ജില്ലാ വിദ്യാഭ്യാസ പരിശീലന ഇൻസ്റ്റിറ്റ്യൂട്ട് (ഡയറ്റ്)

തൃശ്ശൂർ

# ഫോക്കസ് 2021

എസ്. എസ്. എൽ. സി. വിദ്യാർത്ഥികൾക്കുള്ള പഠന സഹായി

## ഗണിതം

**ജില്ലാ വിദ്യാഭ്യാസ പരിശീലന ഇൻസ്റ്റിറ്റ്യൂട്ട് (ഡയറ്റ് ) തൃശൂർ.**

# ഫോക്കസ് 2021.

## എസ്.എസ്.എൽ.സി വിദ്യാർത്ഥികൾക്കുള്ള പഠന സഹായി.

## Support Materials for SSLC March 2021.

**അക്കാദമിക സഹായം.**

ജില്ലാ വിദ്യാഭ്യാസ പരിശീലന ഇൻസ്റ്റിറ്റ്യൂട്ട് കോഴിക്കോട്.

ജില്ലാ വിദ്യാഭ്യാസ പരിശീലന ഇൻസ്റ്റിറ്റ്യൂട്ട് ഇടുക്കി.

നിർമ്മാണ ചുമതല : സി എം ഡി ഇ വിഭാഗം ഡയറ്റ് തൃശൂർ.

പ്രസിദ്ധീകരിച്ചത് : 2021 ഫെബ്രുവരി.

പ്രിയ കുട്ടികളേ,

സ്കൂളിൽ പോയി കൂട്ടുകാരോടൊപ്പമിരുന്ന് അധ്യാപകരുടെ ക്ലാസ്സുകൾ കേൾക്കാനും പഠന പ്രവർത്തനങ്ങൾ നിർവ്വഹിക്കാനും കഴിയാത്ത ഒരു അധ്യയന വർഷമാണ് കടന്നു പോകുന്നത്. ലോകത്തെ മുഴുവൻ ഗ്രസിച്ച കോവിഡ് 19 രോഗബാധ നമ്മുടെ സാധാരണ അധ്യയന രീതികളെയാകെ പുനഃസംവിധാനം ചെയ്യാൻ പ്രേരിപ്പിച്ചു. സംസ്ഥാനതലത്തിൽ ഫസ്റ്റ് ബെൽ ഓൺലൈൻ ക്ലാസ്സുകൾ മുഴുവൻ വിദ്യാർത്ഥികൾക്കും ലഭ്യമാക്കുകയും അധ്യാപകരുടെ പിന്തുണയോടെ പഠന പ്രകിയ പൂർത്തിയാക്കുകയുമാണ് നാം ചെയ്തത്. ഇനിയുള്ളത് കുട്ടികളുടെ സംശയ പരിഹരണത്തിനും പരീക്ഷാ തയ്യാറെടുപ്പിനു മായുള്ള ദിനങ്ങളാണ്. ആത്മവിശ്വാസത്തോടെ എസ്.എസ്.എൽ.സി.പരീക്ഷ നേരിടുന്നതിനായി കുട്ടികളെ സുസജ്ജരാക്കേണ്ടതുണ്ട്. പരീക്ഷയ്ക്കായി കൂടുതൽ ഊന്നൽ നൽകി പഠിക്കേണ്ട പാഠ ഭാഗങ്ങൾ എസ്.സി.ആർ.ടി.നിർദേശിച്ചിട്ടുണ്ട്.

തൃശൂർ ഡയറ്റിന്റെ അക്കാദമിക നേതൃത്വത്തിൽ കോഴിക്കോട്, ഇടുക്കി ഡയറ്റുകളുടെ സഹകരണത്തോട എസ്.എസ്.എൽ.സി. ഫോക്കസ് ഏരിയകളെ ആധാരമാക്കിയുള്ള പ്രവർത്തനങ്ങൾ അടങ്ങിയ പഠന സഹായി തയ്യാറാക്കിയിരിക്കുകയാണ്. പരീക്ഷയുടെ തയ്യാറെടുപ്പുകൾക്കായി നേരിട്ട് അധ്യയനം ആരംഭിച്ച സാഹചര്യത്തിൽ പഠന പ്രവർത്തനങ്ങൾക്കായി അധ്യാപകരും വിദ്യാർത്ഥികളും **"ഫോക്കസ് 2021"** പഠന സഹായി ഫലപ്രദമായി ഉപയോഗിക്കുമെന്നും അത് അവരെ വിജയത്തിലേക്ക് നയിക്കുമെന്നും പ്രതീക്ഷിക്കുന്നു.

**ശ്രീമതി എൻ.ഗീത**

വിദ്യാഭ്യാസ ഉപഡയറക്ടർ

തൃശൂർ

**ശ്രീ. ടി .അബ്ദുൾ നാസിർ**

പ്രിൻസിപ്പൽ,ഡയറ്റ്

തൃശൂർ

# MATHEMATICS

# Revision 2021 SSLC Mathematics

## Chapter 1:

# ARITHMETIC SEQUENCES

* A sequence is determined by the arrangement of numbers or objects with a specific rule .To understand a number sequence properly we need its algebra.

* Algebraic form of a number sequence is the relation between the terms of the sequence and natural numbers

* Arithmetic sequence is a special kind of sequence starting from a number and adding a number repeatedly.The algebra of an arithmetic sequence is generally in the form $x_n = an + b$ .Here $a$ is the common difference and $a + b$ is the first term.

* If $f$ is the first term and $d$ is the common difference then $x_n = dn + (f - d)$ will be its algebraic form.

* The difference between any two terms of an arithmetic sequence will be a multiple of its common difference.

* When a multiple of common difference is added to a term of an arithmetic sequence we get another term of the same sequence.

* In an arithmetic sequence of positive integer terms when the terms are divided by its common difference we get the same number as the remainder.

* In an arithmetic sequence having a definite number of terms, sum of the terms equidistant from both ends are equal. If the number of terms in the sequence is odd we get a middle term . The middle term will be half of this sum.

* Also, the middle term can be obtained by dividing the sum of the terms by the number of terms.

* Algebraic form of a number sequence is the relation between the terms of the sequence and natural numbers in the order.

* We can say all number sequences are genarated from the natural numbers. To understand the sequence properly we use its algebra .

* Terms of the sequence can be written by giving $1, 2, 3 \cdots$ to $n$ in its algebraic form

* Algebraic form can be used to check whether a number is a term of the sequence, to find the number of terms of the sequence , to calculate the sum of the terms and so many other situations.Some of them are discussed below

1) Consider the sequence $1, 3, 5, 7 \cdots$

    a) Write next two more terms of this sequence

    b) Which is the smallest two digit term of this sequence?

    c) Which is the largest two digit term of this sequence

    d) Can the sum of any two terms of this sequence 75

### Answers

    a) $9, 11$

    b) $11$

    c) $99$

    d) No .The terms are odd numbers. the sum of two odd numbers cannot be an odd number. .

2)     a) Write the sequence of numbers in which $1$ comes in the one's place

    b) which is the largest two digit term of this sequence?

    c) How many numbers are there in this sequence below $100$?

### Answers

    a) $1, 11, 21 \cdots$

    b) $91$

    c) $10$

3) In the arithmetic sequence 6, 10, 14, 18...

    a) Write next three terms

    b) What is the tenth term of this sequence

    c) Is $900$ a term of this sequence ? If it is ,what is its position?

    d) Is $206$ a term of this sequence? Why?

4) Consider the numbers which gives the remainder 1 on dividing by 3
   a) Write the sequence of these numbers
   b) Which is the smallest two digit term of this sequence
   c) How many numebrs are there up to 25 in this sequence ?
   d) What are the possible remainders on dividing a number by 3

5) Consider the Arithmetic sequence $\dfrac{1}{5}, \dfrac{2}{5}, \dfrac{3}{5}$ ..............

6) Consider the arithmetic sequence $1, 5, 9, 13 \cdots$
   a) What is the common difference of this sequence ?
   b) How many times common difference should be added to 1 for getting its tenth term?
   c) How many times 3 should be added to the first term 1 to get its $n$th term?
   d) Write the algebraic form of this sequence .

7) Algebraic form of an arithmetic sequence is $3n + 7$.
   a) Write the sequence.
   b) What is the 20 th term of this sequence
   c) What should be added to 5 th term of this sequence to get 10 th term?

a) Give the values $1, 2, 3 \cdots$ to $n$ in its algebraic form.Terms are $10, 13, 16 \cdots$

b) $x_{20} = 3 \times 20 + 7 = 67$

c) $5$times common difference should be added.It is $5 \times 3 = 15$

8) Fifth term of an arithmetic sequence is $10$ and tenth term is $5$.

   a) What is the common difference?

   b) What is its first term?

   c) What is its fifteenth term?

   d) What is the sum of product of $15$ terms?

a) $5d = 5 - 10 = -5, d = -1$

b) $x_1 = x_5 - 4d = 10 - 4 \times -1 = 10 + 4 = 14$

c) $x_{15} = f + 14d = 14 + 14 \times (-1) = 14 - 14 = 0$

d) $0$

9) Consider the arithmetic sequence $1, 8, 15, 22 \cdots$

   a) What is the common difference ?

   b) What will be the remainder when the terms are divided by its common difference?

   c) Which is the first three digit term of this sequence?

   d) Write the algebraic form of this sequence

   e) How many terms are there below $100$in this sequence?

a) $d = 8 - 1 = 7$

b) $1$

c) $106$

d) $x_n = dn + (f - d) = 7n + (1 - 7) = 7n - 6$

e) $7n - 6 < 100, 7n < 106, n < 15.1, n = 15$
   15 terms below $100$.

10) The algebraic form of an arithmetic sequence is $7n + 3$.

   a) What is the common difference?

   b) What is the first term of this sequence ?

   c) What is the position of $171$in this sequence?

   d) What is the position of$248$in this sequence ?

   e) How many terms are there from $171$ to $248$ in this sequence

a) $7$

b) $7 \times 1 + 3 = 10$

c) $7n + (10 - 7) = 171$
   $7n + 3 = 171, 7n = 168, n = 24$
   24th term is $171$

d) $7n + 3 = 248, 7n = 245, n = 37$

e) $37 - 24 + 1 = 14$

11) Insert suitable number in the circle which make the arithmetic sequence

a) $\boxed{5}$, $\bigcirc$, $\boxed{11}$, $\bigcirc$, $\boxed{17}$

b) $\boxed{13}$, $\boxed{9}$, $\bigcirc$, $\bigcirc$, $\boxed{-3}$, $\boxed{-7}$

c) $\bigcirc$, $\bigcirc$, $\boxed{6}$, $\boxed{10}$, $\bigcirc$, $\bigcirc$

d) $\boxed{7}$, $\boxed{15}$, $\bigcirc$, $\bigcirc$, $\boxed{39}$, $\boxed{47}$

**Answers**

a) $5, 8, 11, 14, 17$

b) $13, 9, 5, 1, -3, -7$

c) $-2, 2, 6, 10, 14, 18$

d) $7, 15, 23, 31, 39, 47$

12) Angles of a right triangle are in an arithmetic sequence.

a) Find the middle term of the sequence

b) Write the angles of the triangle

**Answers**

a) $x_2 = \frac{180}{3} = 60$

b) $30°, 60°, 90°$

13) Angles of a quadrilateral are in an arithmetic sequence

a) Find the angle sum

b) What is the sum of first term and fourth term?

c) What is the sum of second term and third term?

d) Suggest a suitable name of this quadrilateral

**Answers**

a) $360°$

b) $x_1 + x_4 = 180°$

c) $x_2 + x_3 = 180°$

d) Since co interior angle sum is $180°$, two sides are parallel. It is a trapezium

14) Angles of a pentagon are in an arithmetric sequence .

a) What is the sum of the terms of this sequence

b) What is the middle term ?

c) If the smallest angle is $40°$ what is the difference between two adjacent angles

d) Write the angles as a sequence

**Answers**

a) $(5 - 2) \times 180 = 540°$

b) $x_3 = \frac{540}{5} = 108°$

c) $108 - 40 = 2d, d = 34°$

d) $40°, 74°, 108°, 142°, 176°$

15) There is an arithmetic sequence having $9$ terms.The sum of the first and $9$ th terms is $32$.

a) What is the sum of second and eighth term?

b) what is the fifth term?

c) If the sixth term is $19$ then what is its common difference?

d) Write the first term of this sequence

a) $x_2 + x_8 = 32$

b) $\frac{32}{2} = 16°$

c) $d = x_6 - x_5 = 19 - 16 = 3$

d) $x_1 = x_5 - 4d = 16 - 12 = 4$

16) 13 th term of an arithmetic sequence is $48$. Its $17$ th term is $64$

a) What is the common difference?

b) What is the $15$ th term?

c) Find the first term of this sequence

d) Write the algebraic form of this sequence

e) What is the product of a number of terms from the beginning of the sequence .

a) $x_{17} - x_{13} = 4d, 4d = 64 - 48 = 16$
$d = 4$

b) $x_{15} = \frac{48+64}{2} = 56$
or
$x_{15} = x_{13} + 2d = 48 + 2 \times 4 = 48 + 8 = 56$
or
$x_{15} = x_{17} - 2d = 64 - 8 = 56$

c) $x_1 = x_{13} - 12 \times d = 48 - 12 \times 4 = 48 - 48 = 0$

d) $x_n = dn + (f - d) = 4n - 4$

e) Since first term is $0$, the product is $0$

17) The sum of first $5$ terms of an arithmetic sequence is $70$. The sum of the first $11$ terms is $286$.

a) What is its third term?

b) What is its sixth term?

c) What is the common difference of this sequence ?

d) What is the first term of this sequence ?

e) Write the algebraic form of this sequence

a) $x_3 = \frac{70}{5} = 14$

b) $x_6 = \frac{286}{11} = 26$

c) $x_6 - x_3 = 26 - 14$
$3d = 12, d = 4$

d) $x_1 = x_3 - 2d = 14 - 2 \times 4 = 14 - 8 = 6$

e) $x_n = dn + (f - d) = 4n + (6 - 4) = 4n + 2$

18) Seventh term of an arithmetic sequence is $21$.

a) What is the sum of sixth and eighth term of this sequence ?

b) What is the sum of first and thirteenth term ?

c) If the eighth term is $25$, what is its common difference

d) What is the first term of this sequence?

e) Can the difference between any two terms $45$?

a) $x_6 + x_8 = 21 \times 2 = 42$

b) $x_1 + x_{13} = 42$

c) $d = x_8 - x_7 = 25 - 21 = 4$

d) $x_1 = x_7 - 6d = 21 - 6 \times 4 = 21 - 24 = -3$

e) 45 is not a multiple of 4.
   $\therefore$ 45 cannot be the difference between two terms

19) Consider the arithmetic sequence $11, 15, 19 \cdots 327$

   a) Write the algebraic form of this sequence

   b) How many terms are there in this sequence?

   c) Calcualte the sum of the terms

   d) Can the sum of any 25 terms of this sequence 2020. Why?

a) $x_n = dn + (f - d)$ , $x_n = 4n + (11 - 4) = 4n + 7$

b) $4n + 7 = 327, 4n = 320, n = 80$

c) sum $= (x_1 + x_n) \times \frac{n}{2} = (11 + 327) \times \frac{80}{2} = 13520$

d) All terms are odd numbers. 25odd numbers cannot make the sum 2020 an even number.

20) Consider the sequence of numbers which leaves the remainder 3on dividing by 4

   a) Write the sequence

   b) Write the algebraic form of this sequence

   c) Which is the term just below $100$

   d) How many terms are there below $100$in this sequence?

   e) Calculate the sum of all terms below $100$

a) $3, 7, 11 \cdots$

b) $x_n = dn + (f - d) = 4n + (3 - 4) = 4n - 1$

c) 99

d) $4n - 1 = 99, 4n = 100, n = 25$

e) sum$= (x_1 + x_n) \times \frac{n}{2} = (3 + 99) \times \frac{25}{2} = 1275$

21) Write the sequence and make calculations as given below

   a) First term 2,common difference 4. Write the arithmetic sequence and calculate the sum of 2 terms

   b) First term 3, common difference 3 . Write the arithmetic sequence and calculate the sum of three terms.

   c) First term 4, common difference 8 .Calculate the sum of first 4 terms.

   d) First term 5,common difference 10. Write the arithmetic sequence and find the sum of first 5 terms

   e) If the first term of an arithmetic sequence is $n$ and common difference $2n$ then prove that the sum of first $n$ terms is $n^3$

a) $2, 6$

    sum $= 2 + 6 = 8$

b) $3, 9, 15$

    sum $= 3 + 9 + 15 = 27$

c) $4, 12, 20, 28$

    sum $= 4 + 12 + 20 + 28 = 64$

d) $5, 15, 25, 35, 45$

    sum $= 5 + 15 + 25 + 35 + 45 = 125$

e) $n, 3n, 5n, 7n \cdots (2n - 1) \times n$

    $x_1 = n, x_n = (2n - 1)n$

    sum $= (x_1 + x_n) \times \frac{n}{2} = n^3$

22) This is the sequence of fine dots which make square pattern



  a) Write the number of dots in each square as a sequence.

  b) How many dots are there in sixth square

  c) What position in the sequence a square with $100$ dots comes

  d) Write the algebraic form of this sequence

  e) How many dots are there in the square just below $1000$?

