square}{360} = \frac{r}{\square}$$

r × 360 = □ × □

$$r = \frac{\square \times \square}{360} = \square$$



Worksheet 10

Base radius and height of a cone are 20cm and 15cm then find its volume.

r = _____ cm h = _____cm

$$v = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times \_\_\_\_^2 \times \_\_\_\_$$

= ____ × ____ = ____ cm^3

Worksheet 11

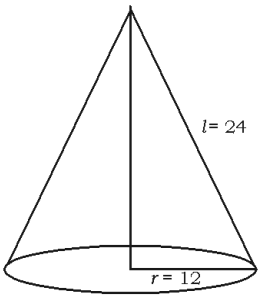
A sector is rolling to make a cone of slant height 24cm and base radius 12cm then.

a) Radius of the sector

b) Area of the sector

a) Radius of the sector

= slant height of the cone = 24cm



b) $$\frac{x}{360} = \frac{r}{R}$$

$$\frac{x}{360} = \frac{\square}{\square}$$

$x \times \square = 360 \times \square$

$$x = \frac{360 \times \square}{\square} = \square$$

Area of sector = Curved surface area of cone

$= \pi rl$

$= \pi \times$ ——— $\times$ ———

$=$ ——— π cm^2

Worksheet 12

A solid cylinder of diameter 20cm, height 30cm. What is the volume of a cone with maximum size can be cut out from the cyliner. Also find the volume of the remaning part of the cylinder.

Cylinder

d = 20cm $\therefore$ r = ——— cm h = ——— cm

Volume of the cylinder $= \pi r^2 h$

$= \pi \times$ ———$^2 \times$ ———

$= \pi \times$ ——— $\times$ ———

$=$ ——— π cm^3

Volume of the cone = ——— × volume of the cyclinder

= ——— × ———

= ——— cm^3

Volume of the remaining part

= ——— × volume of the cyliner

= ——— × ———

= ——— cm^3

Worksheet 13

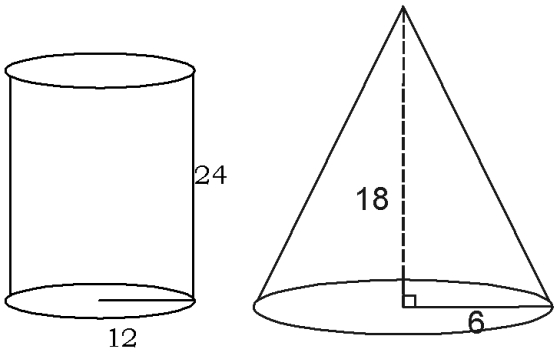
A iron metalic cylinder of height 24cm and base radius 12cm. This cyliner in metled and recast into iron cone of height 6cm and radius 18cm. How may cones are obtained?

Cyliner

r = ——— cm h = ——— cm

v = $\pi \times r^2 h$

= $\pi \times$ ——— $\times$ ——— cm^3



Cone

r = ——— cm

h = ——— cm

v = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \pi$ ———$^2 \times$ ———

$= \pi \times$ ——— $\times$———

No.of the cone = $\frac{\text{Volume of cycliner}}{\text{Volume of cone}} = \frac{\pi \times \_\_\_\_ \times \_\_\_\_}{\pi \times \_\_\_\_ \times \_\_\_\_} =$ _____

Worksheet 14

Now fill the boxes suitably.

Sl. No	**Stant height (*l*)**	**Height (*h*)**	**Radius (*r*)**	**Curved Surface Area**	**Total surface Area**	**Volume**
1	5	__	3	_____	_____	_____
2	15	9	__	_____	_____	_____
3	__	16	12	_____	_____	_____
4	25	__	20	_____	_____	_____
5	__	24a	18a	_____	_____	_____

Worksheet 15

The height and slant height of a cone is 16cm and 20cm respectively. Find

a) Base area

b) Curved surface area

c) Total surface area

d) Volume

Height (h) = _____

Slant height (1) = _____

Radius (r) = $\sqrt{l^2 - h^2}$

= _____ = _____

a) Base area = Area of the circle

= πr^2

= $\pi \times$ _____$\times$_____

= _____

b) Curved surface area = π rl

= $\pi \times$ _____$\times$_____

c) Total surface area = Base area + Curved surface area

= _____ + _____ = _____

d) Volume = $\frac{1}{3}\pi r^2 h$

= _____= _____

Worksheet 16

A sector of central angle 60° is cut from a circle of radius 12cm and rolled up into a cone. Find

a) Slant height

b) Radius of cone

c) Curved surface area

d) Slant height (l) = Radius of sector = _____

b) $x = 60°$

$$\frac{x}{l} = \frac{x}{360}$$

$$\frac{r}{12} = \frac{60}{360}$$

r × _____ × _____

r = _____

c) Curved surface area = π rl = _____ × _____ = _____

Worksheet 17

A cone of base radius 10cm and slant height 25 cm is made up of a sector. Find the central angle of the sector.

Slant height (l) = _____

Base radius (r) = _____

Central angle = x

$$\frac{r}{l} = \frac{x}{360}$$

$$\frac{\square}{\square} = \frac{x}{360}$$

x = _____

Worksheet 18

A sector is rolled to form a cone with radius 15cm and slant height 25cm. Then find.

a) Radius of sector

b) Central angle of a sector

c) Volume of cone

a) Radius of sector = Slant height of cone = _____

r = _____

b) Let central angle = x

$\frac{r}{l} = \frac{x}{360}$

$x \times$ _____ = _____ $\times$ _____

x = _____

c) Let height of cone = h

h = $\sqrt{l^2 - r^2}$

= _____

= _____

Volume = $\frac{1}{3}\pi r^2 h$

= _____

Sphere

If We slice a sphere, we get a circle. If we slice a sphers into exact halves, we get a circle whose centre, radius and diameter are those of the sphers itself. A sphere has only one face.

If the radius of sphere is 'r'

Surface Area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

Hemisphere

If we slice a sphere into exact halves, we get two hemispheres. A hemispherse has two faces. One flat face and one curved face.

If the radius of a hemisphere is 'r'

Surface Area = $3\pi r^2$

Volume = $\frac{2}{3}\pi r^3$

Worksheet 19

Radius of a sphere is 9cm. Then

a) Find its surface Area

b) Find its volume

radius = □

Surface Area = $4\pi \times r^2$

$= 4\pi \times \square$

$= \square$cm

Volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times \square$

$= \square$ cubic cm

Worksheet - 20

Radius of hemisphere is 12cm. Then find its surface area and volume.

Radius of hamisphere, r = $\square$

Surface Area $= 3\pi \times r^2$

$= 3\pi \times \square$

$= \square$

Volume $= \frac{2}{3}\pi \times r^3$

$= \frac{2}{3}\pi \times \square$

$= \square$

Worksheet - 21

The surface area of a solid sphere is 120 square centimeters. If it cut into two halves, what would be the surface area of each hemisphere?

Surface Area of the shere = $\square$

$4\pi r^2 = \square$

$\pi r^2 = \frac{\square}{4}$

$= \square$sq.cm

Surface Area of Hemisphere $= 3\pi r^2$

$= 3 \times \square$

$= \square$ sq.cm

Worksheet - 22

What is the surface area of the largest sphere that can be curved from a cube of edge 8 centimeters?

Diameter of the sphere = length of side of the cube = $\square$cm

radius of the sphere = r = $\square$cm

Surface area of the sphere = $4\pi r^2$

$= 4\pi \times \square$

$= \square$ sq.cm

Worksheet - 23

If the ratio of the radii of two spheres is 2:3 find the ratio of their volumes.

Ratio of radius of two spheres = 2 :3

Let the radius of firs sphers = 2r

Then radius of second sphere = $\square$

Volume of the first sphere = $\frac{4}{3}\pi\ (2r)^3$

Volume of the second sphere = $\frac{4}{3}\pi\ \square$

Ratio of volumes $= \frac{4}{3}\pi\ (2r)^3 : \frac{4}{3}\pi\ (3r)^3$

$= (2r)^3 : (3r)^3$

$= 8r^3 : 27r^3$

$= \square : \square$

Worksheet - 24

The base radius and length of metal cylinder are 4cm and 10cm. If it is melted and recast into spheres of radius 2cm each, how many spheres can be made?

Metal cylinder

radius = r = $\square$cm

height = h = $\square$ cm

volume = $\pi r^2 h$

$= \pi \times \square \times \square$

$= \square$ cubic cm

Sphere

radius, r = $\square$ cm

Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times \square$

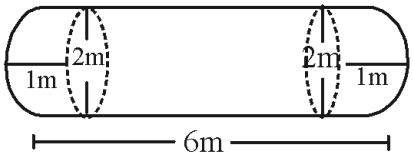
$= \frac{4}{3}\pi \times \square$

= □ cubic cm

Number of spheres = $\frac{\text{Volume of cylinder}}{\text{Volume of one sphere}} = \frac{\square}{\square} = \square$

Worksheet - 25

The picture shows the dimensions of a petrol tank. How many litres of petrol can it hold?



This shape is one cylinder and two hemispheres

Hemisphere

radius = r = 1m

volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3} \times \pi \times \square$

= $\frac{2}{3}\pi$ Cubic metre

Cylinder

radius = r = 1cm

height = h = 6 ___ (1 + 1)

= 6 – □

= □

Volume = $\pi r^2 h$

= $\pi \times \square \times \square$

= □ Cubic metre

Volume of petrol tank = volume of 2 hemispheres + volume of cylinder

= $2 \times \frac{2}{3}\pi + 4\pi$

= $\frac{4\pi}{3} + \frac{12\pi}{3}$

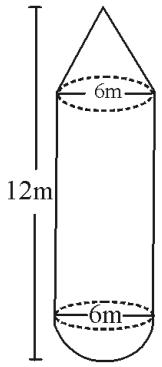
= $\frac{16\pi}{3}$ Cubic metre

$= \frac{16\pi}{3} \times 1000$ litre

$= \frac{16000\pi}{3}$ litre

Worksheet - 26

The picture shows the shape of a boiler. Total height of the boiler is 12 m and diameter is 6 meters, height of the cylindrical part is 6 meters.



a) What is the height of the cone?

b) How many literes can the boiler hold?
($1m^3$ = 1000 litres)

a) Height of the cone = 12– (6+3)
= 12 – □
= □ m

b) This shape contains a cyclinder a cone and a hemisphere

Cone

radius = r = □ m

height = h = □m

volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times \square \times \square$

$= \frac{1}{3}\pi \times \square$

$= 9\pi\ m^3$

Cylinder

radius = r = □ m

height = h = □m

volume = $\pi r^2 h$

$= \pi \times \square \times \square$

$= 54\pi\ m^3$

Hemisphers

radius = r = □ m

volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \times \square$

$= 18\pi\ m^3$

Total volume of boiler = volume of cone + volume of cylinder + volume of hemsphere

radius = $9\pi + 54\pi + 18\pi = \square m^3$

$= 81\pi \times 1000$ litre

$= \square$ litre

ANSWERS

Worksheet 1

Complete the table

Length of Base edge	half the length of base edge	height	Slant
12	6	8	10
6	3	4	5
24	12	5	13
16	8	15	17
30	15	20	25

Worksheet 2

Length of diagonal	Half the length of base edge	Slant height	Height
24	12	13	5
18	9	41	40
22	11	61	60
16	8	17	15
24	12	15	9

Worksheet 3

Length of one basic edge (a)	Half the length of Basic edge	Slant height	Lateral edge
8	4	3	5
12	6	8	10
24	12	5	13
40	20	15	25
32	16	12	20

Worksheet 4

a = 6cm

$l = \frac{\sqrt{3}}{2} \times a$ $\qquad$ $h = \frac{a}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2} \times 6$ $\qquad$ $= \frac{6}{\sqrt{2}}$

$= 3 \times \sqrt{3}$ cm $\qquad$ $= 3 \times \sqrt{2}$ cm

Worksheet 5

a =12 cm $\quad$ l = 15 cm

Basic are (a^2) = 12^2 = 144

Lateral Surface Area = (2al) = $2 \times 12 \times 15 = 360 cm^2$

Total Surface Area = $a^2 + 2al = 144 + 360 = 504 cm^2$

Worksheet 6

a) What is shape of the fold shape is = square pyramid

b) Area of square (a^2) = 10^2

= $100 cm^2$

Height of the triangle (l) = $\sqrt{13^2 - 5^2}$

= $\sqrt{109 - 25}$

= $\sqrt{144}$

= 12 cm

Area of one triangle = $\frac{1}{2} al$

= $\frac{1}{2} \times 10 \times 12$

= $60 cm^2$

Area of the cutout portion = Area of square + 4× Area of triangle

= 100 + (4× 60)

= 100 + 240

= $340 cm^2$

Worksheet 7

a = 10 cm

h = 30 cm

V = $\frac{1}{3} a^2 \times h$

$= \frac{1}{3} \times 10^2 \times 30$

$= 100 \times 10 = 1000 \text{ cm}^3$

Capacity of the square pyramid container = $\frac{V}{1000} = \frac{1000}{1000} =$ 1 liter

Volume of square pyramid = $\frac{1}{3}$ × volume of square prism

$\therefore$ Capacity of square prism cantainer = 3 ×1 = 3 liter

Cone

Worksheet 8

Complete the table

Basic diameter (d)	**Basic radius (r)**	**height (h)**	**slant leight (l)**
6	3	4	5
12	6	8	10
40	20	15	25
24	12	5	13
12	8	15	17
60	30	40	50