**Answers**

  a) $1, 4, 9, 16, 25 \cdots$

  b) $36$

  c) 10th position

  d) $x_n = n^2$

  e) $961$ dots . It is $31^2$

23) We have seen the sequence of interior angle sum of polygons in the class. The sequence is given below

$$180, 360, 540, 720 \cdots$$

  a) This is the sequence of the multiples of $180$. Write the algebraic form of this sequence

  b) What is the fifth term of this sequence.

  c) If the polygons are regular , all interior angles are equal . Write the sequence of interior angles

  d) Think about the algebraic form of the sequence of intelor angles and send your answer to the teacher.

a) $x_n = 180n$

b) $x_5 = 180 \times 5 = 900$

c) $60, 90, 108, 120 \cdots$. This is obtained by dividing the angle sum by the number of sides.

d) Look at the pattern given below

$$60 = \frac{180 \times 1}{3} = \frac{180 \times 1}{1+2}$$

$$90 = \frac{180 \times 2}{4} = \frac{180 \times 2}{2+2}$$

$$108 = \frac{180 \times 3}{5} = \frac{180 \times 3}{3+2}$$

$$120 = \frac{180 \times 4}{6} = \frac{180 \times 4}{4+2}$$

Proceeding like this we get

$$x_n = \frac{180n}{n+2}$$

24) Complete the addition :

a) Calculate the sum of counting numbers from $1$ to $10$

b) Calculate the sum of counting numbers from $1$ to $100$

c) Calculate the sum of the $48$ terms of the sequence $3, 4, 5 \cdots$ obtained by adding $1$ repeatedly

d) Calcualte the sum of counting numbers from $10$ to $20$

a) Sum $= (n+1) \times \frac{n}{2} = (10+1) \times \frac{10}{2} = 55$

b) Sum $= (100+1) \times \frac{100}{2} = 5050$

c) Sum $= (1 + 2 + 3 + 4 \cdots 50) - (1 + 2) = (50 + 1) \times \frac{50}{2} - 3 = 1272$

d) Sum of n numbers from $10$ to $20$ = sum from $1$ to $20$ − sum from $1$ to $9$
$= (20 + 1) \times \frac{20}{2} - (9 + 1) \times \frac{9}{2} = 165$

25) The sum of first $n$ counting numbers is $(n+1) \times \frac{n}{2}$. Use this to complete the following calculations

a) Consider the sequence of the multiples of $3$
$3, 6, 9 \cdots$. Which is the largest two digit term of this sequence

b) Calculate the sum of all terms of this sequence below $100$

c) Find the sum of all even numbers below $100$

d) Calculate the sum of all multiples of 7 below $100$

a) $99$

b) $3 + 6 + 9 + \cdots 99 = 3(1 + 2 + 3 + \cdots 33) = 3 \times (33 + 1) \times \frac{33}{2} = 1683$

c) $2 + 4 + 6 + \cdots 98) = 2(1 + 2 + 3 + \cdots 49) = 2 \times (49 + 1) \times \frac{49}{2} = 2450$

d) $7 + 14 + 21 + \cdots 98 = 7(1 + 2 + 3 \cdots 14) = 7 \times (14 + 1) \times \frac{14}{2} = 735$

26) Answer the following questions

a) Find the sum of first $10$ odd numbers

b) Find the sum of first $25$ odd numbers

c) How many odd numbers from $1$ makes the sum $1225$?

d) $1 + 3 + 5 + 7 \cdots + 2n - 1$ is in between $900$ and $1000$. What is $n$?

a) $s_n = n^2, s_{10} = 10^2 = 100$

b) $s_n = n^2, s_{25} = 25^2 = 625$

c) ആദ്യത്തെ $n$ ഒറ്റസംഖ്യകളുടെ തുക $= n^2$.
$n = \sqrt{1225} = 35$

d) Perfect square in between 900 and 1000 is 961. $n^2 = 961, n = 31$

27) Have you heard of triangular numbers?The sequence of triangular numbers is given below

$$1, 3, 6, 10, 15, 21 \cdots$$

$n$ th term of this sequence can be obtained from the pattern

$$1 = 1$$
$$3 = 1 + 2$$
$$6 = 1 + 2 + 3$$
$$10 = 1 + 2 + 3 + 4$$
$$15 = 1 + 2 + 3 + 4 + 5$$

a) What is the 10th triangular number?

b) Write the algebraic form of the sequence of triangular numbers

c) Which is the largest two digit triangular number?

d) Find the 50 th triangular number

a) $T_{10} = 1 + 2 + 3 \cdots + 10 = (10 + 1)\frac{10}{2} = 55$

b) $(n + 1) \times \frac{n}{2}$

c) $T_{13} = (13 + 1) \times \frac{13}{2} = 91$

d) $T_{50} = (50 + 1) \times \frac{50}{2} = 1275$

28) Look at the pattern given below

$$
\begin{array}{ccccc}
 & & 1 & & \\
 & 2 & 3 & 4 & \\
5 & 6 & 7 & 8 & 9
\end{array}
$$
------------------------------

a) Write the sequence of number of numbers in each line

b) Write the algebraic form of this sequence

c) How many numbers are there in 20 th line ?

d) Which number comes in the right end of 20 th line ?

d) Which number comes in the left end of 20 th line ?

e) Calculate the sum of all numbers upto the end of 20 th line

**Answers**

a) $1, 3, 5, 7 \cdots$

b) $x_n = 2n - 1$

c) $20 = 2 \times 20 - 1 = 40 - 1 = 39$

d) $20^2 = 400$
(Sequence of numbers in the right end : $1, 4, 9, 16 \cdots$)

e) $19^2 + 1 = 362$

f) $1, 2, 3, 4 \cdots 400$ Sum of numbers $: = (400 + 1) \times \frac{400}{2} = 80200$

29) The first term of an arithmetic sequence is $3$ and common difference $2$.

a) Write the sequence

b) How many times common difference should be added to its first term to its tenth term

c) What is the tenth term of this sequence?

d) What is the $101$ th term of this sequence?

e) Is $100$ a term of this sequence ? How can we realize it?

**Answers**

a) $3, 5, 7, 9 \cdots$

b) $9$ times common difference should be added

c) $x_{10} = 3 + 9 \times 2 = 3 + 18 = 21$

d) $x_{101} = 3 + 100 \times 2 = 203$

e) All terms are odd numbers . Even number $100$ cannot be the term

30) $24, \square, 34, 39, \square \cdots$ is a number sequence .If it is an arithmetic sequence

a) What is its common difference

b) Write two missing terms of this sequence

c) Write three more terms of this sequence

d) What should be a added to the first term to get its tenth term

e) Write the algebaric form of this sequence

a) $2d = 34 - 24 = 10$
$d = 5$

b) Number in the first box is $\boxed{29}$. Number in the second box $39 + 5 = 44$

c) $49, 54, 59$

d) $9$ common difference$= 9 \times 5 = 45$should be added

e) $x_n = dn + (f - d) = 5n + 19$

31) 15 th term of an arithmetic sequence is $40$ഉം and its $20$ th term is $60$

a) What is the common difference?

b) What is the first term?

c) What is the tenth term of this sequence?

d) Write the $n$ th term or algebraic form of this sequence

a) $5d = 60 - 40 = 20, d = 4$

b) $x_1 = x_{15} - 14 \times 4 = 40 - 14 \times 4 = 40 - 56 = -16$

c) $x_{10} = f + 9d = -16 + 9 \times 4 = -16 + 36 = 20$

d) $x_n = dn + (f - d) = 4n - 20$

# Chapter 2:

# CIRCLES

**Focus Points**

* Angle in the semicircle is $90°$

* Angle outside the semicircle is less than $90°$

* Angle inside the semicircle is greater than $90°$

* An arc of a circle can make three type of angles. Angle on the arc, angle at the centre and angle in the complement

* Angle formed by the arc in the complement is half the angle at the centre

* Sum of the angles at the centre and in the complement is $180°$

* Angles on an arc are equal

* If the vertices of a qudrilateral are on a circle we call it cyclic quadrilateral.

* The sum of the opposite angles of a cyclic quadrilateral is $180°$.

* The converse of the above statement is also true. If the sum of the opposite angles of a quadrilateral is $180°$ it will be a cyclic quadrilateral.

* Square, rectangle and isosceles trapezium are cyclic .

* Two chords of a circle $AB$ and $CD$ intersect at the point $P$ inside the circle .It can be proved that $PA \times PB = PC \times PD$

* This relation can be used to construct a rectangle having equal area of another rectangle.

* If the chords intersect outside the circle ,the same relation holds. $PA \times PB = PC \times PD$

* In the case of the intersecting chords of a circle,if one chord $AB$ is the diametre of the circle and other chord $CD$ is perpendicular to the diametre ,then $PA \times PB = PC^2$

* This relation is used to construct a square with same area of a rectangle.It can be used to draw the lines of irrational lengths.

1) $ABC$ is a triangle in which $AB = AC = BC$
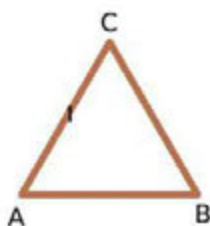
a) What are the measure of its angles?

b) What is the position of the vertex $C$ based on the circle with diametre $AB$? Is it inside, outside or on the circle.

**Answers**

a) $60°$

b) Outside the circle

2) In the figure $AB$ is the diametre of a semicircle. Three angles $x, y, z$ are marked outside, on the semicircle and inside the semicircle.
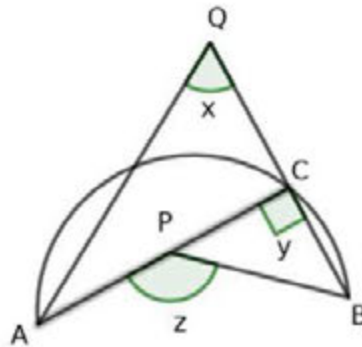


a) What is the value of $y$?

b) If $x, y, z$ are in an arithmetic sequence, then what is $x + z$?

c) If the common difference of the sequence is $50$ then find $x$ and $z$

**Answers**

a) $y = 90°$

b) $x + z = 2 \times 90 = 180°$ (Refer the propery of arithmetic sequence)

c) $d = 50 \therefore x = 90 - 50 = 40°, z = 90 + 50 = 140°$

3)    a) Draw a circle of radius 3cm. Construct a square with vertices are on the circle.

b) What is the length of its side?

c) Calculate the area of the square.

**Answers**

a) Draw a circle of radius 3cm and diametre $AB$.

Draw another diametre $CD$ perpendicular to $AB$. Join the end points of the diametre $ACBD$ will be a square

b) Side of the square $AC = \sqrt{3^2 + 3^2} = 3\sqrt{2}$cm.

c) Area $= 3\sqrt{2} \times 3\sqrt{2} = 18$sq.cm

4) Triangle $OAB$ is an equilateral triangle
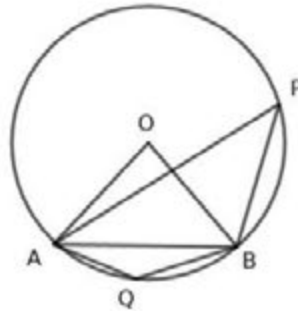


a) What is the measure of angle $AOB$?

b) What is the measure of angle $APB$?

c) What is the measure of angle $AQB$?

a) $\angle AOB = 60°$

b) $\angle APB = \frac{1}{2} \times 60° = 30°$

c) $\angle AQB = 180 - 30 = 150°$

5) Draw a circle of radius 3cm.Construct the angles $30°$ and $150°$ with vertices on the circle using compasses and scale only.

★ Draw a circle of radius 3cm.Mark the center of the circle as $O$

★ Mark a point $A$ on the circle.Draw the radius $OA$.

★ With $A$ as the centre and $OA$ as radius , draw an arc which cut the circle at $B$. Join $OB, \angle AOB = 60°$

★ Mark a point $P$on the complement of the arc $AB$,which makes $60°$ at the center.$\angle APB = \frac{1}{2} \times 60 = 30°$

★ Mark a point $Q$ on the arc $AB$. $\angle AQB = 180 - 30 = 150°$

6) In the figure $BDE = 40°$
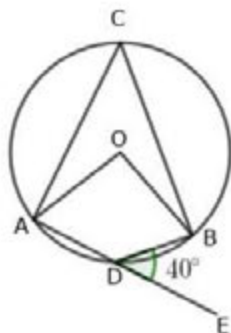


a) What is the measure of $\angle ADB$ ?

b) What is the measure of $\angle ACB$?

c) What is the measure of $\angle AOB$

1+1+1

**Answer**

a) $\angle ADB = 180 - 40 = 140°$

b) $\angle ACB = 180 - 140 = 40°$

c) $\angle AOB = 2 \times 40 = 80°$

7) $ABCD$ is a square .The diagonals $AC$ and $BD$ intersect at $O$.
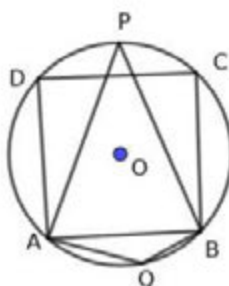


a) What is the measure of angle $AOB$?

b) What is the measure of angle $APB$?

c) What is the measure of angle $AQB$

4

a) Diagonals of a square are perpendicular to eachother. $\angle AOB = 90°$

b) $\angle APB = 45°$

c) $\angle AQD = 180 - 45 = 135°$

8) $OABC$ is a parallelogram. Three vertices are on a circle and one at the centre. $P$ is a point on the circle
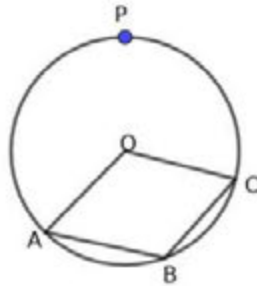


a) Draw $AP$ and $CP$, mark the angle $APC = x$. What is $\angle AOC$

b) What is angle $ABC$?

c) Find $x$

d) Find the angles of the parallelogram

a) $\angle AOC = 2x$

b) $\angle ABC = 2x$ Opposite angles of a parallelogram are equal

c) $\angle APC + \angle ABC = 180° \, x + 2x = 180, 3x = 180, x = 60$

d) Angles are $120°, 60°, 120°, 60°$

9) In the figure $O$ is the centre of the circle, $\angle BAO = 20°$, $\angle BCO = 10°$
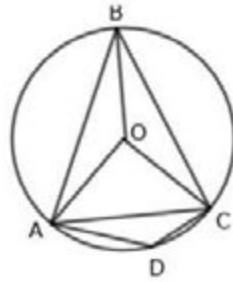


a) What is the measure of angle $ABC$?

b) What is the measure of angle $AOC$?

c) What is the measure of angle $ADC$?

d) Find the angles of triangle $AOC$

e) If the diametre of the circle is $10$cm then find the length of the chord $AB$

**Answers**

a) In triangle $OAB$, $OA = OB$. Angles opposite to the equal sides are equal. Similarly in the case of triangle $OBC$ also.
$\angle ABC = 20 + 10 = 30°$

b) $\angle AOC = 2 \times 30 = 60°$

c) $\angle ADC = 180 - 30 = 150°$

d) Triangle $AOC$, $OA = OC$, $\angle OAC = \angle OCA = \frac{180-60}{2} = 60°$ $\triangle OAC$ is an equilateral triangle. Angles are $60°$ each.

e) $OA = AC = OC = 5$cm, radius $5$cm.

10) In the figure $O$ is the centre of the circle, $\angle OAC = 45°$ then
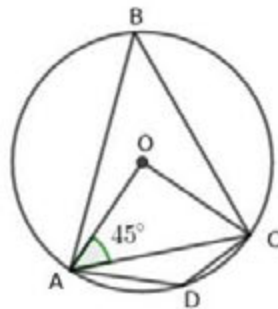


a) What kind of triangle is $OAC$?

b) What is the measure of angle $ABC$?

c) What is the measure of angle $ADC$?

d) If the radius of the circle is 6cm then what is the length of the chord $AC$.

---

**Answers**

a) $OA = OC, \angle OAC = \angle OCA = 45°, \angle AOC = 90°$. $\triangle OAC$ is an isosceles right triangle

b) $\angle ABC = \frac{1}{2}AOC = 45°$

c) $\angle ADC = 180 - 45 = 135°$

d) $AC = \sqrt{6^2 + 6^2} = 6\sqrt{2}$cm

---

11) Draw a circle of radius 3cm , construct an equilateral triangle with vertices on the circle. What is the length of the side?

---

**Answers**

⋆ Draw a circle with centre $O$ and radius 3cm. Mark a point $A$ on the circle and radius $OA$.

⋆ Divide the angle around the centre $120°$ each and mark the points $B, C$ on the circle

⋆ Draw triangle $ABC$ .

⋆ Length of side $= 3\sqrt{3}$cm

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12) $O$ is the centre of the circumcircle of triangle $ABC$.
If $\angle BAC = y, \angle OBC = x$ then
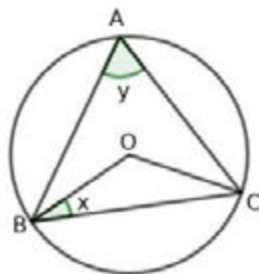


a) What is the measure of $\angle BCO$?

b) What is the measure of $\angle BOC$?

c) Prove that $x + y = 90°$

---

**Answers**

a) Since $OB = OC$ opposite angles of these sides in triangle $OBC$ are equal.
$\angle BCO = x$.

b) $\angle BOC = 180 - 2x$

c) We know that $\angle BOC = 2 \times \angle BAC$
$180 - 2x = 2y, 2x + 2y = 180, x + y = 90°$

---

13) In triangle $ABC$, $\angle A = 70°$, $\angle B = 80°$. The vertices of the triangle are on the circumcircle of the triangle .Radius of the circumcircle is $3$cm.Construct the triangle.

> **Answers**
>
> ★ Draw a circle of radius $3$ , mark its centre as $O$ and a point $A$ on the circle. Draw the radius $OA$
>
> ★ Mark a point $B$ on the circle such that $\angle AOB = 2 \times 70 = 140°$
>
> ★ Mark a point $C$ on the circle such that $\angle BOC = 2 \times 80 = 160°$
>
> ★ Draw triangle $ABC$ .

14) $P$ and $Q$ are the centre of the circles shown in the figure. Circles intersect at $B$ and $C$.If $\angle AQB = 130°$ then
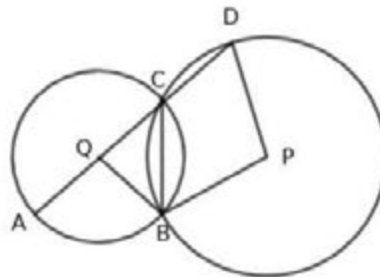


a) What is the measure of $\angle ACB$?

b) What is the measure of $\angle BCD$?

c) What is the measure of $\angle BPD$

> **Answers**
>
> a) $\angle ACB = \frac{1}{2} \times 130 = 65°$
>
> b) $\angle BCD = 180 - 65 = 115°$
>
> c) The central angle of the complement of the arc$BCD$ is $2 \times 115 = 230°$.Therefore $\angle BPD = 360 - 230 = 130°$

15) In the figure $\angle BAC = x, \angle CBO = y, O$ is the centre of the circle.
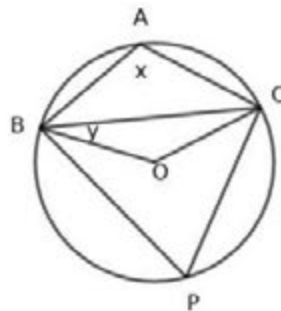
a) What is the measure of $\angle BCO$?

b) What is the measure of $\angle BOC$?

c) What is the measure of $\angle BPC$?

d) Prove that $x - y = 90°$?

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**Answers**

a) $\angle BCO = y$

(In triangle $BCO$, $OB = OC$. Angles opposite to equal sides are equal.)

b) $\angle BOC = 180 - 2y$

c) $\angle BPC = \frac{1}{2} \times \angle BOC = \frac{1}{2} \times (180 - 2y) = 90 - y$

d) $\angle BAC + \angle BPC = 180$, $x + 90 - y = 180$, $x - y = 180 - 90 = 90°$

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16) In the figure $O$ is the centre of the circle .If $\angle BCO = 55°$, $\angle BAO = 20°$ then
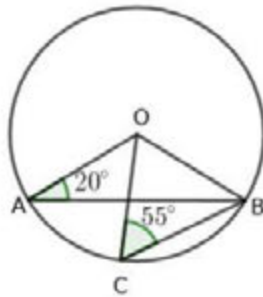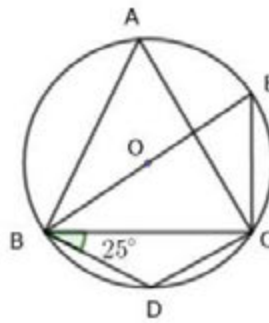


a) What is the measure of $\angle OBC$?

b) What is the measure of $\angle BOC$?

c) What is the measure of $\angle AOC$?

d) What is the measure of $\angle ABC$?

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**Answers**

a) $\angle OBC = 55°$

(In triangle $OBC$, $OB = OC$. Angles opposite to equal sides are equal.)

b) $\angle BOC = 180 - (55 + 55) = 180 - 110 = 70°$

c) In triangle $AOB$, $\angle B = 20°$, $\angle ACB = 180 - 40 = 140°$

$\angle AOC = 140 - 70 = 70°$

d) $\angle ABC = \frac{1}{2} \times 70 = 35°$

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17) In the figure $O$ is the centre of the circle.If $\angle BAC = 32°$ then
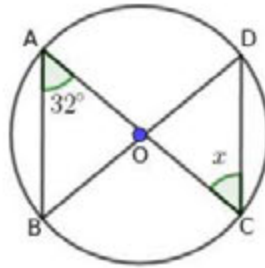
a) Find the angles of triangle $OAB$

b) What is the measure of $\angle DOC$?

c) Find $x$

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**Answers**

a) In triangle $OAB, OA = OB$. Therefore $\angle B = 32°, \angle AOB = 180 - 64 = 116°$

b) $\angle DOC = 116°$
(Opposite angles are equal)

c) In triangle $OCD, \angle D = x$
$x + x + 116 = 180, 2x = 64, x = 32$

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18) This is the picture of a clock face.$1, 8, 5$ are joined to make a triangle.Find the angles of this triangle.