Worksheet 9

a) l = R = 24cm

b) $\frac{x}{360} = \frac{r}{R}$

$\frac{60}{360} = \frac{r}{24}$

$\frac{1}{6} = \frac{r}{24}$

6 ×r = 1 × 24

r = $\frac{24}{6}$ = 4cm

Worksheet 10

r =20cm h =15cm

v = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 20^2 \times 15$

$= \pi \times 400 \times 5 = 2000\pi \text{ cm}^3$

Worksheet 11

a) Radius of the sector (R) = slant height of the cone

b) $\frac{x}{360} = \frac{r}{R}$

$\frac{x}{360} = \frac{12}{24}$

$\frac{x}{360} = \frac{1}{2}$

$2 \times x = 360 \times 1$

$x = \frac{360}{2} = 180°$

Area of sector = Curved surface area of cone

$= \pi rl$

$= \pi \times 12 - \times 24$

$= 288\pi \text{ cm}^2$

Worksheet 12

r = 10cm, h =30cm

Volume of the cylinder $= \pi r^2 h$

$= \pi \times 10^2 \times 30$

$= 3000\pi \text{ cm}^3$

Volume of the cone = $\frac{1}{3}$ = volume of the cyclinder

$= 3000\pi = 1000\pi \text{ cm}^3$

Volume of the remaining part= $\frac{2}{3} = 3000\pi \text{ cm}^3$

Worksheet 13

Volume of cylinder = $\pi r^2 h$

$= \pi \times 12^2 \times 24$

$= 3456\pi \text{ cm}^3$

Volume of cone $= \frac{1}{3} \times \pi r^2 h$

$$\frac{1}{3} \times \pi \times 6^2 \times 18$$

$$= 216\pi$$

No. of cones = $\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{3456\pi}{216\pi} = 16$

Worksheet 14

Tabular column.

Sl. No	**Stant height (*l*)**	**Height (*h*)**	**Radius (*r*)**	**Curved Surface Area**	**Total surface Area**	**Volume**
1	5	4	3	15π	24π	12π
2	15	9	12	180π	32π	43π
3	20	16	12	240π	384π	76π
4	25	15	20	500π	900π	2000π
5	30a	24a	18a	$540\pi a^2$	$864\pi a^2$	$2592\pi a^3$

Worksheet 15

. a) Height (h) = 16 cm

Stant height (l) = 20 cm

Radius (r) = $\sqrt{l^2 - h^2}$

$= \sqrt{20^2 - 16^2} = \sqrt{400 - 256}$

$= \sqrt{144} = 12$ cm

Base area = πr^2

$= \pi \times 12 \times 12 = 144\pi$ sq. cm

b) Curved surface area = πrl

$= \pi \times 12 \times 20 = 240\pi$ sq. cm

c) Total surface area = Base area + Curved surface area

$= 144\pi + 240\pi = 384$ sq. cm

d) Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 12^2 \times 16$

$= 768\pi$ cubic cm

Worksheet 16

a) l = 12 scm

b) $x = 60°$

$\frac{r}{12} = \frac{60}{360}$

r = $\frac{12 \times 60}{360} = 2$ cm

c) Curved surface area = πrl

$= \pi \times 2 \times 12 = 24\pi$ sq. cm

Worksheet 17

Stant height (l) = 25 cm

Base redius (r) = 10 cm

Central angle = $x°$

$\frac{10}{25} = \frac{x}{360}$

$x = \frac{10 \times 360}{25} = 144°$

Worksheet 18

a) Radius of sector (R) = l = 25 cm

b) $\frac{r}{l} = \frac{x}{360}$

$x = \frac{360 \times 15}{25}$

$= 216^0$

c) h = $\sqrt{l^2 - r^2}$

$= \sqrt{25^2 - 15^2}$

$= \sqrt{625-225} = \sqrt{400} = 20$ cm

Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 15^2 \times 20$

$= 1500\pi$ cubic cm

Worksheet 19

r = 9cm.

Surface Area $= 4\pi \times 9^2$

$= 4\pi \times 81$

$= 324\pi$ sq;cm

Volume $\frac{4}{3}\pi \times 9^3$

$= \frac{4}{3}\pi \times 9 \times 9 \times 9$

$= 972\pi$ cubic cm

Worksheet - 20

radius r = 12cm

Surface Area $= 3\pi \times 12^2$

$= 3\pi \times 144$

$= 432\pi$ sq.cm

Volume $= \frac{2}{3}\pi \times 12^3$

$= \frac{2}{3} \times \pi \times 12 \times 12 \times 12$

$= 8\pi \times 144$

$= 1152$ cu.cm

Worksheet - 21

The surface area of a solid sphere is 120 square centimeters. If it cut into two halves, what would be the surface area of each hemisphere?

Surface Area of the shere = 120 sq.cm

$4\pi r^2 = 120$

$\pi r^2 = 30$ sq.cm

Surface Area of Hemisphere $= 3\pi r^2$

$= 3 \times 30$

$= 90$ sq.cm

Worksheet - 22

Diameter of the sphere = length of side of the cube = 8cm

radius of the sphere = r = 4cm

Surface area of the sphere = $4\pi r^2$

$= 4\pi \times 4^2$

$= 64\pi$ sq.cm

Worksheet - 23

Ratio of radii of two spheres = 2 :3

Let the radius of first spher = 2r

Then radius of second sphere = 3r

Volume of the first sphere = $\frac{4}{3}\pi\ (2r)^3$

Volume of the second sphere = $\frac{4}{3}\pi\ (3r)^3$

Ratio of volumes $= \frac{4}{3}\pi\ (2r)^3 : \frac{4}{3}\pi\ (3r)^3$

$= (2r)^3 : (3r)^3$

$= 8r^3 : 27r^3$

$= 8 : 27$

Worksheet - 24

cylinder

radius = r = 4cm

height = h = 10 cm

volume = $\pi r^2 h$

$= \pi \times 4^2 \times 10$

$= \pi \times 160$

$= 160\pi$ cubic cm

Sphere

radius, r = 2 cm

Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 2^3$

$= \frac{4}{3}\pi \times 8$

$= \frac{32\pi}{3}$ cubic cm

Number of spheres $= \frac{\pi \times 160}{\frac{4}{3}\pi \times 18}$

$= \frac{3}{4} \times 20$

$= 15$

Worksheet - 25

Hemisphere

radius = r = 1m

volume $= \frac{2}{3} \times \pi \times 1^3$

$= \frac{2}{3}\pi$ Cubic metre

Cylinder

radius = r = 1cm

height = h = 6 – (1 + 1)

= 6 – 2

= 4m

Volume $= \pi r^2 h$

$= \pi \times 1^2 \times 4$

$= 4\pi$ Cubic metre

Volume of petrol tank $= 2 \times \frac{2}{3}\pi + 4\pi$

$= \frac{4\pi}{3} + \frac{12\pi}{3}$

$= \frac{16\pi}{3}$ Cubic metre

$= \frac{16\pi}{3} \times 1000$ litre

$= \frac{16000\pi}{3}$ litre

Worksheet - 26

a) Height of the cone = 12– (6+3)

= 12 – 9

= 3 m

Cone

radius = r = 3 m

height = h = 3m

volume $= \frac{1}{3}\pi \times 3^2 \times 3$

$= \frac{1}{3}\pi \times 9 \times 3$

$= 9\pi \text{ m}^3$

Cylinder

radius = r = 3 m

height = h = 6m

volume = $\pi r^2 h$

$= \pi \times 3^2 \times 6$

$= 54\pi \text{ m}^3$

Hemisphers

radius = r = 3 m

volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \times 3^3$

$= 18\pi \text{ m}^3$

Total volume of boiler $= 9\pi + 54\pi + 18\pi$

$= 81\pi \text{ m}^3$

$= 81\pi \times 1000$ litre

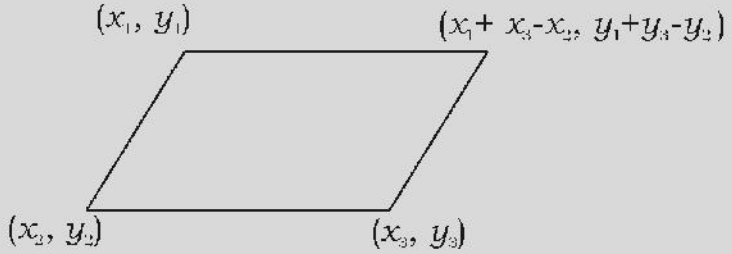
= 81000 litre

Unit 9

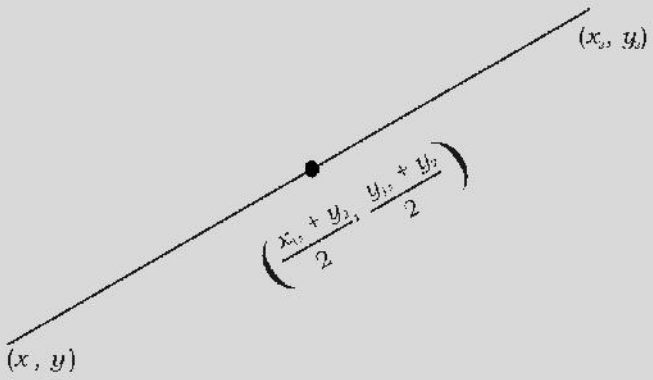
GEOMETRY AND ALGEBRA

Points to Remember

- If (x_1, y_1), (x_2, y_2), (x_3, y_3) are three vertices of a parallelogram, then co-ordinates of its fourth vertex is $(x_1 + x_3 - x_2,\ y_1 + y_3 - y_2)$



- The co-ordinates of the midpoint of the line joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$



- If the point P(x, y) divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m:n$, then

$$x = x_1 + \frac{m}{m+n}(x_2 - x_1)$$

$$y = y_1 + \frac{m}{m+n}(y_2 - y_1)$$

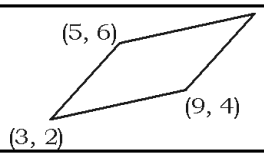
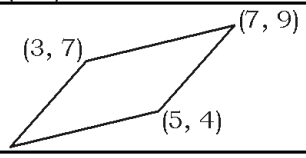
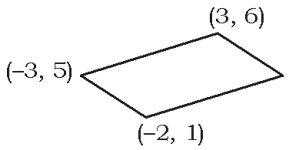
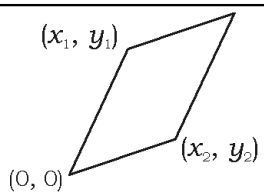
- If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are three vertices of a triangle then

co-ordinates of the centroid = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

- In any line which is not parallel to the axes, the change in y is proportional to the change in x. In this case, the proportionality constant is the slope of this line.
- Slope of the line joining (x_1, y_1) and (x_2, y_2) is $\frac{y_2-y_1}{x_2-x_1}$
- The constant relation between the x-cordinate and y-coordinate of any point on a line is the equation of the line.
- The equation of the line joining (x_1, y_1) and (x_2, y_2) is $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$
- The constant relation between the x-coordinate and y -coordinate of any point on a circle is the equation of the circle.
- The equation of the circle with centre at the origin and radius 'r' is $x^2+y^2=r^2$.
- The equation of the circle with centre (x_1, y_1) and radius 'r' is $(x-x_1)^2+(y-y_1)^2=r^2$

Worksheet - 1

In the following table, write the co-ordinates of the fourth vertex.

Parallelogram	Co-ordinates of fourth vertex
	(5+9–3, 6+4 – 2) = (11, 8)
	
	
	

Worksheet - 2

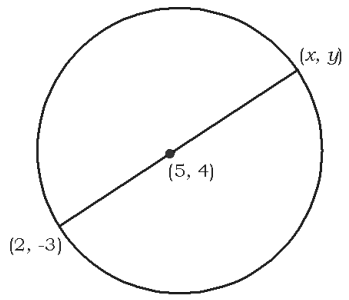
In the following table, given the co-ordinates of the points A and B. Find the co-ordenates of the midpoint of the line AB

Co-ordinats of the points A and B	Co-ordinates the midpoints of the line AB		
	x- Co-ordinates	*y*-Co-ordinates	Co-ordinates of the midpoints
(2, 7)[(4, 5)	$\frac{2+4}{2}=3$	$\frac{7+5}{2}=6$	(3, 6)
(5, –2) , (3, 8)			
(–2, –6). (–4, –10)			
(4, 3), (7, 5)			
$\left(\frac{1}{2}, \frac{1}{3}\right)\left(\frac{3}{2}, \frac{5}{3}\right)$			

Worksheet - 3

If (5, 4) is the centre of the circle and co-ordinates of a point on the circle is (2, –3). Write the co-ordinates of the other end of the diameter through (2, –3)

Co-ordinates of the other end = (*x*, *y*)



$\frac{x+2}{2}=5$, $\frac{y+(-3)}{2}=4$

$x + 2 = \square$, $y - 3 = \square$

$x = \square - \square$, $y = \square + \square$

$= \square$ $= \square$

So co-ordinates of the other end = ($\square$, $\square$)

Worksheet - 4

In the following table the co-ordinates of A and B are given, P divides the line AB in the given ratio. Find the co-ordinates of P.