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**Answers**

★ The central angle of the arc in between two numbers(say $1, 2$) is $\frac{1}{12} \times 360 = 30°$.
The central angle of the arc between $1$ and $5$ is $4 \times 30 = 120°$.
The angle of the triangle with vertex at $8$ is $\frac{1}{2} \times 120 = 60°$

★ The central angle of the arc between $8$ and $5$ is $3 \times 30 = 90°$.
The angle of the triangle with vertex at $1$ is $\frac{1}{2} \times 90 = 45°$

★ The central angle of the arc between $8$ and $1$ is $5 \times 30 = 150°$.
The angle of the triangle with vertex at $5$ is $\frac{1}{2} \times 150 = 75°$

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19) In the figure $PQRS$ is a cyclic quadrilateral.$\angle P = x, \angle Q = y, \angle R = 3x, \angle S = 5y$.
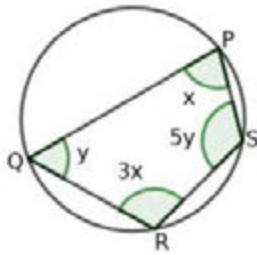
a) Find $x$ and $y$

b) Find the angles of the quadrilateral.

a) $\angle P + \angle R = 180°$
   $x + 3x = 180, 4x = 180, x = 45$
   $\angle Q + \angle S = 180°, y + 5y = 180, 6y = 180, y = 30$

b) $\angle P = 45°, \angle R = 3 \times 45 = 135° \; \angle Q = 30°, \angle S = 5 \times 30 = 150°$

20) $ABCD$ is a cyclic quadrilateral. $AB$ is the diametre of the circle , $AD = CD$ and $\angle ADC = 130°$.
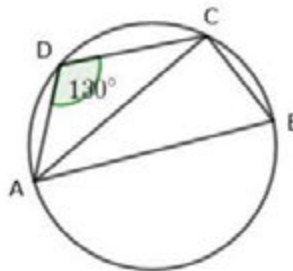


a) What is the measure of $\angle ACB$?

b) What is the measure of $\angle ABC$?

c) Find $\angle DCB$.

d) What is the measure of $\angle BAD$?

a) $\angle ACB = 90°$(Angle in the semicircle)

b) $\angle ABC = 180 - 130 = 50°$

c) Since $CD = AD$,the angles opposite to the equal sides of triangle $ADC$ are equal.
   $\angle DCA = 25°, \angle DCB = 90 + 25 = 115°$

d) $\angle BAD = 180 - 115 = 65°$

21) In the figure $BD = CD, \angle DBC = 25°$



    a) What is the measure of $\angle BDC$?

    b) What is the measure of $\angle BAC$?

    c) What is the measure of $\angle EBC$?

**Answers**

    a) In triangle $BDC, BD = CD$. Angle opposite to these sides are equal $.\angle BCD = 25°$

        $\angle BDC = 180 - (25 + 25) = 130°$

    b) $\angle BAC = 180 - 130 = 50°$

    c) $\angle BEC = \angle BAC = 180 - 130 = 50°, \angle EBC = 180 - (90 + 50) = 180 - 140 = 40°$

22) In the figure $ABCD$ is a quadrilateral in which $AB$ is parallel to $CD$ and $AD = BC$
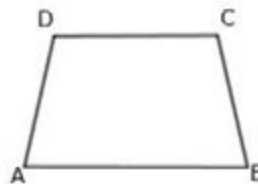


Prove that $ABCD$ is a cyclic quadrilateral.

**Answers**

    ★ The line $AB$ is parallel to $CD$. Therefore $\angle A + \angle D = 180°$

    ★ Since $AD = BC$ then $ABCD$ is an isosceles trapezium $\angle A = \angle B$

    ★ Therefore $\angle B + \angle D = 180°$. $ABCD$ is a cyclic quadrilateral .

23) $C, D$ are two points in a semicircle of diametre $AB$.
    If $\angle BAD = 70°, \angle DBC = 30°$ then
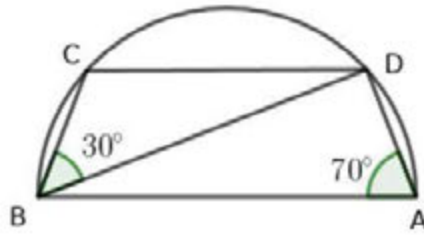
a) What is the measure of $\angle BCD$?

b) What is the measure of $\angle CDB$?

c) What is the measure of $\angle ADC$?

d) What is the measure of $\angle ABD$?

---

**Answers**

a) $\angle BCD = 180 - 70 = 110°$

b) $\angle CDB = 180 - (30 + 110) = 180 - 140 = 40°$

c) $\angle ADC = \angle ADB + \angle BDC = 90 + 40 = 130°$

d) $\angle ABD = 180 - (90 + 70) = 180 - 160 = 20°$

---

24) The parallelogram which is not a rectangle is not cyclic. Justify this statement

**Answers**

★ $ABCD$ is a parallelogram(Draw a parallelogram and name it as $ABCD$ in an order. $\angle A = \angle C$

★ Since $ABCD$ is not a rectangle $\angle A \neq 90°, \angle C \neq 90°$

★ $\angle A + \angle C \neq 180°$.
∴ $ABCD$ is not cyclic

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25) In the figure $ABCDE$ is a regular pentagon.Prove that $ABCE$ is a cyclic quadrilateral.
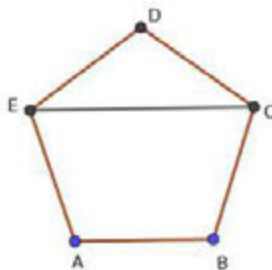
26) In the figure the chord $AB$ has length 8cm and $OA = 5$cm.
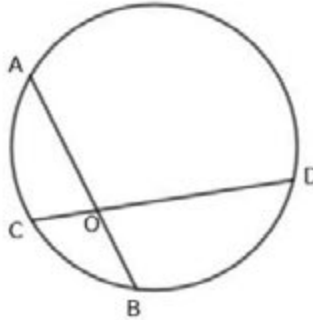


a) What is the length of $OB$?

b) If $OC = 2.5$cm, what is the length of $OD$?
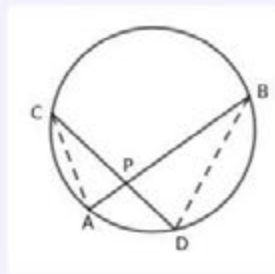
27) The chords $AB$ and $CD$ intersect at $O$ .This point divide each chord into two segments
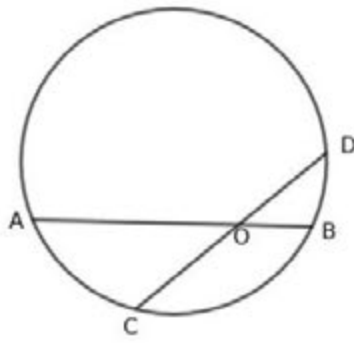
a) What is the relation between these segments?

b) If $CD = 10$cm and $OD = 4$cm then what is the length $OC$?

c) If $OA = 8$cm, $OC = 6$cm and $OD = 4$cm then what is the length $OB$?

**Answers**

a) $OA \times OB = OC \times OD$

b) $OC = CD - OD = 10 - 4 = 6$ cm

c) $8 \times OB = 6 \times 4, OB = 3$cm

28) The chords $AB$ and $CD$ intersect at $P$ outside the circle.
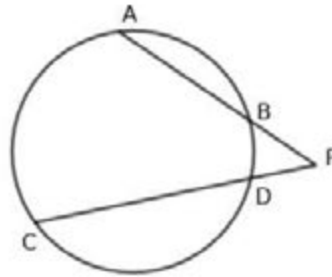


a) What is the relation between $PA, PB, PC$ and $PD$?

b) If $AB = 5$cm , $PB = 3$cm , $PD = 2$cm then what is the length $CD$?

**Answers**

a) $PA \times PB = PC \times PD$

b) $(5 + 3) \times 3 = (PD + CD) \times PD$
   $(5 + 3) \times 3 = (2 + CD) \times 2$
   $2 + CD = 12, CD = 10$cm

29) $AB$ is the diametre of a semicircle, $P$ is a point on $AB$ and $PC$ is perpendicular to $AB$
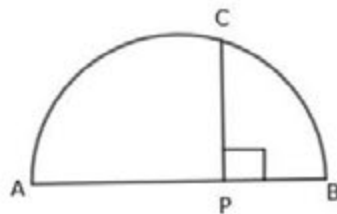


a) Prove that $PA \times PB = PC^2$

b) If $PA = 9$cm , $PB = 4$ cm then what is the length $PC$?

c) What is the area of the square with side $PC$?

**Answers**

a) Draw $AC, BC$.Consider triangle $APC$ and triangle $BPC$.If $\angle PAC = x$ then
   $\angle PCA = 90 - x, \angle PCB = 90 - (90 - x) = x, \angle PBC = 90 - x$
   These are similar triangles .Sides opposite to the equal angles are proportional .
   $\dfrac{PC}{PB} = \dfrac{PA}{PC}$
   $PA \times PB = PC^2$.

b) $PC^2 = 9 \times 4 = 36, PC = 6$cm

c) Area $PC^2 = 36$sq.cm

16

30) ]$AB$ is the diametre of a semicircle, $P$ is a point on $AB$ and $PC$ is perpendicular to $AB$
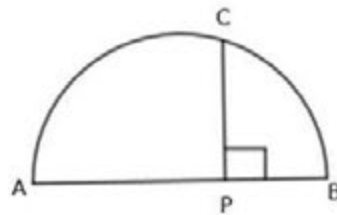


a) If $PC = 6$cm,and $PB = 3$cm then what is the length of $PA$

b) What is the radius of the circle ?

c) What is the area of the square drawn with side $PC$?

**Answers**

a) $PA \times PB = PC^2$
   $PA \times 3 = 6^2, PA = 12$cm

b) $AB = 12 + 3 = 15$cm ,Radius 7.5cm.

c) Area $PC^2 = 36$sq.cm

31) In the figure $AB$ is the diametre of the circle and $PC$ is perpendicular to the diametre. $PA : PB = 2 : 1$ and $PC = 6$cm.
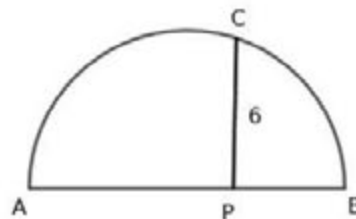


a) Write the relation between $PA, PB$ and $PC$?

b) Find the lengths $PA$ and $PB$

c) What is the radius of the circle?

**Answers**

a) $PA \times PB = PC^2$

b) If $PB = x$ ,$2x \times x = 6^2$,$2x^2 = 36, x^2 = 18, x = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$
   $PA = 6\sqrt{2}, PB = 3\sqrt{2}$

c) $AB = 6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$
   Radius $= \frac{9\sqrt{2}}{2}$ cm

32) Draw a rectangle of sides $5$cm and $3$cm .Construct a square whose area is same as the area of the rectangle

**Answers**

★ Draw the quadrilateral $ABCD$. $AB = 5$cm , $BC = 3$cm.

★ Produce $AB$ and mark the point $E$ such that $BC = BE$

★ Draw a semicircle of diametre $AE$.Produce $BC$, meet the semixircle at $F$.

★ $BA \times BE = BF^2$ can be written as $AB \times BC = BF^2$. $AB \times BC$ is the area of the rectangle

★ Draw a square of side $BF$.Area of rectangle is equal to the area of the square as per the relation $AB \times BC = BF^2$

33) Draw an equilateral triangle of one side $\sqrt{18}$cm

★ $18 = 6 \times 3, 6 + 3 = 9$
  Draw a line $AB$of length 9cm

★ Draw a semicircle with diameter $AB$. Mark a point $P$ at the diatance 6cm from $A$. Draw a perpendicular from $P$ to $AB$. This line cut the circle at $C$. $PC = \sqrt{18}$.

★ Draw an equilateral triangle with $PC$ as side

1

# Chapter 3:

# MATHEMATICS OF CHANCE

**Focus Points**

★ Probability or mathematical chance is measured as the ratio of favourable outcome and possible outcome .

★ The experiments whose resut or outcome cannot be predected are called probability experiment

★ Probability can be measuered as the ratio of areas in the calculation of the probability relating to the area of geometric figures . Here facourable outcome is the overlapping area and the possible outcome is the larger area.

★ A square is drawn inside a circle.A fine dot is placed without looking into the figure. The probability of falling the dot into the square is the ratio of the area of the square to the area of the circle.

1) A vessel contains 3 black beads and 2 white beads. One is taken from the vessel without looking into the vessel.

   a) What is the probability of getting black bead?
   b) What is the probability of getting white bead?

   **Answers**

   a) Probability of getting black bead $= \frac{3}{5}$

   a) Probability of getting white bead$= \frac{2}{5}$

2) A box contains $10$ cards on which one of the numbers $1, 2, 3 \cdots 10$ is written in each card.One card is taken from the box at random.

   a) What is the probability of getting a an even numbered card
   b) What is the probability of getting an odd numbered card?
   c) What is the probability of getting a card on which a prime number is written ?
   d) What is the probability of getting a perfect square on the card.

   **Answers**

   a) Probability of egtting even $= \frac{5}{10}$

   b) Probability of getting odd$= \frac{5}{10}$

   c) Probability of getting prime number$= \frac{4}{10}$
      Prime number 2, 3, 5, 7

   d) Probability of getting perfect square$= \frac{3}{10}$
      Per feet square 1, 4, 9

3) Each of the numbers from $1$ to $100$ are written on small paper pieces .One is taken from the card at random.

   a) How many perfect squared cards are there in the box?
   b) What is the probability of getting a perfect squared card?
   c) What is the probability of getting an even perfect squred card

d) What is the probability of getting an odd perfect squared card?

e) What is the probability of not getting a perfect numbered card?

4) A die in which the numbers 1 to 6are written on the faces is thrown

   a) What is the probability of falling an even numbered face?

   b) What is the probability of getting an odd numbered face ?

   c) What is the probability of getting a prime numbered face?

5) Two digit numbers are written in small paper pieces and placed in a box.One is taken from the box at random

   a) How many multiples of $5$ are there in the box?

   b) What is the probability of getting a multiple of 5?

   c) What is the probability of not getting a multiple of 5?

6) Numbers $1, 2, 3 \cdots 17$ are written in small paper cards and placed in a box.One card is taken from the box at random.

   a) What is the probability of getting odd numbered card?

   b) What is the probability of getting prime numbered card?

   c) What is the probability of getting a multiple of $3$?

   d) What is the probability of getting a multiple of $2$ and $3$?

7) A die numbered 1 to 6 are thrown.

   a) What is the probability of falling a number less than 4?

   b) What is the probability of getting a multiple of 2?

   c) What is the probability of falling a multiple of both 2 and 3

   d) What is the probability of not falling a prime number?

   a) $\frac{3}{6}$

   b) $\frac{3}{6}$

   c) $\frac{1}{6}$

   d) $\frac{3}{6}$

8) What is the probability of getting 5 Mondays in the month December?

There are 31 days in December. 28days decide 4weeks ,so four Mondays.
The combinations are(Sunday, Monday,Tuesday),(Monday,Tuesday ,Wednesday),(Tuesday, Wednesday ,Thursday ),
(Wednesday, Thursday , Friday),(Thursday ,Friday ,Saturday),(Friday, Saturday,Sunday ),
(Saturday,Sunday, Monday) .
There are three combinations in which Moday occur.Probability of occuring five Mondays is $\frac{3}{7}$

9) From all two digit numbers with either digit 1, 2, 3 or 4 are written in small paper card and placed in a box.

   a) How many cards are there in the box?

   b) If one card is taken from the box at random , what is the probability of getting an even numbered card?

   c) What is the probability of getting an odd numbered card?

   d) What is the probability of getting a card with equal digits?

   a) Numbers are
      $11, 12, 13, 14$
      $21, 22, 23, 24$
      $31, 32, 33, 34$
      $41, 42, 43, 44$
      Total number $= 16$

   b) Eight of them are even. Probability of getting even is $\frac{8}{16} = \frac{1}{2}$

   c) Eight of them are odd . Probability of getting odd is $\frac{8}{16} = \frac{1}{2}$

   d) $11, 22, 33, 44$ are the numbers with equal digits .Probability $\frac{4}{16} = \frac{1}{4}$

10) Two digit numbers are written in small paper pieces and placed in a box.

   a) How many paper slips are there in the box?

   b) If one is taken from the box, what is probability of getting a number with digits same?

   c) If one is taken from the box, what is probability of getting a number in which the product of the digits a prime number.

   d) What is the probability of getting a prime number?

a) $10, 11, 12 \cdots 99$ are two digit numbers . There are 90 such numbers

b) Numbers with same digits are $11, 22, 33, 44, 55, 66, 77, 88, 99$
Total number of these numbers is 9
Probability= $\frac{9}{90} = \frac{1}{10}$

c) In the two digit numbers with product of the digits a prime , one digit is 1 and other digit is one of the numbers $2, 3, 5, 7$
Numbers are $12, 13, 15.17, 21, 31, 51, 71$.
Probability $= \frac{8}{90}$

d) There are 25 prime numbers below 100. 4 of them are one digit primes and the rest of the 21 numbers are two digit primes .Probability is $= \frac{21}{90}$

11) Two dices numbered $1, 2, 3, 4, 5, 6$ are thrown together.The outcome faces are written as pairs.

a) How many pairs are there ?

b) Make the list of pairs with sum 2 , 3 ,4 , 5 and 6 seperately

c) What is the probability of occuring the maximum sum

a) number of pairs $6 \times 6 = 36$

b) $(1, 1) \rightarrow$ sum $= 2$
$(1, 2), (2, 1) \rightarrow$ sum $= 3$
$(1, 3), (2, 3), (3, 1) \rightarrow$ sum$= 4$
$(1, 4), (4, 1), (2, 3)(3, 2) \rightarrow$ sum $= 5$
$(1, 5), (5, 1), (2, 4)(4, 2), (3, 3) \rightarrow$ sum $= 6$
$(1, 6), (6, 1), (2, 5)(5, 2), (3, 4), (4, 3) \rightarrow$ sum$= 7$
Sums are $2, 3, 4, 5, 6, 7 \cdots 12$.Largest is 12.

c) Proceeding like this there is only 1 pair with the largest sum 12.Probability $= \frac{1}{36}$

12) Two dice numbered 1 to6 are thrown together.

a) Write the outcomes as pairs

b) What is the probability of the occurence of equal numbers ?

c) What is the probability of the occurence of perfect squares ?

d) What is the probability of the occurence of multiple of 2 in one die and multiple of 3 in other die ?

a) $(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$
$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$
$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$
$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$
$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$
$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)$

b) $\frac{6}{36}$

c) $(1, 1), (1, 4), (4, 1), (4, 4)$.
Probability$\frac{4}{36}$

d) $(2, 3), (4, 3), (6, 3), (2, 6), (4, 6), (6, 6), (3, 2), (3, 4), (3, 6), (6, 2), (6, 4)$
Probability$\frac{11}{36}$

13) There are two circles in the picture.One is inside other.Radius of the small circle is half of the radius of the big circle.
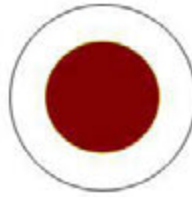


a) If the radius of the small circle is $r$ then what is the area of the small circle and big circle ?

b) If a fine dot is placed into the figure, what is the probability of falling the dot in the small circle?

c) What is the probability of falling the dot the white shaded part in the figure.

**Answers**

a) Area of small circle $\pi r^2$
   Area of big circle$\pi \times (2r)^2 = 4\pi r^2$

b) Probability of falling the dot in the small circle is $\frac{\pi r^2}{4\pi r^2} = \frac{1}{4}$

c) Praobability of falling the dot in the yellow shaded part is $1 - \frac{1}{4} = \frac{3}{4}$

14) Triangle $PQR$ is drawn by joining the mid points of the sides of triangle $ABC$.
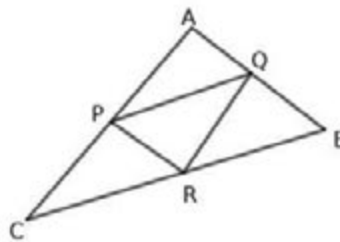


a) How many equal triangles are there in the figure?

b) A fine dot is placed into the figure. What is the probability of falling the dot in triangle $PQR$?

c) How many parallelograms are there in the picture?

d) A fine dot is placed into the figure. What is the probability of falling the dot in the parallelogram $PQRC$?

**Answers**

a) There are 4 equal triangles .
   $\triangle PQR, \triangle APQ, \triangle PCR, \triangle QRB$ are .

b) $\frac{1}{4}$ (Area of equal triangles are equal)

c) There are 3 parallelograms .
   $PQRC, PQBR, PRQA$ are equal parallelograms .

d) To fall the dot in the parallelogram $PQRC$ it should either be in triangle $PCR$ or triangle $PQR$
   Probability is $\frac{2}{4} = \frac{1}{2}$

15) A square is drawn in a circle . The vertices of the square are on the circle. A fine dot is placed into the figure at random. What is the probability of falling the dot in the shaded square.
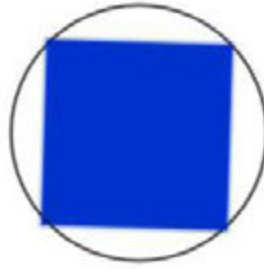
* If one side of the square $a$ the diagonal is $d = \sqrt{a^2 + a^2} = \sqrt{2}a$
  Radius of the circle is $\frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$

* Area of the square $a^2$, area of the circle $\pi \times (\frac{a}{\sqrt{2}})^2 = \frac{\pi a^2}{2}$

* Probability of falling the dot in the shade is $= a^2 \div \frac{\pi a^2}{2} = \frac{2}{\pi}$

16) $O$ is the center of the circle of diametre $AB$.
There is another circle with diametre $OB$. If $r$ is the radius of the small circle
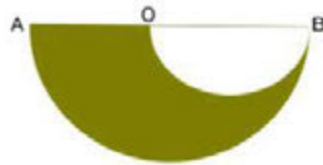


a) What is the radius of the big circle ?

b) Find the area of big circle and small circle.

c) If a fine dot is placed into the figure what is the probability of falling the dot in the shaded part.

a) $2r$

b) Area of small semicircle $\frac{\pi r^2}{2}$
   Area of big semicircle $\frac{\pi (2r)^2}{2} = 2\pi r^2$

c) Area of coloured part $2\pi r^2 - \frac{\pi r^2}{2} = 3\frac{\pi r^2}{2}$
   Probability $= 3\frac{\pi r^2}{2} \div 2\pi r^2 = \frac{3}{4}$

17) A box contains three paper slips carrying numbers $2, 3, 4$ . Another box contains paper slips carrying fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. One is taken from each box at random

a) How many pairs are possible?

b) What is the probability of getting the product of numbers in each pair a natural number?

c) Wha is the probability of not getting the numbers in the pair whose product is not a natural number?

a) Number of pairs$= 3 \times 3 = 9$

$(2, \frac{1}{2}), (2, \frac{1}{3}), (2\frac{1}{4})$

$(3, \frac{1}{2}), (3, \frac{1}{3}), (3\frac{1}{4})$

$(4, \frac{1}{2}), (4, \frac{1}{3}), (4\frac{1}{4})$

b) Pairs with the product a natural number are $(2, \frac{1}{2}), (3, \frac{1}{3}), (4, \frac{1}{4}), (4, \frac{1}{2})$

There are four such pairs.

Probability of getting the product a natural number is $= \frac{4}{9}$

c) Probability of not getting the product a natural number is $1 - \frac{4}{9} = \frac{5}{9}$

18) Manju has three ornaments :Green , Red and Blue ear rings and chains.She ware it in different ways.

a) How many ways she can ware the ornaments?

b) What is the probability of waring ornaments of same colour?

c) What is the probability of wearing the ornaments of different colours?

a) Number of pairs $3 \times 3 = 9$

(Green, Green),(Green ,Red),(Green,Blus)

(Blue, Green),(Blue ,Red),(Blue ,Blue)

(Red, Green),(Red,Red),(Red,Blue)

b) (Green,Green),(Red,Red),(Blue,Blue)

Probability$= \frac{3}{9} = \frac{1}{3}$

c) Probability of wearing different colours is $1 - \frac{1}{3} = \frac{2}{3}$

19) A box contains $4$ black balls and $3$ white balls. Another box contains $5$ black balls and $3$ white balls. One from each box is taken at random.

a) How many pair of balls are possible ?

b) What is the probability of getting both balls black?

c) What is the probability of getting both balls white?

d) What is the probability of getting balls of different colours?

a) Total number of possible selections $= (3 + 4) \times (5 + 3) = 7 \times 8 = 56$

b) Probability of getting both black$\frac{4 \times 5}{56} = \frac{20}{56}$

c) Probability of getting both white$\frac{3 \times 3}{56} = \frac{9}{56}$

d) Probability of getting balls of different colours$\frac{(4 \times 3) + (3 \times 5)}{56} = \frac{27}{56}$

20) A box contains four paper slips carrying numbers $1, 2, 3, 4$.Another box contains paper slips carrying numbers $1, 2, 3$. One from each box is taken at random and entered as pairs.

a) How many pairs are possible ?

b) What is the probability of getting a pair with the product of the digits odd?

c) What is the probability of getting a pair with the product of the digits even?

a) Number of pairs $4 \times 3 = 12$

$(1,1), (1,2), (1,3)$
$(2,1), (2,2), (2,3)$
$(3,1), (3,2), (3,3)$
$(4,1), (4,2), (4,3)$

b) Pairs of getting product odd are $(1,1)(1,3), (3,1)(3,3)$
Probability $\frac{4}{12} = \frac{1}{3}$

c) Probability of getting product even $= 1 - \frac{1}{3} = \frac{2}{3}$

21) There are $30$boys and $20$girls in $10A$. There are $15$boys and $25$girls in $10B$.
One student is selected from each class at random.

a) How many ways the selections can be made ?

b) What is the probability of getting both boys?

c) What is the probability of getting both girls?

a) Total number of pairs $(20 + 30) \times (15 + 25) = 50 \times 40 = 2000$

b) Probability of selecting both boys$= \frac{30 \times 15}{2000} = \frac{450}{2000} = \frac{9}{40}$

c) Probability of getting both girls$= \frac{20 \times 25}{2000} = \frac{500}{2000} = \frac{1}{4}$

# Chapter 4:

# SECOND DEGREE EQUATIONS

## Focus Points

* ⋆ Equations of the form $ax^2 + bx + c = 0, a \neq 0$ are second degree equations. In this unit we dicuss varoius situations of forming second degree equations.

* ⋆ The values of $x$ satisfying the equation are called solutions of the equations.

* ⋆ There are many methods to solve the equation. Completing the square method is a suitable way to solve the second degree equation.

1) Form the equations in the following cases.

    a) The sum of a number and its square is $12$

    b) When a number is subtracted from its square results $20$

    c) The sum of the square of a number and two times that number is $63$

    d) Product of two consecutive odd numbers is $63$.

### Answers

    a) If the number is $x$ then $x^2 + x = 12$

    b) If the numebr is $x$ then $x^2 - x = 20$

    c) If the numebr is $x$ then $x^2 + 2x = 63$

    d) Numbers are $x, x+2$ then $x(x+2) = 63, x^2 + 2x = 63$

2) The square of a number is $16$.

    a) What are the numbers ?

    b) Take the number as $x$ and form an equation

    c) Can the square of a real number $-16$? Explain.

### Answers

    a) Numbers are $4, -4$

    b) If the number is $x$ then $x^2 = 16$

    c) No real number exist with its square a negative number. The square of $-4$ and the square of $+4$ is $16$.

3) $x$ is an odd number greater than 1.

    a) What are the odd numbers nearer to $x$

    b) If the product of those numbers is $45$, form an equation.

    c) Find the numbers.

> **Answers**
>
>     a) Odd number is $x$. The numbers nearer to it are $x-2, x+2$
>
>     b) $(x-2)(x+2) = 45$
>         $x^2 - 4 = 45, x^2 = 49$
>
>     c) $x = 7$
>         Numbers are $5, 9$

4) If the sides of a square are reduced by $1$, the area becomes $100$.

    a) If the side of the first square before reducing is $x$, form an equation.

    b) Find the side of the square.

    c) What will be the perimetre of the new square.

> **Answers**
>
>     a) If one side is $x$ then $(x-1)^2 = 100$
>
>     b) $x - 1 = \sqrt{100} = 10, x = 11$
>
>     c) Perimetre decreases by $4$. Perimetre of the new square is $40$

5) The chords $AB$ and $CD$ meet at a point $P$ inside the circle.
$CD = 21$cm, $PC = 5$cm.
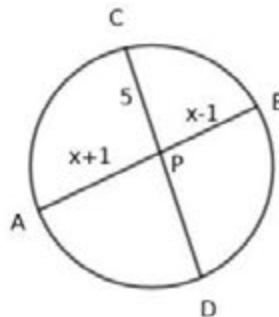


    a) What is $PD$?

    b) If $PA = x + 1$ and $PB = x - 1$ then form an equation

    c) Find the lenght $PA$ and $PB$.

> **Answers**
>
>     a) $PD = 21 - 5 = 16$cm
>
>     b) $PA \times PB = PC \times PD$
>         $(x+1)(x-1) = 5 \times 16 = 80$
>         $x^2 - 1^2 = 80, x^2 - 1 = 80$
>
>     c) $x^2 - 1 = 80 \rightarrow x^2 = 81, x = 9$
>
>         $PA = 9 + 1 = 10$cm, $PB = 9 - 1 = 8$cm

6) The product of two consecutive even numbers is $360$

    a) If the odd number in between these numbers is $x$ then write the numbers .

    b) Form an equation using the given condition.

    c) Find the numbers.

**Answers**

    a) Numbers are $x - 1, x + 1$

    b) $(x - 1)(x + 1) = 360, x^2 - 1 = 360$

    c) $x^2 = 361, x = \sqrt{361} = 19$. Numbers $19 - 1 = 18, 19 + 1 = 20$

7) Consider the arithmetic sequence $5, 9, 13, 17, 21 \cdots$.

    a) Write the algebraic form of this sequence.

    b) What is the position of the term in the sequence whose square is $625$?

    c) Is $36$ a term of this sequence . How can you realize it ?

    d) What is the position of $49$in this sequence ?

**Answers**

    a) $x_n = dn + (f - d) = 4n + (5 - 4) = 4n + 1$

    b) $(4n + 1)^2 = 625, 4n + 1 = \sqrt{625} = 25, 4n = 24, n = 6$

    c) All terms are odd numbers . The even number $36$cannot be a term of this sequence

    d) $4n + 1 = 49, 4n = 48, n = 12$.
    $12$ th term is $49$

8) Consider two adjacent even numbers

    a) If one of them is $x$ then what is the other?

    b) If the product is $120$ then write a second degree equation.

    c) Convert this equation as a completed square by suitable changes

    d) Find the numbers .

**Answers**

    a) $x + 2$

    b) $x(x + 2) = 120$
    $x^2 + 2x = 120$

    c) Add $1$on both sides $x^2 + 2x + 1 = 120 + 1$
    $(x + 1)^2 = 121$

    d) $x + 1 = \sqrt{121} = 11, 11, x + 1 = 11, x = 10$
    Even numbers are $10, 12$

9) Length of a rectangle is $8$ more then its breadth.

    a) If the breadth is $x$ then what is its length?

    b) If the area is $240$ sq.cm form a second degree equation.

    c) Calculate the lenght and breadth

a) Length $= x + 8$

b) $x(x + 8) = 240, x^2 + 8x = 240$

c) Add $(\frac{8}{2})^2$ on both sides. It is 16
$x^2 + 8x + 16 = 240 + 16$
$(x + 4)^2 = 256, x + 4 = \sqrt{256} = 16, x = 16 - 4 = 12$
Breadth 12 cm,Length $12 + 8 = 20$cm

10) In the figure $AB$ is the diametre of the semicircle. $AB$ is perpendicular to $PC$.Also,$AP = BP + 5$, $PC = 6$.
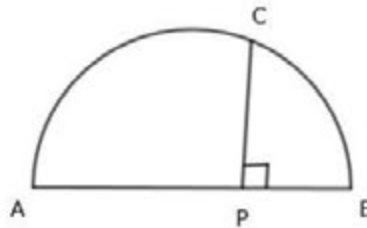


a) Write the relation between the lenghts $PA, PB$ and $PC$

b) If $PB = x$ then write an equation connecting the lenghts $PA, PB$ and $PC$

c) What is the length of $PB$?

d) What is the radius of this circle.

Answers

a) $PA \times PB = PC^2$

b) $(x + 5) \times x = 6^2, x^2 + 5x = 36$
$x^2 + 5x + (\frac{5}{2})^2 = 36 + (\frac{5}{2})^2$
$(x + \frac{5}{2})^2 = 36 + \frac{25}{4}$
$(x + \frac{5}{2})^2 = \frac{169}{4}$
$(x + \frac{5}{2}) = \sqrt{\frac{169}{4}} = \frac{13}{2}$
$x = \frac{13}{2} - \frac{5}{2} = 4$

c) $PB = 4$
$AP = 4 + 5 = 9, AB = 9 + 4 = 13$
Radius $= \frac{13}{2}$ cm

11) Consider the sequence of even numbers $2, 4, 6, 8 \cdots$.

a) What is its algebraic form?

b) How many terms from the beginning in the order makes the sum 210?

a) $x_n = 2n$

b) $n(n+1) = 210, n^2 + n = 210$
$n^2 + n + \frac{1}{4} = 210 + \frac{1}{4}$
$(n + \frac{1}{2})^2 = \frac{841}{4}$
$n + \frac{1}{2} = \sqrt{\frac{841}{4}} = \frac{29}{2}$
$n = \frac{29}{2} - \frac{1}{2} = 14$
The sum of the first $14$ even numbers is $210$

12) The smallest side of a right angled triangle is $4$ less than its hypotenuse. Third side is $2$ more than the smallest side.

    a) If the smallest side is $x$ what are the other two sides.

    b) Write an equation connecting the length of the sides .

    c) What is the length of the smallest side?

    d) Find the length of other sides of the triangle.

    a) If the smallest side is $x$ then hypotenuse is $= x + 4$, third side is $x + 2$

    b) $(x+4)^2 = (x+2)^2 + x^2, x^2 + 8x + 16 = x^2 + 4x + 4 + x^2$
       $x^2 - 4x - 12 = 0$

    c) $x^2 - 4x = 12, x^2 - 4x + 4 = 12 + 4$
       $(x-2)^2 = 16, x - 2 = 4, x = 6$
       Smallest side is $6$

    d) Sides are $6, 8, 10$

13) In traingle $ABC$, $AB = AC$
$AD$ is the perpendicular from $A$ to $BC$. This perpendicular distance from $A$ to $BC$ is $2$ cm more than $BC$. Area of the triangle is $60$ sq.cm
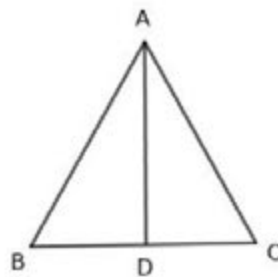


    a) If $BC = x$ then what is the langth $AD$?

    b) Form an equation connecting the lengths $BC, AD$ and area of the rectangle

    c) Find the length of $BC$.

    d) What is the lenghth of $AD$?

    e) Calculate the perimetre of the triangle $ABC$

a) $AD = x + 2$

b) $\frac{1}{2} \times x \times (x + 2) = 60$
$x(x + 2) = 120, x^2 + 2x = 120$

c) $x^2 + 2x + 1 = 121, (x + 1)^2 = 121, (x + 1) = \sqrt{121} = 11, x = 11 - 1 = 10cm$

d) $AD = 10 + 2 = 12$

e) $AB^2 = BD^2 + AD^2$
$AB^2 = 5^2 + 12^2 = 169, AB = \sqrt{169} = 13cm$
Perimetre $= 13 + 13 + 10 = 36cm$

14) Length of a rectangle is $4$ more than its breadth .Area of the rectangle is $357$ sq.cm

   a) If the breadth is $x$ then what is its length?

   b) Write an equation connecting length , breadth and area

   c) Find the lenghth and breadth of the rectangle .

a) Length $x + 4$

b) $x(x + 4) = 357, x^2 + 4x = 357$

c) $x^2 + 4x + 4 = 357 + 4 = 361, (x + 2)^2 = 361, x + 2 = \sqrt{361} = 19, x = 19 - 2 = 17$

d) Breadth $17cm$ ,length $17 + 4 = 21cm$

15) Hypotenuse of a right angled triangle is $1$ less than twice its small side.Third side is $1$ more than its small side

   a) If the small side is $x$ what is the length of other two sides .

   b) Form an equation connecting the length of the sides .

   c) Calculate the length of the sides of the triangle.

a) Hypotenuse $= 2x - 1$, Third side $= x + 1$

b) $(2x - 1)^2 = x^2 + (x + 1)^2, 4x^2 - 4x + 1 = x^2 + x^2 + 2x + 1$
$2x^2 - 6x = 0$

c) $x = 3$.Sides are :
Hypotenuse$2x - 1 = 6 - 1 = 5cm$.
Other two sides are $3cm$ ,$4cm$.

16) Two chords $AB$ and $CD$ intersect at $P$ inside the circle.If $AB = 13$cm
and $PC = 12$ cm ,$PD = 3$ cm
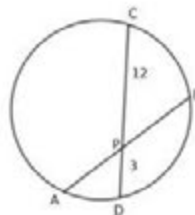
a) Write the relation between $PA, PB, PC$ and $PD$

b) If $PA = x$ form an equation

c) Find the lengths of $PA$ and $PB$

**Answers**

(a)     PA x PB = PC x PD

(b)     x(13-x) = 12 x 3

        $x^2$ - 13x = -36

(c)     $x^2 - 13x + \left(\frac{13}{2}\right)^2 = -36 + \left(\frac{13}{2}\right)^2$

        $\left(x - \frac{13}{2}\right)^2 = \frac{25}{4}$

        $x - \frac{13}{2} = \frac{5}{2}$

        x = 9    PA = 9    PB = 4

17) The sum of the first $n$ terms of the arithmetic sequence $7, 9, 11, 13 \cdots$ is $40$.

a) Form a second degree equation using this condition.

b) How many terms make the sum $40$ ?

c) Find $n$ in another method .

**Answers**

(a)   xn = dn + (F-d) = 2n + 5

     $\frac{n}{2}[x_1 + x_n] = 40$

     $\frac{n}{2}[7 + 2n + 5] = 40$

     $n^2 + 6n = 40$

(b)   $n^2 + 6n + \left(\frac{6}{2}\right)^2 = 40 + \left(\frac{6}{2}\right)^2$

     $(n + 3)^2 = 4$

     $n + 3 = 7$

     $n = 4$

(c)   7 + 9 + 11 + 13 = 40

     n=1

18) The product of the digits of a two digit number is $18$. When $63$ is subtracted from the number we get the two digit number with digits in the reversed order.

a) If the digit in the tens place is $x$ when what will be the digit in the one's place ?

b) Write the number using the place value of the digits.

c) Form a second degree equation using the given condition.

d) Find the number.

**Answers**

a)   length of smaller piece = 16 − x

b)   $2x^2 = 164 + (16 - x)^2$

     $x^2 + 32x = 420$

c)   $x^2 + 32x + \left(\frac{32}{2}\right)^2 = 420 + \left(\frac{32}{2}\right)^2$

     $x^2 + 32 + 256 = 676$

     $(x + 16)^2 = 676$

     $x + 16 = \sqrt{676}$

     x+16 = 26

     x = 10

     length of pieces = 10, 6

19) A rod of 16cm length is cut into two pieces. Two times the square of the length of the larger piece is equal to 164 more than the square of the smaller piece.

    a) If the length of the larger piece is $x$ then what is the length of the smaller piece.

    b) Form an equation using the given conditions.

    c) Find the length of the pieces .

Answer

(a) Digit in the tens place = x

Digit in the one's place = $\dfrac{18}{x}$

(b) Number = $10x + \dfrac{18}{x}$

(c) $10x + \dfrac{18}{x} - 63 = 10 \times \dfrac{18}{x} + x$

$9x^2 - 63x = 162$

$x^2 - 7x = 18$

(d) $x2 - 7x + \left(\dfrac{7}{2}\right)^2 = 18 + \left(\dfrac{7}{2}\right)^2$

$\left(x - \dfrac{7}{2}\right)^2 = \dfrac{121}{4}$

$x - \dfrac{7}{2} = \dfrac{11}{2}$

$x = 9$

∴ number = 92

# Chapter 5:

# TRIGONOMETRY

★ There are some special right triangles. The diagonal of a square makes two right triangles of angles $45°, 45°, 90°$

★ If the side opposite to $45°$ is 1 then the side opposite to $90°$ will be $\sqrt{2}$. The sides are in the ratio $1 : 1 : \sqrt{2}$

★ The altitude of an equilateral triangle makes two right triangles.The angles of these triangles are $30°, 60°, 90°$.
If the side opposite to $30°$ is 1, the side opposite to $90°$ will be 2, side opposite tos $60°$ will be $\sqrt{3}$

★ A triangle can be scaled without altering its angles.While doing so length of the sides changes keeping the ratio of the sides constant.This is what we have studied in similar triangles.