Co-ordinates of A	Co-ordinates of B	ratio	Coordinates of the point P		
			x coordinate	*y* coordinate	coordinaets of P
			$1+\frac{2}{5}(6-1)$	$2+\frac{2}{5}(7-2)$	
(1, 2)	(6, 7)	2:3	$=1+\frac{2}{5}\times 5$ $= 1 + 2$ $= 3$	$=2+\frac{2}{5}\times 5$ $= 2 + 2$ $= 4$	(3, 4)
(2, 3)	(10, 15)	3:1			
(3, 1)	(5, 8)	3:2			
(–3, 2)	(5, –3)	1:2			

Worksheet - 5

The point P divides the line joining the points A (2, 3) and B (9, 7) in the ratio 4:3. Find the coordinates of the point P.

Let coordinates of P be (x, y)

$$x_1 = x_1 + \frac{m}{m+n}(x_2 - x_1)$$

$$= \square + \frac{4}{7}(\square - \square)$$

$$= \square + \square$$

$$= \square$$

$$y_1 = y_1 + \frac{m}{m+n}(y_2 - y_1)$$

$$= \square + \frac{4}{7}(\square - \square)$$

$$= \square + \square$$

$$= \square$$

Worksheet - 6

The points (3, 2) , (8, 3) and (5, 6) are the vertices of a triangle. Find the centroid of the triangle?

$$\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= (\square, \square)$$

$$= (\square, \square)$$

Worksheet - 7

Find the slope of the line joining the points (3, 5), (6, 7)

$$\text{Slope of the line} = \frac{7-5}{6-3} = \frac{2}{3}$$

Complete the table given below

The points on a line	***x*- difference**	***y*- difference**	**slope of the line**
(2, 5) (6, 7)	4	2	$\frac{2}{4} = \frac{1}{2}$
(3, 7), (6, 9)			
(3, 6), (4, 9)			
(–1, 4), (1, 2)			

Worksheet - 8

Find the equation of the line joining the points (2, 4), (5, 6)

Slope of the line = $\frac{6-4}{5-2} = \frac{2}{3}$

If (x, y) is a point on this line, then the slope of the line joining the points $(2, 4)$ and (x, y) in $\frac{y-4}{x-2} = \frac{2}{3}$

$\therefore$ the equation of this line is

$$\frac{y-4}{x-2} = \frac{2}{3}$$

$$2(x-2) = 3(y-4)$$

$$2x-4 = 3y-12$$

$$2x-3y-4+12=0$$

$$2x - 3y + 8 = 0$$

Complete the following table.

Two points on a line	**Slope of the line**	**Equation of the line**
(2,5), (4,6)	$\frac{1}{2}$	$\frac{y-5}{x-2} = \frac{1}{2}$ $1(x-2) = 2(y-5)$ $x-2 = 2y-10$ $x-2y-2+10=0$ $x-2y+8=0$
(2, 3), (4, 6)		
(1, 3), (5, 4)		
(–2, 4), (4, 5)		

Worksheet - 9

Find the equation of the circle with centre (1, 4) and radius 2 unit.

$$(x-1)^2+(y-4)^2=2^2$$

$$x^2-2x+1+y^2-8y+16=4$$

$$x^2+y^2-2x-8y+17-4=0$$

$$x^2+y^2-2x-8y+13=0$$

Complete the following table.

Centre of the ratio	Radius of the circle	Equation of the circle
(2, 3)	4	$(x-2)^2+(y-3)=4^2$ $x^2-4x+4+y^2-6y+9=16$ $x^2+y^2-4x-6y+13-16=0$ $x^2+y^2-4x-6y-3=0$
(3, 4)	3	
(0, 0)	2	
(1, 2)	5	

Worksheet - 10

Check whether the points A (1, 3), B (2, 5), C (3, 7) are lie on a line?

x - co-ordinate of A = □

x = co-ordinate of B = □

Difference between the x -coordinates of A and B

= □ – 1

= □

y -cordinate of A = □

y – cordinate of B = □

Difference between the y-coordinates of A and B

= □ – □

= □

∴ Slop of AB = 2

Difference between the x-coordinates of B and C

= □ – □

= □

Difference between the y - coordinates of B and C

= □ – □

= □

∴ Slope of the line BC = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

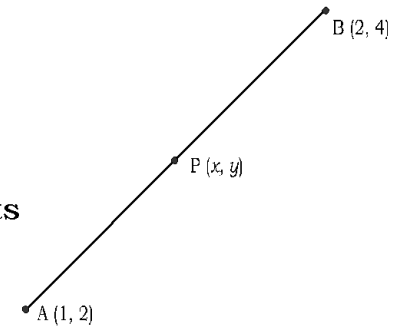
Slopes of the lines AB and BC are equal □ /unequal □

∴ the points A, B and C lie on a line □

Worksheet - 11

Find the equation of the line joining the points (1, 2) and (2, 4)

Let A (1, 2) and B (2, 4) be the points



When we move from A to B the x-coordinates is increased by $\square$ y co-ordinate is increased by $\square$

When x - coordinates is increased by $\square$ the y-coordinates is increased by $\square$

The rate of increase in y with the increase in x, slope of the line = $\frac{\square}{\square}$

Now consider the points A and P the x-coordinates is increased by $x - \square$

y - coordinate is incresed by $y - \square$

Since the rate of increase in y - coordinate with the increase in x-coordinate is always some at every where in a line.

$$\frac{y-\square}{x-\square} = \frac{\square}{\square}$$

From the weight $\square$ $(x - \square) = \square$ $(y - \square)$

Simplifying this weget $x - \square y + \square = 0$

This is the equation of the line.

Worksheet - 12

Find the equation of the circle with centre (2, 5) and radius 3 unit

Let P (x, y) is a point on this circle.

Difference between the x-coordinates of the centre O and the point

P = $x - \square$

Length of the line OP = 3 unit

$$\sqrt{(x-\square)^2 + (y-\square)^2} = 3$$

$$(x-\square)^2 + (y-\square)^2 = 9$$

$$x^2 - \square x + \square + y^2 - \square y + \square = 9$$

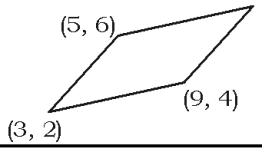
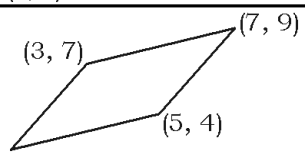
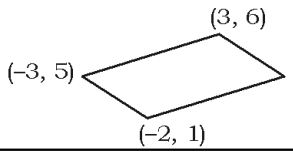
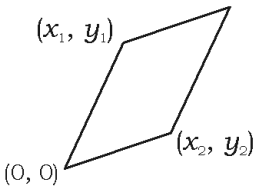
Simplifying this we get.

$$x^2 + y^2 - \square x - \square y - \square = 0$$

This is the equation of the circle.