★ Unchaging angles and unchanging ratio of the sides make a special type of angle measurements.These are known as trigonometric measurement of the angles.

★ We define trigonometric measurement of angles on the acute angles of a right triangle. In $45° - 45° - 90°$right triangle the sides opposite to these angles are in the ratio$1 : 1 : \sqrt{2}$.Whatever be the size of the triangle this ratio remains unchanged.

★ The sides opposite to $30° - 60° - 90°$ angles of a right triangle are in the ratio$1 : \sqrt{3} : 2$. This ratio is independent of the size of the triangle.This leads to the measurement of angles,known as $\sin, \cos, \tan$.

★ In triangle $ABC, A, B, C$are the angles and $a, b, c$ are the opposite sides. If$B = 90°$then $\sin A = \frac{a}{b}, \cos A = \frac{c}{b}, \tan A = \frac{a}{c}$

★ $\sin 30° = \frac{1}{2}, \sin 60° = \frac{\sqrt{3}}{2}, \sin 45° = \frac{1}{\sqrt{2}}.$
$\cos 30° = \frac{\sqrt{3}}{2}, \cos 60° = \frac{1}{2}, \cos 45° = \frac{1}{\sqrt{2}}.$
$\tan 30° = \frac{1}{\sqrt{3}}, \tan 60° = \sqrt{3}, \tan 45° = 1$

1) Consider a square of perimetre $40$cm

  a) What is the length of its side?

  b) What is the length of its diagonal

  c) What is the area of the square drawn on its diagonal?

a) **Length of one side** $= \frac{40}{4} = 10$cm

b) **Two sides and the diagonal form a** $45°, 45°, 90°$ **right triangle .**
   **The side opposite to** $45°$ **is** $10$cm.
   $\therefore$ **the side opposite to** $90°$ **is** $10\sqrt{2}$cm

c) **Area** $= (10\sqrt{2})^2 = 100 \times 2 = 200$ **sq.cm**

2) In traingle $ABC$ $\angle A = 30°$, $BC = 10$cm
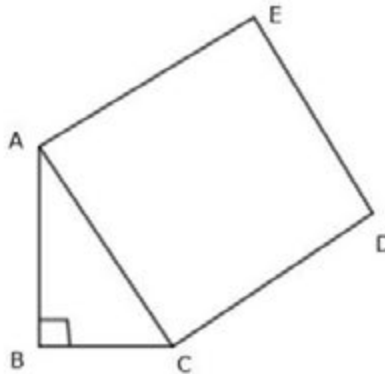


a) What is the length $AB$?
b) What is the length of the side $AC$?
c) What is the length of the diagonal of the square drawn on $AC$?
d) What is the perimetre of the square?

a) **In a** $30 - 60 - 90$ **triangle ,side opposite to** $30°$ **is** $10$cm
   **Therefore side opposite tos** $60°$ **is** $10\sqrt{3}$cm

b) **Side opposite to** $90°$ **is** $20$cm

c) **Length of diagonal of the square is** $20\sqrt{2}$cm

d) **Perimetre** $= 4 \times 20 = 80$ **cm**

3) Consider an equilateral triangle of side $10$cm

   a) What is its altitude?
   b) Draw a rough diagram of the square drawn on the altitude
   c) What is the area of this square.
   d) What is the length of its diagonal?

a) $\triangle ABD$ is a $30-60-90$ triangle. $AD$ is the altitude. $AD = 5\sqrt{3}$cm

b) Figure
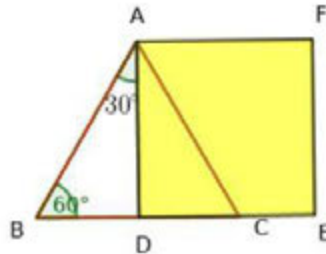


c) Area $= (5\sqrt{3})^2 = 25 \times 3 = 75$ sq.cm

d) Length of the diagonal is $5\sqrt{3} \times \sqrt{2} = 5\sqrt{6}$cm

4) In the figure $O$ is the centre of the circle. $\angle ACB = 30°$
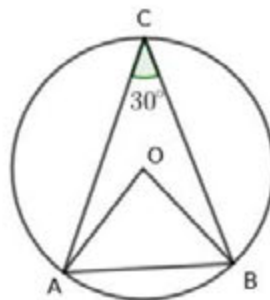


a) What is the measure of $\angle AOB$?

b) What kind of triangle is $OAB$?

c) If the radius of the circle is $12$cm then what is the altitude of triangle $OAB$?

d) What is the area of triangle $OAB$?

**a)** $\angle AOB = 60°$

**b)** $OA = OB$, Angles opposite to the equal sides are equal. All angles are $60°$ .
This is an equilateral triangle

**c)** Look at the picture
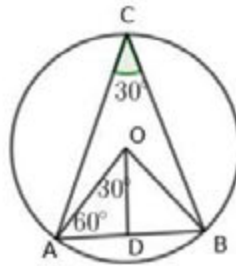


Draw $OD$ perpendicular to $AB$
Triangle $ODA$ is a $30° - 60° - 90°$ triangle .
Side opposite to $90°$ is $12$ cm ,Side opposite to $30°$ is $6$ cm , $OD = 6\sqrt{3}$ cm

**d)** Area $\frac{1}{2} \times 12 \times 6\sqrt{3} = 36\sqrt{3}$ sq.cm

5) The diagonal of the rectangle $ABCD$ is $12$ cm , $\angle BAC = 30°$
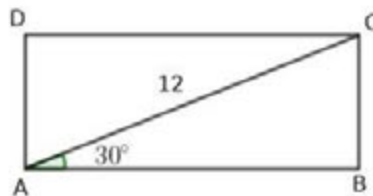


a) What is the length of the side $AB$?
b) What is the length of the side $BC$?
c) Calculate the area of the rectangle

**a)** $\triangle ABC$ is a $30° - 60°, 90°$ triangle.
Side opposite to $90°$ is $12$ cm
Side opposite to $30°$ is $6$ cm $AB = 6\sqrt{3}$ cm

**b)** $BC = 6$ cm

**c)** Area $= AB \times BC = 36\sqrt{3}$ sq.cm

6) In triangle $ABC$, the line $AD$ is perpendicular to $BC$, $AB = 12$cm
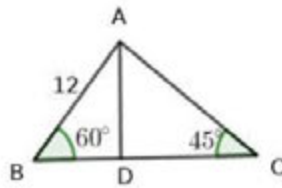


a) What is the length of $AD$?

b) What is the length of $AC$?

c) What is the length of $BC$?

d) Calculate the area of triangle $ABC$?

a) $\triangle ADB$ is a $30° - 60° - 90°$ triangle.
$\angle A = 30°$. The side opposite to $90°$ is 12cm.
The side opposite to $30°$ is $6$cm
$BD = 6$cm, $AD = 6\sqrt{3}$cm

b) $\triangle ADC$ is a $45° - 45° - 90°$ triangle.
$AC = 6\sqrt{3} \times \sqrt{2} = 6\sqrt{6}$cm

c) $CD = 6\sqrt{3}$cm, $BC = 6 + 6\sqrt{3}$cm

d) Area $A = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times (6 + 6\sqrt{3}) \times 6\sqrt{3} = 18\sqrt{3}(1 + \sqrt{3})$sq.cm

7) In the fogure $O$ is the centre of the circle. $\angle BAC = 60°, BC = 10$ then
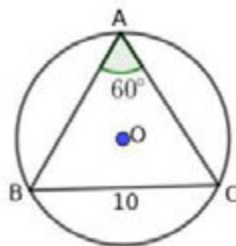


a) Draw the diametre from $B$ which meet the circle at $P$

b) Draw triangle $BPC$, write the measure of $\angle BPC$?

c) What is the diametre of the circle? What is its radius?

d) Calculate the area of triangle $BPC$?

5

**a) Picture is given below**
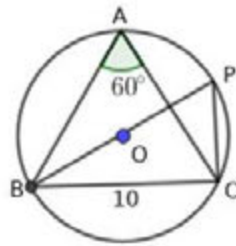


**Draw $BP$ as in the figure. Draw $PC$**

**b) $\angle BPC = 60°$. (Angle in the same arc)**

**c) $\triangle BPC$ is a $30° - 60° - 90°$ triangle.**
**Side opposite to $60°$ is $10$cm**
**Side opposite to $30°$ is $\frac{10}{\sqrt{3}}$ cm**
**Side opposite to $90°$ is $2 \times \frac{10}{\sqrt{3}}$**
**Diametre $= \frac{20}{\sqrt{3}}$ cm, Radius $= \frac{10}{\sqrt{3}}$ cm**

**d) Area $= \frac{1}{2} \times BC \times PC = \frac{50}{\sqrt{3}}$ sq.cm**

8) In the figure $AP$ is perpendicular to $BC$, $\angle ACD = 150°$, $\angle BAP = 60°$
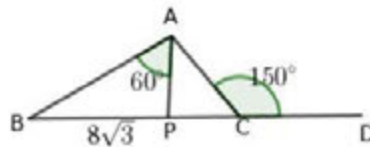


a) What is the length of $AP$ and $AB$.

b) What is the measure of angle $ACP$?

c) What is the length of $PC$?

d) What is the area of the triangle $ABC$

**a)** $\triangle APB$ is a $30° - 60° - 90°$ triangle.
Side opposite to $60°$ is $8\sqrt{3}$cm
Side opposite to $30°$ is $8$cm
Side opposite to $90°$ is $16$cm.
$AP = 8$cm, $AB = 16$cm

**b)** $\angle ACP = 180 - 150 = 30°$

**c)** $AP = 8$cm, $PC = 8\sqrt{3}$cm

**d)** Area $= \frac{1}{2} \times BC \times AP = \frac{1}{2} \times 16\sqrt{3} \times 8 = 64\sqrt{3}$ sq.cm

9) In the figure $ABCD$ is a **rectangle** $\angle DPA = 30°, \angle CPB = 45°, PB = 4$ cm then
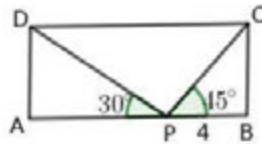


a) What is the length of $BC$?

b) What is the length of $AP$?

c) Find the area of the rectangle ?

d) Find the length of $PD$ and $PC$

**a)** $\triangle PBC$ is a $45° - 45° - 90°$ triangle.
$BC = 4$cm

**b)** $\triangle APD$ is a $30° - 60° - 90°$ triangle.
$AD = 4$cm, $AP = 4\sqrt{3}$cm.

**c)** $AB = 4 + 4\sqrt{3}$cm, $BC = 4$cm
Area $= 4(4 + 4\sqrt{3}) = 16(1 + \sqrt{3})$ sq.cm

**d)** $PD = 8$cm, $PC = 4\sqrt{2}$cm

10) In triangle $ABC$, if $\angle B = 90°$, $\sin A = \frac{3}{5}$ then

    a) Draw a rough diagram

    b) Write $\cos A$ and $\tan A$

    c) Write $\cos C$ and $\tan C$
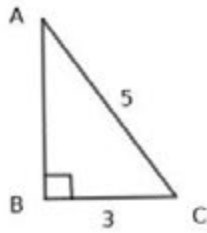


**a) See the diagram**

**b)** $AB = \sqrt{5^2 - 3^2} = 4$

    $\cos A = \frac{4}{5}$ , $\tan A = \frac{3}{4}$

**d)** $\cos C = \frac{3}{5}$, $\tan C = \frac{4}{3}$

11) If in triangle $ABC$, $\angle B = 90°$, $\sin A = 0.8$ then

    a) Draw a rough diagram.

    b) Write $\cos A$, $\tan A$

    c) Write $\cos C$, $\tan C$
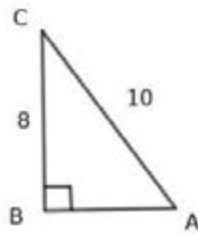
**a) See the diagram**

**b)** $\sin A = 0.8 = \frac{8}{10}$
$AB = \sqrt{10^2 - 8^2} = 6$
$\cos A = \frac{6}{10} = 0.6$ , $\tan A = \frac{8}{6}$

**d)** $\cos C = \frac{8}{10}$, $\tan C = \frac{6}{8}$

12) In the figure $AB = 10$cm, the altitue from $A$ to $BC$ is 8cm ,$\angle C = 45°$.
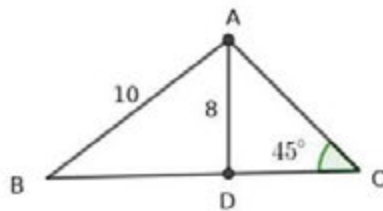


a) What is the length of $BD$?

b) Write $\sin B, \cos B, \tan B$

c) Write the length of $BC$

d) Find the area of triangle $ABC$

e) Find the perimetre of triangle $ABC$.

**a)** $BD = \sqrt{10^2 - 8^2} = 6$**cm**

**b)** $\sin B = \frac{8}{10}$
$\cos B = \frac{6}{10}$
$\tan B = \frac{8}{6}$

**c)** $\triangle ADC$ **is a** $45° - 45° - 90°$ **triangle.**
$AD = CD = 8$**cm**
$BC = 6 + 8 = 14$**cm**

**d) Area** $= \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 14 \times 8 = 56$ **sq.cm**

**e) Perimetre** $= AB + AC + BC = 10 + 8\sqrt{2} + 14 = 24 + 8\sqrt{2}$**cm**

9

13) In triangle $ABC$ , $AB = 8$cm , $BC = 12$cm , $\angle B = 120°$
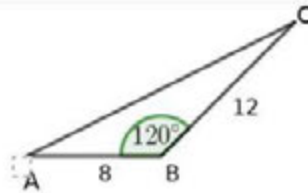


a) What is the altitude from $C$ to $AB$?

b) Find the area of triangle $ABC$.
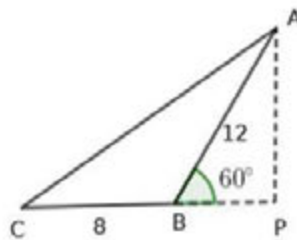


a) $\angle ABP = 180 - 120 = 60°$

$\sin 60° = \frac{AP}{AB}$

$\frac{\sqrt{3}}{2} = \frac{AP}{12}$

$AP = 6\sqrt{3}$cm

b) Area $= \frac{1}{2} \times 8 \times 6\sqrt{3} = 24\sqrt{3}$ sq.cm

14) $ABCD$ is a trapezium. $\angle A = 60°, \angle B = 30°, AB = 14$cm , $AD = 4$cm
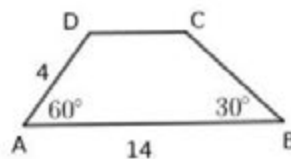


a) What is the diatance between parallel sides

b) What is the length of $CD$

c) Find the perimetre of the trapezium

d) Claculate the area of the trapezium.
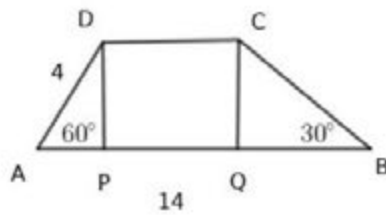
a) Draw $DP$ and $CQ$ perpendicular to $AB$
$\triangle APD$ is a $30° - 60° - 90°$ triangle.
Side opposite to $90°$ is $4$cm.
$AP = 2$cm, $PD = 2\sqrt{3}$cm.

b) $\triangle CQB$ is a $30 - 60 - 90$ triangle.
$QC = 2\sqrt{3}$cm, $QB = 2\sqrt{3} \times \sqrt{3} = 6$cm
$PQ = 14 - (2 + 6) = 6$cm
$CD = 6$cm

c) $CB = 4\sqrt{3}$cm.
perimetre $= 14 + 4 + 6 + 4\sqrt{3} = 24 + 4\sqrt{3}$cm

d) Area $= \frac{1}{2} \times h \times (a + b) = \frac{1}{2} \times 2\sqrt{3} \times (14 + 6) = 20\sqrt{3}$sq.cm

15) From a point on the ground $40$ metre away from the foot of the tower sees the top of the tower at an angle of elevation $30°$ and sees the top of the water tank on the top of the tower at an angle of elevation $45°$.

   a) Draw a rough diagram.

   b) Find the height of the tower.

   c) Find the height of the water tank

**a) Diagram**
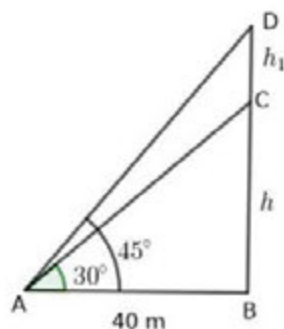


**b) In triangle** $ABD$ **we have** $\tan 45° = \frac{BD}{AB}$

$1 = \frac{h+h_1}{40}, h + h_1 = 40$

$\tan 30° = \frac{BC}{AB}, \frac{1}{\sqrt{3}} = \frac{h}{40}$

$h = \frac{40}{\sqrt{3}} = 23.1\text{m}$

**Height of the tower** $23.1\text{m}$

**c)** $23.1 + h_1 = 40, h_1 = 40 - 23.1 = 16.9$**metre.**

16) The shadow of a vertical tower on level ground increases by $10$m when the altitude of the Sun changes from the angle of elevation $45°$ to $30°$.

    a) Draw a rough diagram

    b) Calculate the height of the tower.

**a) Diagram**
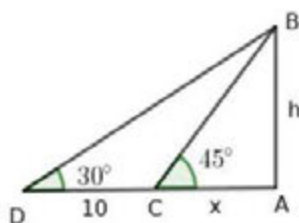


**b)** $\tan 45 = \frac{AB}{AC}, 1 = \frac{h}{x}, h = x$

$\tan 30 = \frac{1}{\sqrt{3}} = \frac{h}{x+10}$

$x + 10 = \sqrt{3}h, h = 13.65$**metre.**

17) A tall building and a short building are standing on a level ground.The angle of elevation of the top of the short building from the foot of the tall building is $30°$.

The angle of elevation of the top of the tall building from the foot of the short building is $60°$. The tall building has height 50m.

a) Draw a rough diagram

b) What is the diatance between the buildings.

c) Calculate the height of the short building.

a) **Diagram**
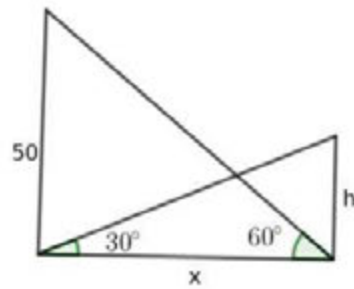


b) $\tan 60° = \frac{50}{x}, \sqrt{3} = \frac{50}{x}$

$x = \frac{50}{\sqrt{3}} = \frac{50}{1.73} = 28.9$m

**The distance between the buildings $= 28.9$m**

c) $\tan 30 = \frac{h}{x}, \frac{1}{\sqrt{3}} = \frac{h}{28.9}$ , $\sqrt{3}h = 28.9, h = \frac{28.9}{1.73} = 16.7$m

19) The top of a $30$ high building can be seen from a point at some diatance from the foot of the building is at an angle of elevation $30°$. When the polt of observation is some distance closer to the building the angle become $60°$

    a) Draw a rough diagram

    b) What is the distance from the foot of the building to the second point of observation.

    c) What is the diatance between two points from which the top of the building is observed .

    d) What is the diatance from the foot of the tower to the first point of observation.

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**a) Diagram**
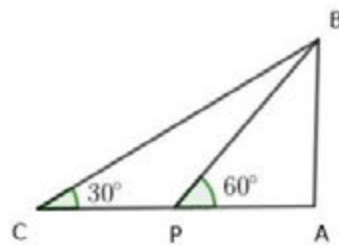


**b)** In $\triangle ABP$, $\tan 60° = \frac{30}{y}$, $\sqrt{3} = \frac{30}{y}$, $y = \frac{30}{\sqrt{3}} = 10\sqrt{3}$m

The second point is at the distance $10 \times 1.732 = 17.32$m away from the building.