ANSWERS

Work Sheet - 1

Parallelogram	Co-ordinates of fourth vertex
	(11, 8)
	(1, 2)
	(4, 2)
	$(x_1 + x_x, y_1 + y_2)$

Work Sheet - 2

Co-ordinats of the points A and B	Co-ordinates the midpoints of the line AB		
	x- Co-ordinate	*y*-Co-ordinate	Co-ordinates of the midpoints
(2, 7)[(4, 5)	3	6	(3, 6)
(5, –2) , (3, 8)	4	3	(4, 3)
(–2, –6). (–4, –10)	–3	–8	(–3, –8)
(4, 3), (7, 5)	$11/2$	4	$(11/2, 4)$
$(1/2, 1/3)(3/2, 5/3)$	1	1	(1, 1)

Worksheet - 3

$x + 2 = 10$ $\quad\quad$ $y - 3 = 8$

$x = 10 - 2 = 8$ $\quad\quad$ $y = 8 + 3 = 11$

Co-ordinates of the other end = (8, 11)

Worksheet - 4

Co-ordinates of A	Co-ordinates of B	ratio	Coordinates of the point P		
			x coordinate	*y* coordinate	coordinaets of P
(1, 2)	(6, 7)	2:3	3	4	(3, 4)
(2, 3)	(10, 15)	3:1	8	12	(8, 12)
(3, 1)	(5, 8)	3:2	$\frac{21}{5}$	$\frac{26}{5}$	$\left(\frac{21}{5}, \frac{26}{5}\right)$
(–3, 2)	(5, –3)	1:2	$\frac{-1}{3}$	$\frac{1}{3}$	$\left(\frac{-1}{3}, \frac{1}{3}\right)$

Worksheet - 5

$x = 2 + \frac{4}{7}(9-2)$ $\qquad$ $y = 3 + \frac{4}{7}(7-3)$

$= 2 + \frac{4}{7} \times 7$ $\qquad$ $= 3 + \frac{4}{7} \times 4$

$= 2 + 4$ $\qquad$ $= 3 + \frac{16}{7}$

$= 6$ $\qquad$ $= \frac{37}{7}$

Co-ordinates of P = $(6, {}^{37}\!/_{7})$

Worksheet - 6

$$\left(\frac{3+8+5}{3}, \frac{2+3+6}{3}\right)$$

$$= \left(\frac{16}{3}, \frac{11}{3}\right)$$

Work Sheet - 7

The points on a line	*x*- difference	*y*- difference	slope of the line
(2, 5) (6, 7)	4	2	$\frac{2}{4} = \frac{1}{2}$
(3, 7), (6, 9)	3	2	$\frac{2}{3}$
(3, 6), (4, 9)	1	3	${}^{3}\!/_{1} = 3$
(–1, 4), (1, 2)	2	2	$\frac{-2}{2} = -1$

Work Sheet - 8

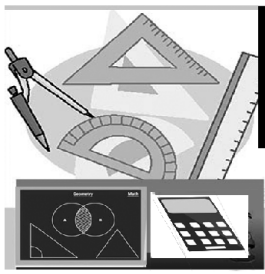
Two points on a line	Slope of the line	Equation of the line
(2,5), (4,6)	$\frac{1}{2}$	$\frac{y-5}{x-2} = \frac{1}{2}$ $1(x-2) = 2(y-5)$ $x-2 = 2y-10$

		$x-2-2+10=0$ $x-2y+8=0$
(2, 3), (4, 6)	$\frac{3}{2}$	$\frac{y-3}{x-2}=\frac{3}{2}$ $3\ (x-2)=2\ (y-3)$ $3x-6=2y-6$ $3x-2y=0$
(1, 3), (5, 4)	$\frac{1}{4}$	$\frac{y-3}{x-1}=\frac{1}{4}$ $1(x-1)=4(y-3)$ $x-1=4y-12$ $x-4y+11=0$
(–2, 4), (4, 5)	$\frac{1}{6}$	$\frac{y-4}{x-(-2)}=\frac{1}{6}$ $1(x+2)=6(y-4)$ $x+2=6y-24$ $x-6y+26=0$

Worksheet - 9

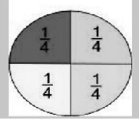
Centre of the ratio	Radius of the circle	Equation of the circle
(2, 3)	4	$(x-2)^2+(y-3)=4^2$ $x^2-4x+4+y^2-6y+9=16$ $x^2+y^2-4x-6y+13-16=0$ $x^2+y^2-4x-6y-3=0$
(3, 4)	3	$(x-3)^2+(y-4)^2=3^2$ $x^2-6x+9+y^2-8y+16=9$ $x^2+y^2-6x-8y+25=9$ $x^2+y^2-6x-8y+25-9=0$ $x^2+y^2-6x-8y+16=0$

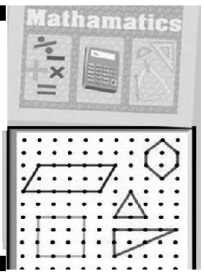
(0, 0)	2	$(x-0)^2+(y-0)^2=1^2$ $x^2+y^2=1$ $x^2+y^2-1=0$
(1, 2)	5	$(x-1)^2+(y-2)^2=5^2$ $x^2-2x+1+y^2-4y+4=25$ $x^2+y^2-2x-4y+5-25=0$ $x^2+y^2-2x-4y-20=0$



Unit
10

POLYNOMIALS





Points to Remember

- If the polynomial p(*x*) = q(*x*) × r(*x*), then the polynomials q(x) and r(*x*) are called the factors of p(*x*).
- If the first degree polynomial (*x* – a) is a factor of the polynomial p(*x*), then p(a) = 0. that is a is a solution of the equation p(*x*) = 0
- If the polynomial p(*x*) can be split into first degree factors as.

  p(*x*) = $(x - a_1)\ (x - a_2)$ ..... $(x\text{-}a_n)$ then the numbers a_1, a_2, ...a_n are the solutions of the equation p(*x*) = 0.
- For any second degree polynomial p (*x*) and for any number a, the polynomial *x* – a is a factor of the polynomial p(*x*) – p(a).
- For any second degree polynomial p(*x*) and for any number a, if p(a) = 0, then the first degree polynomial *x* –a is a factor of the polynomial p(*x*)
- If p and q are the solutions of the second degree equation $ax^2 + bx + c = 0$, then $ax^2 + bx + c = a(x - p)\ (x - q)$

Worksheet - 1

For all numbers *x* and *y*, we have $x^2 - y^2 = (x + y)\ (x - y)$

by using this, $x^2 - 4 = x^2 - 2^2$ = $(x + 2)\ (x - 2)$

(*x*+2) and (*x* –2) are factors of $x^2 - 4$

Fill up the following table.