**c)** $\tan 30 = \frac{30}{x+y}$

$\frac{1}{\sqrt{3}} = \frac{30}{x+y}$, $30\sqrt{3} = x + y = x + 17.3x = 30\sqrt{3} - 17.3 = 34.66$m

The distance between the points is $34.66$m

**d) Distance is** $x + y = 30\sqrt{3} = 51.96$m

---

20) A man sees a car moving towards a building at uniform speed at an angle of depression $30°$ from the top of the building. After $6$ seconds the angle of drepression becomes $60°$.

    a) Draw a rough diagram.

    b) How long the car take to reach near the building?

**a)  Diagram**
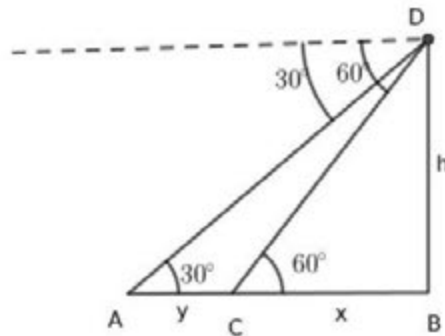


★ $\tan 60° = \frac{h}{x}, h = \sqrt{3}x$

★ $\tan 30 = \frac{h}{x+y}, \frac{1}{\sqrt{3}} = \frac{h}{x+y}$
$x + y = 3x, y = 2x.$
**The car takes $6$ seconds to travel $y$ distance. Since $y = 2x, x = \frac{y}{2}$. Since the car travels equal distance in the equal interval of time the car takes $3$ seconds to travel $\frac{y}{2}$ distance. That is the car takes $3$ more seconds to reach the building**

21)  A man sees a boat approaching the shore at the angle of depression $30°$ from the top of a light house . After $6$ seconds the angle of depression becomes $60°$.

   a)  Draw a rough diagram

   b)  How long will the boat takes to reach the shore .

   c)  If the speed of the boat is $25$kilometre per hour, what is the distance from the shore to the second point of observation.

**a) Diagram**
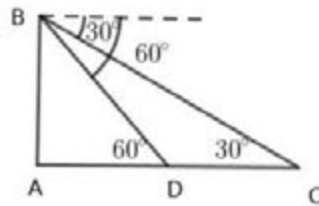


**b)** $\triangle ABC$ is a $30° - 60° - 90°$ triangle.IF $AB = h$, $AC = \sqrt{3}h$
$\triangle ABD$ is a $30° - 60° - 90°$ triangle. $AD = \frac{h}{\sqrt{3}}$
$CD = AC - AD = \sqrt{3}h - \frac{h}{\sqrt{3}} = \frac{2h}{\sqrt{3}}$
$AD : CD = 1 : 2$
The boat takes $6$ minutes to reach $C$ to $D$.So the boat takes $3$ minutes to reach he shore from $D$

**c)** If the speed $25$ km per hour, , $CD = $ speed $\times$ time . That is $CD = 25 \times \frac{6}{60} = 2.5$km.
The diatance from $A$ to $D$ is $1.25$km

22) A man sees the top of a light house of height $100$m at the angle of elevation $60°$. After $2$ minutes the angle becomes $45°$.

   a) Draw a rough diagram.

   b) Calculate the distance between the positions of obaservation.

   c) Calculate the speed of the boat.

**a) Diagram**
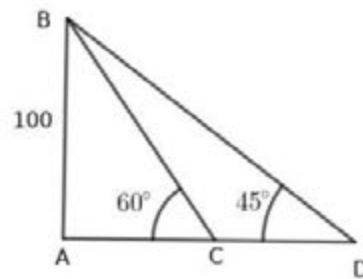


**b)** $\triangle ABD$ **is a** $45° - 45° - 90°$ **triangle.** $AD = 100$m
$\triangle ABC$ **is a** $30° - 60° = 90°$ **triangle.** $AC = \frac{100}{\sqrt{3}} = 57.80$m
$CD = 100 - 57.80 = 42.2$m

**c) Speed** = **distance** ÷ **time** = $42.2 ÷ 2 = 21.1$ **metre per minute .**

23) In the figure $AB = BC = 12$cm , $\angle B = 120°$, $\angle ACD = 45°, CD = 8$cm
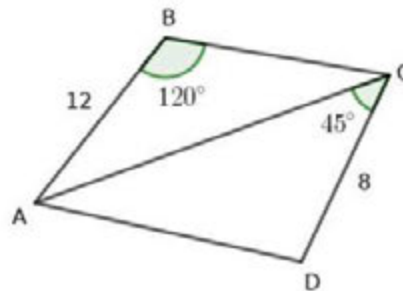


a) What is the length of $AC$?

b) What are the altitudes from $B$ and $D$ to $AC$

c) Calculate the area of $ABCD$.
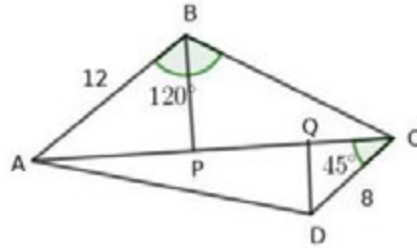
**a)** Since $AB = BC, \triangle ABC$ is an isosceles triangle .
Draw $BP$ perpendicular to $AC$.
$\triangle ABP$ is a $30° - 60° - 90°$ triangle.
$AP = 6\sqrt{3}$cm , $PC = 6\sqrt{3}$cm
$AC = 12\sqrt{3}$cm.

**b)** $PB = 6$cm , $\triangle DQC$ is a $45° - 45° - 90°$ triangle. $DC = 8$cm $DQ = \frac{8}{\sqrt{2}}$cm

**c)** Area $= \frac{1}{2} \times 12\sqrt{3} \times 6 + \frac{1}{2} \times 12\sqrt{3} \times (6 + 4\sqrt{2}, = 6\sqrt{3}(6 + 4\sqrt{2})$ sq.cm

24) In the figure $AB$ is the diametre of the seimicircle. $PC$ is perpendicular to $AB$.
If $BC = 12$cm , $\angle PCB = 30°$ then
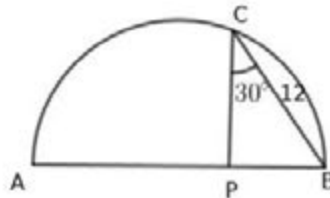


a) Find $PB$ and $PC$

b) What is the length of $AP$

c) What is the radius of the semicircle.

**a)** $\triangle CPB$ is a $30° - 60° - 90°$ triangle. Since $BC = 12$cm, $PB = 6$cm , $PC = 6\sqrt{3}$cm

**b)** $PA \times PB = PC^2$,
$PA \times 6 = (6\sqrt{3})^2$
$6 \times PA = 36 \times 3, PA = 18$cm

**c)** Radius $12$cm

# Chapter 6:

## COORDINATES

**Focus Points**

* Two perpendicular straight lines divide the plane into four parts .The intersecting point of the lines is called origin.

* The position of a point can be determined by a pair of real numbers .In $P(x, y)$ $x$ is called $x$ coordinate and $y$ is called $y$ coordinate

* The coordinates of the origin is $(0, 0)$

* $y$ coordinates of all points on X axis is $0.y$ coodinates of all points on a line parallel to Y axis are equal.

* $x$ coordinates of all points on Y axis are is $0.$ $x$ coordinates of all points on a line parallel to Y axis are equal.

* The distance between two points on X axis or on a line parallel to X axis is the absolute value of the difference between their $x$ coordinates

* The distance between two points on Y axis or on a line parallel to Y axis is the absolute value of the difference between their $y$ coordinates

* If $P, Q$ are two points on a line parallel to X axis ,their $y$ coordinates are equal.In general $P(x_1, y), Q(x_2, y)$ can be considered as two points.

* If $P, Q$ are two points on the line parallel to Y axis, their $x$ coordinates are equal.In general $P(x, y_1), Q(x, y_2)$ can be taken as two points

* $x$ coordinates and $y$ coordinates of points on inclined line are different. $P(x_1, y_1), Q(x_2, y_2)$ can be taken as the points.

* The distance between $P(x_1, y_1), Q(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

1)  a) Draw coordinate axes , mark the points $A(-2, -2), B(-2, 2), C(2, 2), D(2, -2)$
    b) What is the most suitable name to $ABCD$
    c) Calculate the perimetre of $ABCD$

> a) Do yourself
>
> b) Square
>
> c) One side of the square is $4$. Perimetre $= $ **16**

2) $A(1, 1), B(-3, 1), C(-3, -4), D(1, -4)$ are the oordintes of the vertices of a rectangle.

   a) What is the length of the side $AB$?          1
   b) What is the length of the side $AD$?
   c) Calcualte the perimetre and area of the rectangle.

a) $AB =| 1 -^- 3 |= 4$

b) $AD =| 1 -^- 4 |= 5$

c) Perimetre$= 2(4 + 5) = 18$
   Area $= 4 \times 5 = 20$

3) There is a circle with centre at the origin. The circle passes through $(5, 0)$

   a) What is the radius of the circle?
   b) What are the coordinates of the points where the circle cut the axes?
   c) Is $(3, 4)$ a point on the circle? How can we realize it?
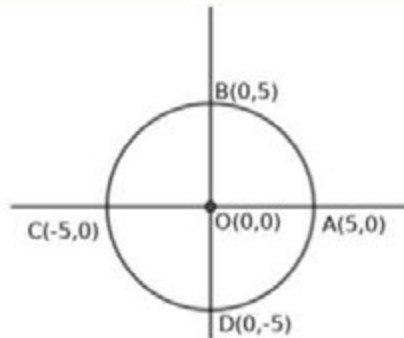


a) $5$

b) $A(5, 0), B(0, 5). C(-5, 0), D(0, -5)$

c) The distance from origin to the point $(3, 4)$ is $= \sqrt{3^2 + 4^2} = 5$, the radius of the circle. This point is on this circle.

4) In $\triangle ABC$, $A(1, 3), B(7, 3), C(4, 11)$ are the vertices

   a) What is the length of $AB$?
   b) What is the altitude to $AB$
   c) Calculate the area of $\triangle ABC$

a) $AB =| 7 - 1 |= 6$

b) $h =| 11 - 3 |= 8$

c) Area $= \frac{1}{2} \times 6 \times 8 = 24$ sq.cm

5) $\triangle ABC$ is an equilateral triangle. Side $AB$ coincides $x$axis. If $A(-1, 0), B(5, 0)$ then

   a) What is the length of $AB$?
   b) What is the altitude of the triangle?
   c) What are the coodinate pairs of $C$?

a) $AB =| 5 -^- 1 |= 6$

b) Altitude $= 3\sqrt{3}$

c) $C(2, 3\sqrt{3}), (C(2, -3\sqrt{3})$

2

6) Three vertices of $ABCD$ are $A(0,0), B(8,0)C(8,4)$

    a) Write the coordiantes of $D$

    b) Find the perimetre of the rectangle.

    c) Calculate the area of the rectangle.

> a) $D(0,4)$
>
> b) $AB = CD = 8, BC = AD = 4$
>    Perimetre$= 2(8+4) = 24$
>
> c) Area $= 8 \times 4 = 32$sq.unit

7) In triangle $ABC$, $A(1,2), B(7,2)$ are two vertices.

    a) What is the length of the side $AB$

    b) In triangle $ABC$, $\angle A = 90°$.Write a pair of coordinates of $C$

    c) What is the length of side $AC$?

    d) Calculate the area of the triangle.

> a) $AB =\mid 7-1 \mid= 6$
>
> b) $C(1,5)$ or any other pair with $x$ coordinate 1
>
> c) If $C(1,5)$, $AC =\mid 5-2 \mid= 3$
>
> d) In the right triangle $ABC$ with $A(1,2), B(7,2)$ and $C(1,5)$
>    Area $= \frac{1}{2} \times 6 \times 3 = 9$ sq.unit

8) $P(3,4)$ is a point on a circle with centre at the origin
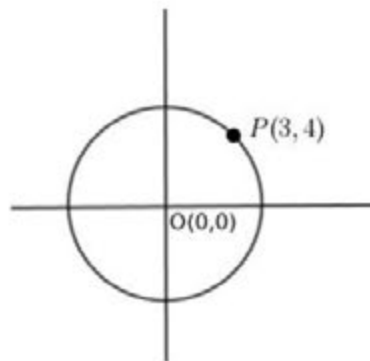


    a) What is the radius of the circle?

    b) $PQRS$ is a rectangle with its vertices are on this circle, sides are parallel to the axes . Write the coordinates of its vertices.

    c) What are the points where the circle cut the axes

    d) Calculate perimetre and area of the rectangle.
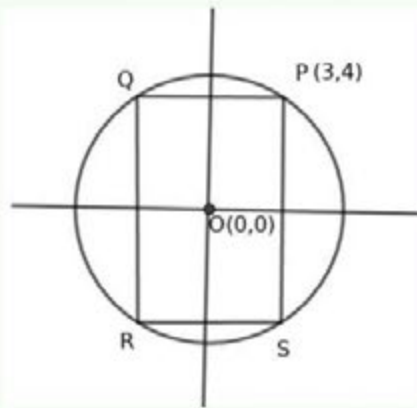
a) Radius $OP = \sqrt{3^2 + 4^2} = 5$

b) $Q(-3, 4), R(-3, -4), S(3, -4)$

c) $(5, 0), (0, 5), (-5, 0), (0, -5)$

d) Length $QR = PS = 8$, breadth $PQ = RS = 6$
   perimetre $= 2(8 + 6) = 28$, Area $= 48$

9) $OABC$ is a parallelogram, $O(0,0), A(4,0), B(6,5)$
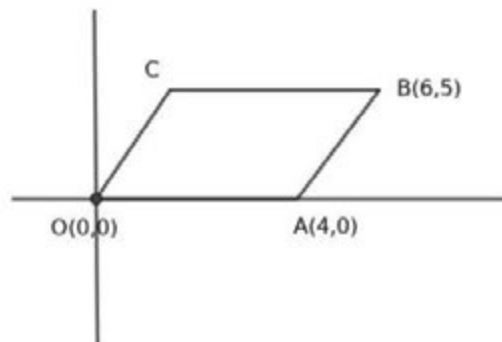


a) Write the coordinates of $C$

b) Write the length of $OA$ and $BC$

c) What is the diatance between the parallel sides $OA$ and $BC$

d) Calculate area and perimetre of the parallelogram

4
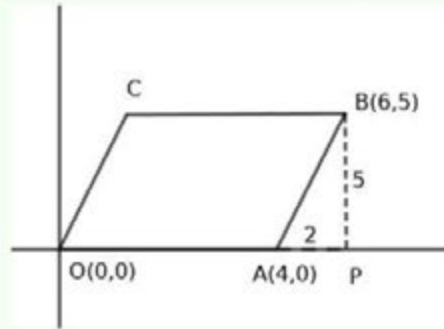
a) $OA = 4 \to BC = 4, C(6 - 4, 5) = C(2, 5)$

b) $OA = 4, BC = 4$

c) The distance between parallel sides $OA$ and $BC$ is 5

d) From the figure,the perpendicular from $B$ to $x$ axis , $AP$ and $AB$ form a right triangle.
Hypotenuse $AB = \sqrt{5^2 + 2^2} = \sqrt{29}$
Perimetre $= 8 + 2\sqrt{29}$
Area of the parallelogram $= 4 \times 5 = 20$

10) $P$ is a point on the circle with centre at the origin and radius 5.If $OP$ makes an angle $30°$ with the positive side pf $x$ axis ,



a) What are the points where the circle cut the axes?

b) Write the coordinates of $P$

c) The vertices of the rectangle $PQRS$, with the sides parallel to the axes are on the circle.Write the coordinates of the vertices.

a) $(5, 0), (0, 5), (-5, 0), (0, -5)$

b) Draw a line perpendicular to $x$ axis .Let it be $PN.\triangle ONP$ is a $30° - 60° - 90°$ triangle.
Side opposite to $90°$ is 5.
$PN = \frac{5}{2}, ON = \frac{5}{2}\sqrt{3}$
$P(\frac{5}{2}\sqrt{3}, \frac{5}{2})$.

c) $Q(-\frac{5}{2}\sqrt{3}, \frac{5}{2}), R(-\frac{5}{2}\sqrt{3}, -\frac{5}{2}), S(\frac{5}{2}\sqrt{3}, -\frac{5}{2})$

11) $ABCD$ is a rectangle ,sides are parallel to the axes .If $A(2, 3), AB = 6, BC = 5$then

5
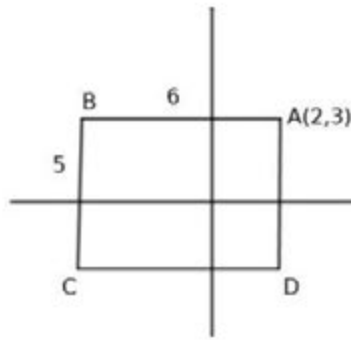
a) Write the coordinates of $B, C, D$

b) Find the perimetre of the rectangle.

c) Calculate the area of the rectangle.

a) $B(2-6, 3) = B(-4, 3)$
$C(-4, -2), D(2, -2)$

b) $AB = 6, BC = 5$
Perimetre $= 22$

c) Area $= 6 \times 5 = 30$

12) The perpendicular sides of the right triangle coincides the axes,right angled is at the origin . The mid point of the hypotenuse is $(6, 8)$.If the sum of the perpendicular sides is $28$

a) What is the radius of the circumcircle.

b) What is the length of its hypotenuse?

c) Find the area of the triangle.
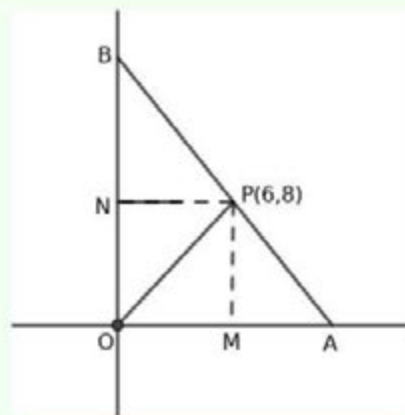


a) Radius of the circumcircle is $\sqrt{6^2 + 8^2} = 10$

b) Length of the hypotenuse is $20$

c) Draw $PM$ perpendicular to $x$ axis and $PN$ perpendicular to $y$ axis.
$OP = PA = 10, \triangle POA$ is an isosceles triangle.Since $OM = 6, OA = 12$
Similarly , $\triangle OPB$ is isosceles triangle, $ON = 8, OB = 16$
Area $= \frac{1}{2} \times 12 \times 16 = 96$ .