Second degree polynomial p(*x*)	$x^2 - y^2$ **from**	**Factor form of p(*x*)** $[(x + y)\ (x - y)$form$]$	**Factors of p(*x*)**
$x^2 - 9$	$x^2 - 3^2$	(*x*+3) (*x* –3)	*x* + 3, *x* – 3
$x^2 - 25$	_____	_____	_____, _____
$x^2 - 5$	$x^2 - \left(\sqrt{5}\right)^2$	_____	_____, _____
$x^2 - 7$	_____	_____	_____, _____
$x^2 - 1/9$	$x^2 - \left(1/3\right)^2$	_____	_____, _____
$x^2 - 1/4$	_____	_____	_____, _____

$4x^2-9$	$(2x)^2-3^2$	_____	_____, _____
$9x^2-25$	_____	_____	_____, _____
$5x^2-16$	$\left(\sqrt{5}x\right)^2-4^2$	_____	_____, _____
$7x^2-25$	_____	_____	_____, _____

Worksheet - 2

$p(x)=x^2+5x+6$

a) Write p(x) as the product of two first degree polynomials.

b) What are the solutions of the equation p(x) = 0

Let $x^2+5x+6 = x^2+(a+b)x+ab$

a + b = 5

ab = 6

a = 3, b = 2

$x^2+5x+6 = x^2+(3+2)x+3\times2$

$= (x+3)\ (x+2)$

b) $p(x) = (x+3)\ (x+2)$

$= [x-(-3)]\ [x-(-2)]$

$\therefore$ The solutions of the equation p(x) = 0 are –3 and –2

Fill up the following table.

Second degree polynomial p(x)	**Factor form of p(x)**	**Factors of p(x)**	**Solutions of the equation p(x) = 0**
$x^2+7x+10$	$(x+2)\ (x+5)$	$x+2,\ x+5$	–2, –5
$x^2+8x+15$	_____	_____, _____	_____, _____
$x^2-7x+12$	$(x-3)\ (x-4)$	_____, _____	3, _____
$x^2-8x+12$	_____	_____, _____	_____, _____
$x^2+12x-13$	$(x+13)\ (x-1)$	_____, _____	–13, _____
$x^2-12x-13$	_____	_____, _____	_____, _____

Worksheet - 3

p(x) = $x^2-6x+10$

a) What number is p(2) ?

b) Write p(x) – p(2) as the product of two first degree polynomials.

a) p(2) = $2^2 - 6 \times \square + 10$

= 4 – 12 + $\square$

= $\square$

b) p(x) – p(2) = $\left(x^2 - 6x + 10\right) - \square$

= $x^2 - 6x + 8$

= (x–2) (x–4)

Fill up the following table.

p(x)	**a**	**p(a)**	**p(x) – p(a)**	**One factor of p(x)–p(a)**	**Second factor of p(x)–p(a)**
$x^2 - 4x + 4$	1	$1^2 - 4 \times 1 + 4 = 1$	$x^2 - 4x + 3$	$x - 1$	$x - 3$
$x^2 - 7x + 13$	2	————	————	————	————
$x^2 + 6x + 13$	–2	$(-2)^2 + 6 \times (-2) + 13$ = 4–12 + 13 = 5	$x^2 + 6x + \square$	$x - (-2)$ = x + 2	x+4
$x^2 + 7x + 16$	–3	————	————	————	————
$x^2 - 4x + 2$	5	$5^2 - 4 \times \square +$ $\square$ = $\square$	$x^2 - 4x - 5$	$x - 5$	————
$x^2 - 2x + 1$	3	————	————	————	————

Worksheet - 4

a) Write $x^2 - 20x + 91$ as the product of two first degree polynomials.

Consider the equation $x^2 - 20x + 91 = 0$

a = $\square$, b = $\square$, c = $\square$

$b^2 - 4ac$ = $\square^2$ – 4 × $\square$ × $\square$

= $\square$ – $\square$

= $\square$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{\square \pm \sqrt{\square}}{2 \times \square}$

$= \dfrac{\square}{\square}$ or $\dfrac{\square}{\square}$

= □ or □

$x^2 - 20x + 91$ = (x – □) (x – □)

Worksheet - 5

Prove that the polynomial $x^2 + x + 1$ cannot be factored into a product of first degree polynomials.

Consider the equation $x^2 + x + 1 = 0$

a = □, b = □, c = □

$b^2 - 4ac$ = □^{2} – 4 × □ ×□

= □ – □

= □

Since $b^2 - 4ac$ <0, the equation $x^2 + x + 1 = 0$ has no solution.

So the polynomial $x^2 + x + 1$ cannot be factored into a product of first degree polynomials.

Worksheet - 6

In the polynomial $x^2 + kx + 6$, what number must be taken as k to get a polynomial for which (x–1) is a factor?

$p(x) = x^2 + kx + 6$

(x–1) is a factor, if p(1) = □

□^{2} + k× □ + □ = 0

1 + k + □ = 0

k + □ = 0

k = □

ANSWERS

Worksheet- 1

Second degree polynomial p(*x*)	$x^2 - y^2$ **form**	**Factor form of p(*x*)** $[(x+y)\ (x-y)$form$]$	**Factors of p(*x*)**
x^2-9	x^2-3^2	$(x+3)\ (x-3)$	$x+3,\ x-3$
x^2-25	x^2-5^2	$(x+5)\ (x-5)$	$x+5,\ x-5$
x^2-5	$x^2-\left(\sqrt{7}\right)^2$	$(x+\sqrt{5})\ (x-\sqrt{5})$	$x+\sqrt{5},\ x-\sqrt{5}$
x^2-7	$x^2-\left(\sqrt{5}\right)^2$	$(x+\sqrt{7})\ (x-\sqrt{7})$	$x+\sqrt{7},\ x-\sqrt{7}$
$x^2-1/9$	$x^2-\left(1/3\right)^2$	$(x+1/3)\ (x-1/3)$	$x+1/3,\ x-1/3$
$x^2-1/4$	$x^2-\left(1/2\right)^2$	$(x+1/2)\ (x-1/2)$	$x+1/2,\ x-1/2$
$4x^2-9$	$(2x)^2-3^2$	$(2x+3)\ (2x-3)$	$2x+3,\ 2x-3$
$9x^2-25$	$(3x)^2-5^2$	$(3x+5)\ (3x-5)$	$3x+5,\ 3x-5$
$5x^2-16$	$\left(\sqrt{5}x\right)^2-4^2$	$\left(\sqrt{5}x+4\right)\ \left(\sqrt{5}x-4\right)$	$\sqrt{5}x+4,\ \sqrt{5}x-4$
$7x^2-25$	$\left(\sqrt{7}x\right)^2-5^2$	$\left(\sqrt{7}x+5\right)\ \left(\sqrt{7}x-5\right)$	$\sqrt{7}x+5,\ \sqrt{7}x-5$

Worksheet - 2

Second degree polynomial p(*x*)	**Factor form of p(*x*)**	**Factors of p(*x*)**	**Solutions of the equation p(*x*) = 0**
$x^2+7x+10$	$(x+2)\ (x+5)$	$x+2,\ x+5$	$-2,\ -5$
$x^2+8x+15$	$(x+3)\ (x+5)$	$x+3,\ x+5$	$-3,\ -5$
$x^2-7x+12$	$(x-3)\ (x-4)$	$x-3,\ x-4$	$3,\ 4$
$x^2-8x+12$	$(x-2)\ (x-6)$	$x-2,\ x-6$	$2,\ 6$
$x^2+12x-13$	$(x+13)\ (x-1)$	$x-1,\ x+13$	$1,\ -13$
$x^2-12x-13$	$(x+1)\ (x-13)$	$x+1,\ x-13$	$-1,\ 13$