13) $P(3, 4)$ is a point in a circle with centre at the origin.
$Q(x, y)$ is another point on this circle ,$\angle AOQ = 30°$ then
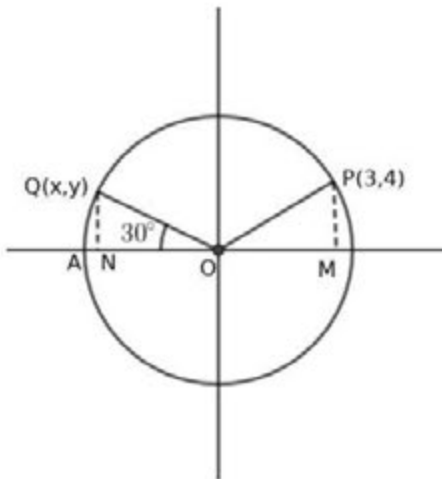
a) What is the radius of this circle?

b) What are the points where the circle cut the axes ?

c) Write the coordinates of $Q$

d) Write the coordinates of three more points on this circle.
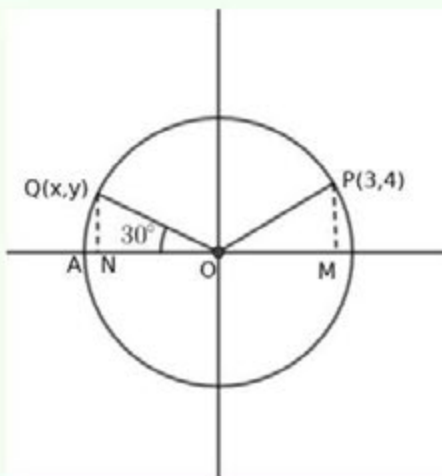


a) $OP = \sqrt{OM^2 + PM^2} = \sqrt{3^2 + 4^2} = 5$

b) $(5,0), (0,5), (-5,0), (0,-5)$

c) $\triangle ONQ$ is a $30° - 60° - 90°$ triangle
$OQ = 5, \therefore QN = \frac{5}{2}, ON = \frac{5}{2}\sqrt{3}$
$Q(-\frac{5}{2}\sqrt{3}, \frac{5}{2})$

d) $(-3,4), (-3,-4), (3,-4)$

14) $ABC$ is an equilateral triangle. If $A(1,1), B(7,1)$ then
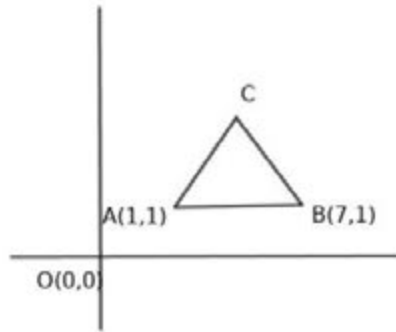
a) What is the length of one side?

b) What is the altitude of this triangle?

c) Write the coordinates of $C$

d) Calculate the area of the triangle.



a) $AB = | 7 - 1 | = 6$

b) $3\sqrt{3}$
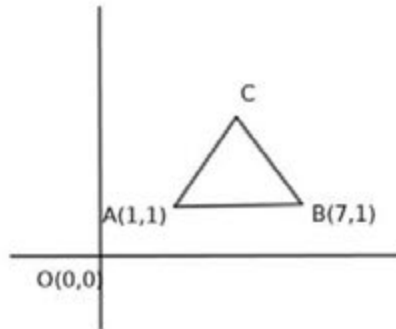
c) $C(1 + 3, 1 + 3\sqrt{3})$

d) Area $= \frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3}$

15) In the figure $ABCD$ is a square. $OD = 10, \angle AOD = 30°$.



a) Write the coordinates of $A$

8
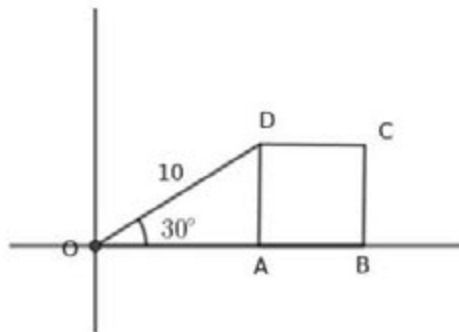
b) What is the length of one side of the square?

c) Write the coordinates of the vertices of the square.

---

a) $OA = 5\sqrt{3}$
$A(5\sqrt{3}, 0)$

b) $AD = 5$, Side is 5 unit

c) $A(5\sqrt{3}, 0), B(5 + 5\sqrt{3}, 0), C(5 + 5\sqrt{3}, 5), D(5\sqrt{3}, 5)$

---

16) Using the distance formula calculate the following.

a) The distance between $P(-6, 7)$ and $Q(-1, -5)$

b) What is the distance from origin to $(-5, 12)$

c) Find the distance between $P(-7, -3)$ and $Q(-5, -11)$

---

a) $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$PQ = \sqrt{(-1 -^- 6)^2 + (-5 - 7)^2}$
$PQ = \sqrt{5^2 + (-12)^2} = \sqrt{169} = 13$

b) $O(0,0), A(-5, 12)$
$OA = \sqrt{(-5 - 0)^2 + (12 - 0)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

c) $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$PQ = \sqrt{(-5 -^- 7)^2 + (-11 -^- 3)^2} = \sqrt{2^2 + 8^2} = \sqrt{68}$

---

17) Consider the points $A(1, -1), B(5, 2), C(9, 5)$

a) Find the distance $AB$, $BC$ and $AC$

b) Prove that these points are on a line.

c) What is the mid point of $AC$?

---

a) $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, AB = \sqrt{(5 - 1)^2 + (2 -^- 1)^2} = \sqrt{16 + 9} = 5$
$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, BC = \sqrt{(9 - 5)^2 + (5 - 2)^2} = \sqrt{16 + 9} = 5$
$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, AC = \sqrt{(9 - 1)^2 + (5 -^- 1)^2} = \sqrt{64 + 36} = 10$

b) $AB + BC = 10, AC = 10 \rightarrow AB + BC = AC$
$A, B, C$ are on a line

c) $AB = 5, BC = 5$. Therefore $B$ is the mid point of $AC$

---

18) Consider the points $A(4, 2), B(7, 5), C(9, 7)$

a) Find the distances $AB, BC$ and $AC$

b) Can we construct $\triangle ABC$ ? why?

c) Write the property of these points.

---

a) $AB = \sqrt{(7 - 4)^2 + (5 - 2)^2} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$
$BC = \sqrt{(9 - 7)^2 + (7 - 5)^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$
$AC = \sqrt{(9 - 4)^2 + (7 - 2)^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$

b) The sum of two sides is not greater than the third side. Triangle cannot be constructed.

c) $AB + BC = AC$. So, the points are on a line.

19) Consider the points $A(0,1), B(1,4), C(4,3), D(3,0)$

    a) Find the sides of $ABCD$

    b) Find the length of diagonals.

    c) Suggest a suitable name to this quadrilateral.

---

    a) $AB = \sqrt{(1-0)^2 + (4-1)^2} = \sqrt{10}$
        $BC = \sqrt{(4-1)^2 + (3-4)^2} = \sqrt{3^2 + (-1)^2} = \sqrt{10}$
        $CD = \sqrt{(3-4)^2 + (0-3)^2} = \sqrt{(-1)^2 + (-3)^2} = \sqrt{10}$
        $AD = \sqrt{(0-3)^2 + (1-0)^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{10}$

    b) $AC = \sqrt{(4-0)^2 + (3-1)^2} = \sqrt{4^2 + 2^2} = \sqrt{20}$
        $BD = \sqrt{(3-1)^2 + (0-4)^2} = \sqrt{2^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20}$

    c) Sides are equal. Diagonals are equal .$ABCD$ is a square .

---

20) Consider the points $A(2,3), B(3,4), C(5,6), D(4,5)$

    a) Calculate the $AB$ and $CD$

    b) Calcualte $AD$ and $BC$

    c) Find the length of diagonals $ABCD$

    d) Suggest a suitable name to $ABCD$.

---

    a) $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$
        $AB = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{2}.$
        $CD = \sqrt{(4-5)^2 + (5-6)^2} = \sqrt{2}$

    b) $AD = \sqrt{(4-2)^2 + (5-3)^2} = \sqrt{8} = 2\sqrt{2}$
        $BC = \sqrt{(5-3)^2 + (6-4)^2} = \sqrt{8} = 2\sqrt{2}$

    c) $AC = \sqrt{(5-2)^2 + (6-3)^2} = \sqrt{18} = 3\sqrt{2}$
        $BD = \sqrt{(4-3)^2 + (5-4)^2} = \sqrt{2}$

    d) $AB = CD, AD = BC$ opposite sides are equal
        $AC \neq BD$ diagonals are not equal . $ABCD$ is a parallelogram .

---

21) $\triangle OAB$ is an equilateral triangle. If $O(0,0), A(0,6)$ then

    a) Draw a rough diagram

    b) Find the length of one side .

    c) Write a pair of coordinates of $B$

    d) How many equailateral triangles are there satisfying this condition.

a) Look at the picture
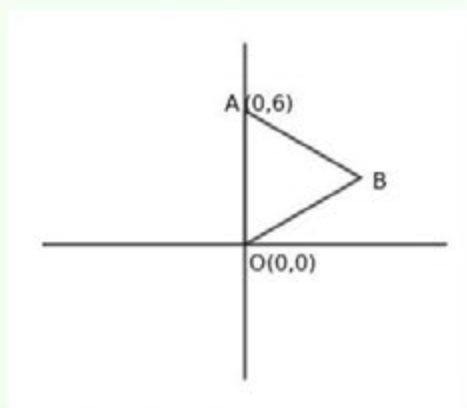


b) $OA = |\, 6 - 0 \,| = 6$

c) Mid point of $OA$ is $P(0,3)$

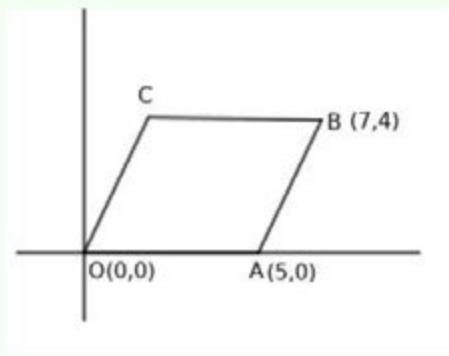   $\triangle BAP$ is a $30° - 60° - 90°$ triangle . $PA = 3, PB = 3\sqrt{3}$
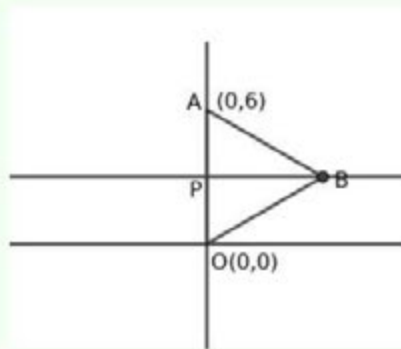


Coordinates of $B$ are $B(3\sqrt{3}, 3)$

d) Two eqilateral triangles are possible.  Triangle with vertices $O(0,0), A(0,6), B(3\sqrt{3},3)$ and triangle with vertices $O(0,0), A(0,6), B(-3\sqrt{3},3)$

22) $OABC$ is a parallelogram . If $O(0,0), A(5,0), B(7,4)$ then

   a) Draw a rough diagram

   b) Write the coordinates of $C$

   c) Calcualte the area of the parallelogram.

a) Look at the picture



b) Side $OA$ is parallel to $BC$. Therefore the difference of $x$ coordinates of $O, A$ is same as the difference of $x$ coordinates of $B$ and $C$
Similarly in the case of $y$ coordinates
$C(7-5, 4) = C(2, 4)$

c) Area $= 5 \times 4 = 20$

23) In the trapezium $ABCD$, $A(8, 5), B(-8, 5), C(-5, -3), D(5, -3)$ then

a) Find the length of parallel sides

b) What is the diatance between parallel sides ?

c) Calculate the area of the trapezium

12

a) $y$ coodinates of $A$ and $B$ are equal. Line $AB$ is parallel to $x$ axis. $y$ coordinates of $C$ and $D$ are equal. $CD$ is parallel to $x$.

That is $AB$ is parallel to $CD$. $AB = \mid 8 - {}^-8 \mid = 16$, $CD = \mid 5 - {}^-5 \mid = 10$

b) Distance between $AB$ and $CD$ is $\mid 5 - {}^-3 \mid = 8$

c) Area $= \frac{1}{2} \times 8(16 + 10) = 4 \times 26 = 104$

25) $P(2, -1), Q(3, 4), R(-2, 3), S(-3, -2)$ are the vertices of a quadrilateral.

a) Find the length of sides .

b) What is the length of its diagonals?

c) Suggest a suitable name to this quadrilateral.

d) Calculate the area .

a) $PQ = \sqrt{(3-2)^2 + (4+1)^2} = \sqrt{26}$
$QR = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{26}$
$RS = \sqrt{(-3+2)^2 + (-2-3)^2} = \sqrt{26}$
$SP = \sqrt{(-3-2)^2 + (-2+1)^2} = \sqrt{26}$

b) Diagonals $PR = \sqrt{(-2-2)^2 + (3+1)^2} = 4\sqrt{2}$
$QS = \sqrt{(-3-3)^2 + (-2-4)^2} = 6\sqrt{2}$

c) $PQ = QR = RS = SP$
$PR \neq QS$ .This is a rhombus

d) Area $= \frac{1}{2} \times d_1 \times d_2 = 24$ sq.unit

25) In the figure $ABCD$ is a parallelogram.If $A(2, 1), B(5, 1), D(3, 3)$ then
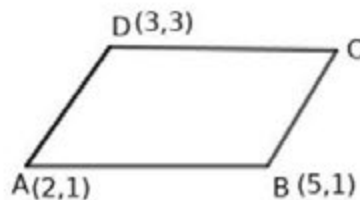


a) Write the coordinates of $C$

b) Find the length of side $AB$ and the distance between the parallel sides $AB$ and $CD$

c) Calculate the area of the parallelogram.

a) $AB$ is parallel to $CD$
The change in the $x$ coordinates of $A$ and $B$ is same as the change in the $x$ coordinates of $C$ and $D$.
Change in the $y$ coordinates of $A$ and $B$ is same as the change in the $y$ coodrinates of $C$ and $D$
$C(3 + 3, 3) = C(6, 3)$

b) $AB = \mid 5 - 2 \mid = 3$
Distance between the parallel sides $= \mid 3 - 1 \mid = 2$

c) Area $= 3 \times 2 = 6$

26) In the parallelogram $ABCD$, if $A(0,1)$, $B(5,3)$, $D(0,7)$ then



D(0,7)

C

B(5,3)

A(0,1)
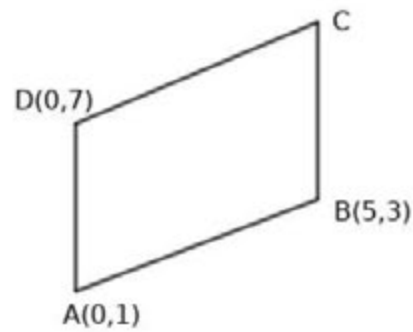
a) Write the coordinates of $C$

b) What is the diatance between the sides $AD$ and $BC$

c) Calculate the area of the parallelogram

a) $AD$ is parallel to $BC$
   The difference in the $x$ coordinates of $A$ and $D$ is same as that of $B$ and $C$. It is zero.
   The difference in the $y$ coordinates of $A$ and $D$ is same as that of $B$ and $C$. It is 6.
   $C(5, 3+6) = C(5,9)$

b) Distance $= \mid 5 - 0 \mid = 5$

c) Area $= 6 \times 5 = 30$

# Chapter 7:

# TANGENTS

1) Construct a tangent to a circle by the steps given below

   a) Draw a circle of radius $3$cm and mark a point $P$ on the circle.

   b) Mark $O$ as the centre of the circle and draw the radius $OP$

   c) Draw the tangent to the circle at $P$

**Answers**
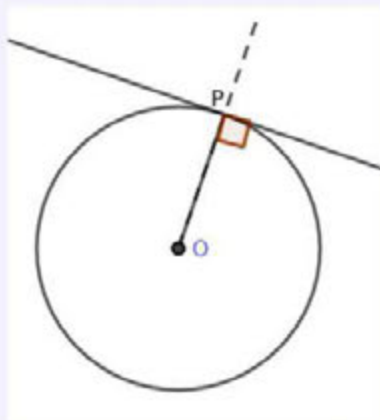


2) Draw a circle of radius 3.5 cm and draw a diameter. Draw tangent through the two end points of the diameter. Are they parallel. Why?
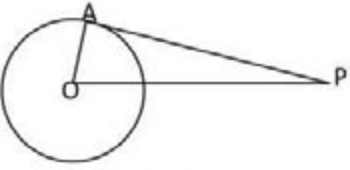
3) Draw suitable figure find the lengths

   a) A tangent of length $12$cm is drawn to a circle from a point outside the circle.If the radius of the circle is $5$cm find the distance from centre to the exterior point from which the tangent is drawn.

   b) What is the length of tangent drawn from a point at the distance $10$ cm away from centre of a circle of radius $6$cm

   c) A tangent is drawn from a point at the distance $26$ cm away from the centre of a circle. If the length of the tangent is $24$cm find the radius of the circle.

a)    AP = 12 cm, OA = 5 cm

$OP^2 = OA^2 + AP^2$

$= 5^2 + 12^2 = 25 + 144 = 169$

$OP = \sqrt{169} = 13$ cm, Distance 13 cm

b)    OP = 10cm, OA = 6 cm

$AP^2 = 10^2 - 6^2 = 100 - 36 = 64$

$AP = \sqrt{64} = 8$ cm

Length of the tangent = 8 cm

c)    OP = 26 cm, AP = 24

$OA^2 = 26^2 - 24^2$

$= 676 - 576 = 100$

$OA = \sqrt{100} = 10$ cm

radius = 10 cm
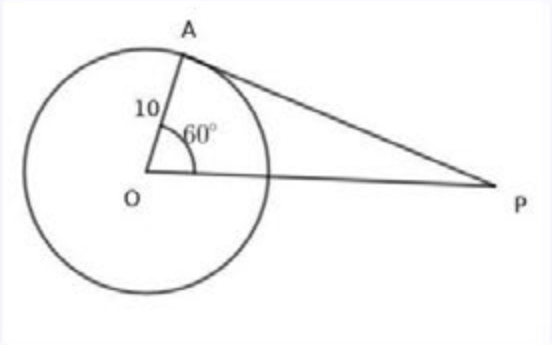


**B)** $O$ is the centre of a circle. A tangent $PA$ is drawn from the outer point $P$ to the circle at $A$

   a) Draw a rough diagram .

   b) If $\angle POA = 60°$ then what are the other angles of $\triangle OAP$

   c) If $\angle POA = 60°$, and the radius of the circle is $10$cm find the length of tangent.

   d) What is the length of the line $OP$

   a) see the diagram



   b) Other angles: $\angle OAP = 90°$, $\angle POA = 60°$, $\angle OPA = 30°$

   c) $\triangle OPA$ is a $30° - 60° - 90°$ triangle.

       Side opposite to $30°$ is $10$cm

       Side opposite to $90°$ is $20$cm.

       Length of the tangent is $10\sqrt{3}$cm

   d) $OP = 20$cm

4) In the figure $PA$ and $PB$ are tangents $O$ is the centre of the circle , $\angle AQB = 50°$ then
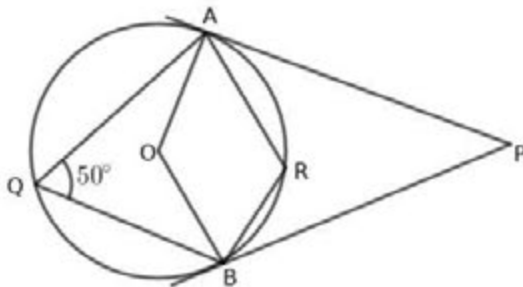
a) What is the measure of $\angle AOB$?

b) What is the measure of angle $\angle ARB$, $\angle APB$?

> **Answers**
>
> a) $\angle AOB = 2 \times 50° = 100°$
>
> b) $QARB$ is cyclic. $\angle ARB = 180 - 50 = 130°$
>
> c) $OAPB$ is cyclic. $\angle APB = 180 - 100 = 80°$

5) Draw a triangle with two angles 40°, 60° with its sides touching a circle of radius 3 cm

> **Answers**
>
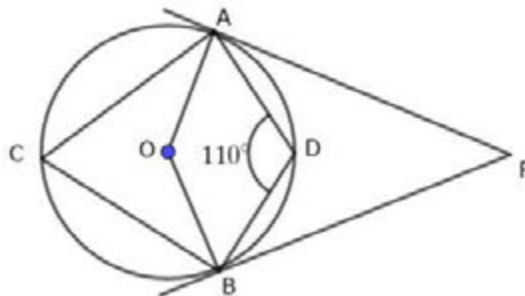> ★ Draw a circle of radius 3 cm
>
> ★ Since two angles are $40°$, $60°$ their supplementary angles are $180-40 = 140°$, $180-60 = 120°$. Draw radii such that it divide the angle around the centre as $140°$, $120°$, $100°$
>
> ★ Draw tangents to the circle at the ends of the radii.

6) The sides of an equilateral triangle touche a circle of radius 3cm .Construct the triangle.