Worksheet - 3

p(x)	a	p(a)	p(x) – p(a)	One factor of p(x)–p(a)	Second factor of p(x)–p(a)
x^2-4x+4	1	$1^2-4\times1+4=1$	x^2-4x+3	$x-1$	$x-3$
$x^2-7x+13$	2	$2^2-7\times2+13=3$	$x^2-7x+10$	$x-2$	$x-5$
$x^2+6x+13$	–2	$(-2)^2+6\times(-2)+13$ = 5	x^2+6x+8	$x+2$	$x+4$
$x^2+7x+16$	–3	$(-3)^2+7\times(-3)+16$ = 4	$x^2+7x+12$	$x+3$	$x+4$
x^2-4x+2	5	$5^2-4\times5+2=7$	x^2-4x-5	$x-5$	$x+1$
x^2-2x+1	3	$3^2-2\times3+1=4$	x^2-2x-3	$x-3$	$x+1$

Worksheet - 4

a = 1, b = –20, c = 91

b^2-4ac = $(-20)^2-4\times1\times91$

= 400 – 364

= 36

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{20\pm\sqrt{36}}{2\times1}$

$= \dfrac{26}{2}$ or $\dfrac{14}{2}$

= 13 or 7

$x^2-20+91=(x-13)\ (x-7)$

Worksheet - 5

a = 1, b = 1, c = 1

b^2-4ac = $1^2-4\times1\times1$

= 1 – 4

= – 3

Worksheet- 6

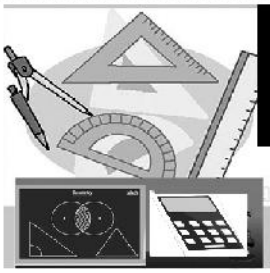
p(1) = 0

$1^2 + k \times 1 + 6 = 0$

1 + k + 6 = 0

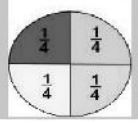
k + 7 = 0

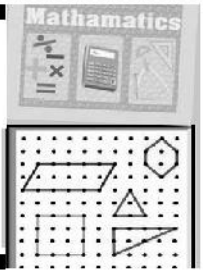
k = – 7



Unit
11

STATISTICS





Points to Remember

- Mean = $\frac{\text{sum of questions}}{\text{No. of questions}}$

Median $\Rightarrow$ The middle most abservation when the observations are arranged in order.

- If the number of observations is 'n'

then the middle most observation, is

$\frac{n}{2}^{th}$, if n is odd

$\left(\frac{n}{2}+1\right)^{th}$, if n is even.

Worksheet - 1

The weight in kg of 9 children are given below. Find the mean and the median.

29, 30, 39, 41. 35, 34, 28, 31, 38

Mean = $\frac{\text{Sum}}{\text{Number}}$

Sum = 29+ 40+ 39+ 41 + 35 + 34+ 28+ 31+ 38 = □

$\therefore$ Mean = $\frac{\square}{\square}$ = □

When the given observations are arranged in order.

□, □, □, □, □, □,□, □, □, □

The middle most observation is □ th observation.

$\therefore$ Median = □

Worksheet - 2

The height of 8 people are given below. Find the median height.

152, 157, 153, 148, 151, 155, 150, 154

If the heights are arranged in ascending order

□, □, □, □, □, □, □, □, □

Middle most observations are □ and □

$\therefore$ The heights in the middle are □ and □

Median = $\frac{\square + \square}{2} = \square$

Worksheet - 2

Daily wages of same labourers are given below in the table. Find the median.

Daily wages (In Rupees)	Number of labourers
600	4
700	6
800	7
900	8
1000	4

When the number of labourers we arranged according to their income.

Worksheet - 3

Daily wages (In Rupees)	No.of labourers
Upto 600	4
Upto 700	10
Upto 800	□
Upto 900	□
Upto 10000	□

Total No.of labourers = □

Position of the middle most labourer = 15^{th} labourer

$\therefore$ Median = □

Worksheet - 4

Monthly income of 30 families are given below. Find the median income.

Monthly Income (In Rupees)	No.of families
6000	4
7000	6
8000	2
9000	3
10000	5
11000	7
12000	3

When the families are arranged according to their income

Monthly income (In Rupees)	**No.of families**
Upto 6000	4
Upto 7000	10
Upto 8000	□
Upto 9000	□
Upto 10000	□
Upto 11000	□
Upto 12000	□

Position of the middle most familes are □th and □th.

Income of that families at are □ and □ .

Median = $\frac{\square + \square}{2}$ = □

Worksheet - 5

The following table shows the daily wages of some workers in a factory. Find the median daily wages.

Daily wages (In Rupees)	**No.of labourers**
400 - 500	5
500 - 600	9
600 - 700	10
700 - 800	12
800 - 900	6
900 - 1000	3

When the labourers are arranged according to their wages.

Daily wages	**No.of Labourers**
Below 500	5
Below 600	14
Below 700	24
Below 800	□
Below 900	□
Below 1000	□

If 45 labourer's are arranged according to their daily wages,

Wage of the 23 rd labourer is the median.

And this wage is between 600 and 700.

Wage from position 15 to 24 are in this class.

Number of labourer's from 15 to 24 = ☐

Difference of wages from 600 to 700 = ☐

Difference of wage from 600 to 700 is 100 and it is divided into 10 equal divisions. Assume that there is one labourer in each division and the daily wage of each subdivision is in the middle of that division.

If 100 rupees is divided into 10 equal parts, each part is $= \frac{100}{10} = 10$

Daily wage of 15th labourer = In the middle of 600 and 610 = ☐

Daily wage of 23rd labourer = ☐

Median daily wage = ☐

ANSWERS

Worksheet - 1

Mean = $\frac{315}{9} = 35$

Median

28, 29, 31, 34, 35, 38, 39, 40, 41

Worksheet - 2

148, 150, 151, 152, 153, 155, 157

Middle most heights, 152, 153

Median = $\frac{152+153}{2} = 152.5$

Worksheet - 3

No.of tolal laboureres = 29

Median = 850

Worksheet - 4

Position, of the middle most families are 15 and 16.

Median Income = $\frac{9000+10000}{2} = 9500$

Worksheet - 5

No.of Labourers from 15 to 24 are 10.

Difference in income from 600 to 700 = 100

(Part of income each will get) $= \frac{100}{10} = 10$

Daily wage of 15th labourer = 605

Daily wage of 23 rd labourer = 685

Median daily wage = 685