> **Answers**
>
> ★ Draw a circle of radius 3cm . Mark the centre as $O$
>
> ★ Since the angles of an equilateral triangle are $60°$ , divide the angle around the centre as three equal parts of $180 - 60 = 120°$
>
> ★ For this draw the radii $OA, OB, OC$
>
> ★ Draw tangents at $A, B, C$ to the circle.The tangents make the triangle $PQR$

7) In the figure $O$ is the centre of the circle. $PA$ and $PB$ are the tangents. If $\angle ADB = 110°$ then



a) Find the measure of $\angle ACB$

b) Find the measure of $\angle AOB$

c) Find the measure of $\angle APB$

> **Answers**
>
> a) $\angle ACB = 180 - 110 = 70°$     (eg. ABCD is cyclic)
>
> b) $\angle AOB = 2 \times 70 = 140°$
>
> c) $\angle APB = 180 - 140 = 40°$

8) Two angles of a triangle are $120°, 40°$.The sides touches a circle of radius 3cm . Construct the triangle.

**Answers**

* Draw a circle of radius 3cm

* Draw radii which divides the angle around the centre as $180 - 120 = 60°$, $180 - 40 = 140°$, $160°$

* Draw tangents at the ends of the radius which makes the triangle.

9) $P$ is a point at the distance 7cm from the centre $O$ of a circle of radius 3 cm

   a) Draw tangents from $P$ to the circle.
   b) Measure the length of radius and write aside.

**Answers**

a) * Draw a circle of radius 3cm, centre $O$ . Mark a point $P$ at the distance 7cm from the centre.

   * Draw a circle with $OP$ as the **diameter** .The second circle intersect the first circle at $A$ and $B$

   * Draw the lines $PA$ and $PB$. These lines are the tangents to the circle of radius 3cm from $P$

b) **Measure the lengths of the Tangents**

10) The sides of $ABCD$ touches the circle at $P, Q, R, S$
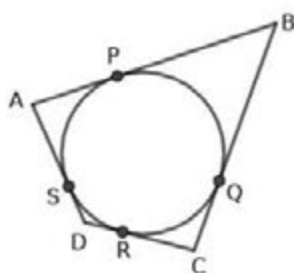


   a) Prove that $AB + CD = AD + BC$
   b) If $AB = 12$cm $CD = 8$cm , $AD = 14$cm then find $BC$.

**Answers**

a)

$$AP = AS \qquad (1)$$
$$BP = BQ \qquad (2)$$
$$DR = DS \qquad (3)$$
$$CR = CQ \qquad (4)$$

Adding these equations , $AP + BP + DR + CR = AS + BQ + DS + CQ$
$(AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ)$
$AB + CD = AD + BC$

b) $12 + 8 = 14 + BC, BC = 20 - 14 = 6$cm

11) $ABC$ is an equilateral triangle.Tangents are drawn at **through** vertices **on** circumcircle.These tangents form another triangle $PQR$.
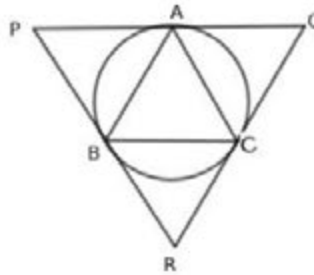


a) Prove that $PQR$ is an equilateral triangle.

b) If the perimetre of $ABC$ is 12cm then what is the perimetre of $\triangle PQR$.

c) How many times the area of triangle $PQR$ is that of triangle. $ABC$?

**Answers**

a) $\triangle ABC$ is an equilateral triangle.
$\angle BAC = \angle CBR = \angle BCR = 60°$ , $\angle R = 60°$
$\angle ABC = \angle CAQ = \angle ACQ = 60°$, $\angle Q = 60°$,$\angle P = 60°$. $\triangle PQR$ is an equilateral triangle.

b) $PACB$ is a parallelogram.$BC = PA \angle \textbf{PAB} = \angle \textbf{ABC}$
$QABC$ is a parallelogram $BC = AQ \therefore PQ = 2 \times BC$,
Similarly $PR = 2 \times AC, QR = 2 \times AB$
Perimetre of $\triangle PQR = 2 \times 12 = 24$cm

c) $PACB, QABC, RBAC$ are equal parallelograms. Each one can be divided into two equal triangles.We can see four equal triangles in the picture.
Area of $\triangle PQR = 4$ times the area of $\triangle ABC$

12) In the figure $AP$ is the diametre of the circle.$AB = 6\sqrt{3}$cm $PB = 6$ cm
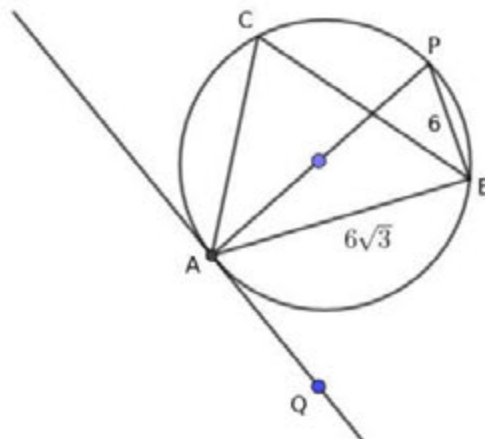


a) What is the radius of the circle?

b) What are the angles of $\triangle APB$?

c) What is the measure of $\angle ACB$?

d) What is the measure of $\angle BAQ$?

13) In the figure $QR$ is the diametre of the circle, $PA$ is the tangent, $\angle RPA = 30°$.
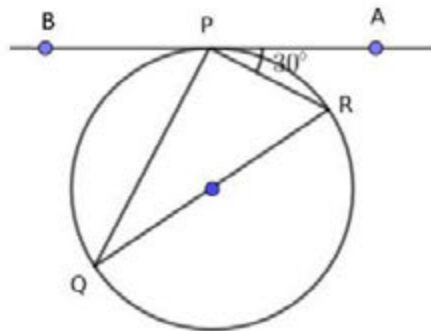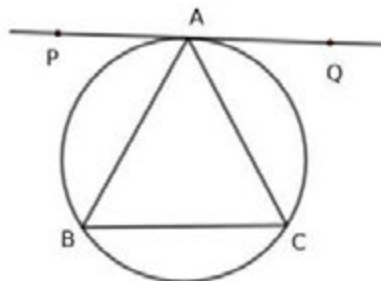


a) What is the measure of $\angle PQR$?

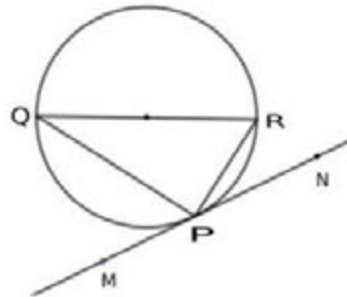b) What is the measure of $\angle PRQ$?

c) What is the measure of $\angle BPQ$

14) In $\triangle ABC$, $AB = AC$, a tangent $PQ$ is drawn through $A$ to its circumcircle. Prove that $PQ$ is parallel to $BC$.



6

15) In the figure $QR$ is the diametre of the circle , $MN$ is the tangent to the circle at $P$.
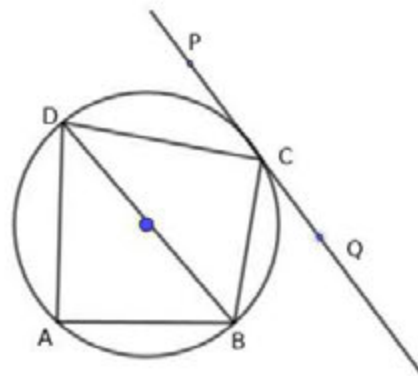If $\angle RPN = 50°$



a) What is the measure of $\angle PQR$?

b) What is the measure of $\angle PRQ$?

c) What is the measure of $\angle QPM$?

**Answers**

a) $\angle PQR = 50°$
( In a circle the angle between a chord and tangent at its end is equal to the angle in the other side of the chord on the circle.)

b) $\angle QPR = 90°$, $\angle PRQ = 90 - 50 = 40°$

c) $\angle QPM = \angle PRQ = 40°$

16) $ABCD$ is a cyclic quadrilaeral . $PQ$ is a tangent at $C$. $BD$ is the diametre of the circle.
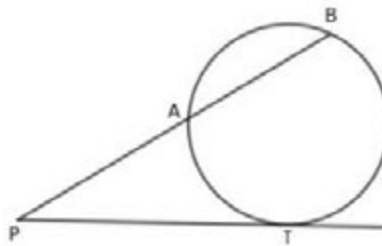$\angle DCP = 40°$, $\angle ABD = 60°$



a) What is the measure of angle $DBC$?

b) What is the measure of angle $BCQ$?

c) What is the measure of angle $BDC$?

d) What is the measure of $ADB$?

17) $PT$ is a tangent from an outer point $P$ to the circle.
Another line from $P$ intersect the circle at $A$ and $B$. If the length of the chord $PB$ is 16cm and $AB = 7$ cm then
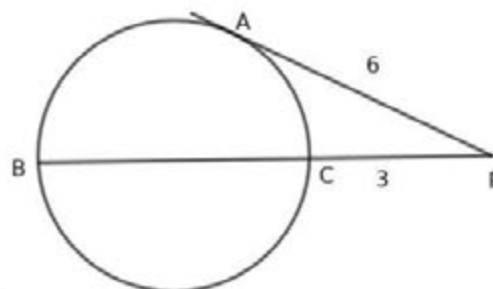


a) What is the length $PA$?

b) What is the relation between $PA, PB, PT$?

c) What is the length of the tangent $PT$?

d) What is the length of the other tangent from $P$ to the circle.

18) $BC$ is the diametre of the circle. $P$ is a point on $BC$ produced.
Tangent $PA$ is drawn from $P$ to the circle. If $PA = 6$cm and $PC = 3$cm then



a) What is the length $PB$?

b) Find the radius of the circle.

# Chapter 8:

## SOLIDS

★ Cone can be made by rolling a sectorol sheet.While doing so the arc length of the sector becomes the base perimetre of the cons.Area of the sector becomes lateral surface area of the cone. Lateral surface area is also known as curved surface area .

★ The radius of the sector becomes slant height of the cone . It can be denoted by $l$

★ Since arc length of the sector is equal to base perimetre of the cone we can make the relation given below

$$\frac{2\pi l}{360}x = 2\pi r$$

$l$ radius of the sector , $x$ central angle of the sector and $r$ is the radius of the cone , $lx = 360r$.

★ Area of the sector becomes the lateral surface area of the cone.We can make a formula to calculate lateral surface area of the cone
Curved surface area= Area of the sector
Curved surface area $= \frac{\pi r^2}{360}x = \frac{\pi \times l \times l \times x}{360}$
$lx = 360r$
Curved surface area $= \frac{\pi \times l \times 360r}{360} = \pi rl$

* If we denote the base radius of a cone as 'r', slant height as l height as h, then l, rol h from a right triangle $l^2 = r^2 + h^2$
* Base perimeter of a cone $= 2\pi r$ base area $= \pi r^2$
* Curved surface are $= \pi rl$
surface area $= \pi rl + \pi r^2$
* Volume of the cone $= \frac{1}{3}\pi r^2 h$

1) A sectoral sheet of central angle $120°$ is cut off from a circular sheet of radius 12cm . It is rolled in such a way as to get a cone.

    a) What is the slant height of the cone?

    b) What is the radius of the cone ?

    c) Find the curved surface area of the cone.

**Answers**

    a) Slant height$l = 12$cm

    b) $lx = 360r$
        $12 \times 120 = 360 \times r, r = 4$ cm

    c) Lateral surface area $= \pi rl = \pi \times 4 \times 12 = 48\pi$ sq.cm

2) A cone is made by rolling a semicircular metal sheet of radius 10cm

    a) What is the slant height of the cone.

    b) What is the radius of the cone.

    c) Find the curved surface area of the cone.

    d) Base is made by a suitable circular sheet. What is its total surface area ?

3) A circular sheet of card board of radius $12$ cm   is cut off into two sectors of central angle $120°$ and $240°$. Both of them are rolled into cones.

   a) What is the measue common to both cones

   b) What is the radius of small cone ?

   c) What is the radius of the big cone.

   d) How **the** radii of the cones are related to the radius of the circular sheet.

4) A sector of central angle $90°$ is cut off from a circular sheet of radius $16$ cm .It is rolled in such a way as to get a cone.

   a) What is the lateral surface are of the cone?

   b) What is the radius of the cone?

   c) The remaining part of the circular sheet is also rolled to get a cone . What is its base radius?

   d) Which cone has **less** height ? Explain

5) A cone is made by a sectoral sheet taken from a circular sheet. The slant height of the cone is two times its radius.

   a) What is the relation between lateral surface area and base area?

b) If the base perimetre is $20\pi$ cm then what will be its lateral surface area ?

c) What is the central angle of this sector?

d) The remaining part is also rolled to get a cone. What is the ratio of the heights of cones so formed

> **Answers**
>
> a) $l = 2r \rightarrow$ lateral surface area $= \pi r l = \pi \times r \times 2r = 2\pi r^2 = 2\times$ base area
>
> b) $2\pi r = 20\pi \rightarrow r = 10$ cm
> $l = 20$ cm .Lareral surface area $\pi \times 10 \times 20 = 200\pi$ sq.cm
>
> c) $lx = 360r \rightarrow 2r \times x = 360 \times r$
> $x = 180°$
>
> d) This is a semicircle. Remaining part is also a semicircle. Ratio of the height is $1 : 1$

6) A cone is made by taking a sector from a circular sheet.The slant height of the cone is $25$ cm and its radius $110$ cm

   a) What is the radius of the circular sheet?

   b) What is the central angle of the sector?

   c) What is the central angle of the remaining part?

   d) What is the radius of the cone made by rolling the remaining part?

> **Answers**
>
> a) $25$ cm
>
> b) $lx = 360r \rightarrow 25 \times x = 360 \times 10, x = \frac{360 \times 10}{25} = 144°$
>
> c) Central angle of sector $= 360 - 144 = 216°$
>
> d) Radius of the cone $25 - 10 = 15$ cm

7) The base perimetre of a cone is $20\pi$ cm, slant height $18$ cm . It is rolled to get a cone.

   a) What is the radius of the sector?

   b) What is the radius of the cone?

   c) What is the central angle of the sector?

   d) Find the lateral surface area of the cone?

> **Answers**
>
> a) $18$ cm
>
> b) $2\pi r = 20\pi, r = 10$ cm
>
> c) $lx = 360r \rightarrow 18 \times x = 360 \times 10, x = \frac{360 \times 10}{18} = 200°$
>
> d) Lateral surface area $\pi r l = 180\pi$ sq.cm

8) A sector of central angle $288°$ and radius $25$ cm is taken from a circulat sheet .

   a) What is the radius of the cone?

   b) What is the height of the cone ?

   c) Find the lateral surface area of the cone?

   d) What is the radius of the cone made by rolling the remaining part?

a) $lx = 360r \rightarrow 288 \times 25 = 360 \times r, r = \frac{288 \times 25}{360} = 20$ cm

b) $l^2 = h^2 + r^2$ , $25^2 = h^2 + 20^2 \rightarrow h^2 = 625 - 400 = 225, h = \sqrt{225} = 15$cm

c) Lateral surface area $\pi rl = \pi \times 20 \times 25 = 500\pi$ sq.cm

d) Radius of the remaining part $25 - 20 = 5$cm

9) Radius of a cone is $5$cm, height $12$cm

    a) what is the slant height of the cone?

    b) What is the total surface area of the cone?

    c) What is the volume of the cone ?

    d) In a cone , radius and height are equal. If the volume and curved surface area are equal then what is its radius ? What is its slant height?

a) $l^2 = r^2 + h^2 \rightarrow l = \sqrt{r^2 + h^2} = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$cm

b) Total surface area $\pi r^2 + \pi rl = \pi \times 5^2 + \pi \times 5 \times 13 = 25\pi + 65\pi = 90\pi$

c) Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 5^2 \times 12 = 100\pi$ cubic cm

d) If $r = h$ then slant height $l = \sqrt{2}r$ .Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times r^2 \times r = \frac{1}{3}\pi \times r^3$
Total surface area of the cone $= \pi \times r \times \sqrt{2}r$.
$\frac{1}{3}\pi r^3 = \pi r \times \sqrt{2}r \rightarrow r = 3\sqrt{2}$cm   (volume = Total surface area)
slant height $= \sqrt{2} \times 3\sqrt{2} = 6$cm

10) The base perimetre of a cone is $30\pi$ cm, height $20$ cm

    a) What is the radius and slant height of the cone ?

    b) What is the total surface area?

    c) Find the volume of the cone?

    d) What is the volume of a cyclidrical vessel of radius and height equal to that of the cone.

a) $2\pi r = 30\pi, r = 15$cm
slant height $l = \sqrt{r^2 + h^2} = \sqrt{15^2 + 20^2} = \sqrt{625} = 25$cm

b) Total surface area $\pi r^2 + \pi rl = 225\pi + 375\pi = 600\pi$ sq.cm

c) Volume $= \frac{1}{3}\pi r^2 \times h = \frac{1}{3} \times \pi \times 15^2 \times 20 = 1500\pi$ cubic cm

d) Volume becomes $3$ times .Volume $= 4500\pi$ cubic cm

11) Diametre and height of a cone are equal.

    a) What is the relation between radius and slant height?

    b) What is the ratio of radius , height and slant height?

    c) If the radius is $6$ cm then what is its volume?

    d) If the radius is $6$ cm then what is the total surface area ?

a) $h = 2r, l = \sqrt{r^2 + (2r)^2} = \sqrt{5r^2} = \sqrt{5}r$

b) $r : h : l = r : 2r : \sqrt{5}r$
   $r : h : l = 1 : 2 : \sqrt{5}$

c) If radius is 6cm then height 10cm .Volume $= \frac{1}{3}\pi \times 6^2 \times 10 = 360\pi$ cubic cm

d) If radius is 6cm then $l = \sqrt{5} \times 6 = 6\sqrt{5}$cm.
   Total surface area $= \pi \times 6^2 + \pi \times 6 \times 6\sqrt{5} = 36 + 36\sqrt{6}$
   $= 36(1 + \sqrt{6})$sq.cm

12) Radius of a cone is $r$ and height $h$.

   a) What will be the change in volume if radius and height are doubled?

   b) What will be the change in volume if radius is doubled and height is halved?

   c) How many solid cones can be made by melting a solid cone of radius 10cm and height 6cm with half the radius and height of the melted cone?

a) $V = \frac{1}{3}\pi r^2 h$.
   radius $2r$, height $2h$ Volume $= \frac{1}{3}\pi(2r)^2 \times (2h) = 8 \times \frac{1}{3}\pi r^2 h = 8V$
   Volume becomes 8times

b) If radius is $2r$ and height $\frac{h}{2}$ then
   volume $\frac{1}{3}\pi \times (2r)^2 \times \frac{h}{2} = 2 \times \frac{1}{3}\pi r^2 h = 2V$
   Volume becomes 2 times .

c) When radius and height become halved then volume become $\frac{1}{8}$ part . 8 cones can be made.

13) A conical measuring vessel is made by rolling a sectoral sheet of central angle $288°$ and radius 10cm.

   a) What is the radius of the vessel?

   b) What is the height of the vessel?

   c) What is the capacity of the vessel in litres ?

a) $lx = 360r \rightarrow 10 \times 288 = 360 \times r$
   $r = \frac{10 \times 288}{360} = 8$ cm

b) $h = \sqrt{l^2 - r^2} = \sqrt{10^2 - 8^2} = 6$cm

c) Volume $= \frac{1}{3}\pi \times r^2 \times h = \frac{1}{3} \times \pi \times 8^2 \times 6 = 128\pi$ cubic cm
   1000cubic cm $=$ 1litre
   Volume $= \frac{128 \times 3.14}{1000} = 0.4$litre

14) Radius of a cone is 21cm , height 28cm.

   a) Calculate slant height.

   b) Find the leteral surface area .

   c) Calculate the total surface area

   d) Calcualte the volume of the cone..

5

15) Ratio of radius and height of a cone are $3:4$. Volume of the cone is $301.44$ cubic cm

    a) Find the radius of the cone.

    b) Find the height of the cone.

    c) Calculate the slant height of the cone.

    d) Calculate the lateral surface area of the cone.

16) Lateral surface area of a cone is $4070$ sq.cm , diametre $70$ cm

    a) Find the slant height of the cone .

    b) Find the height of the cone?

    c) Calcualte the volume of the cone.

17) The height of a cone is $24$cm , its lateral surface area $550$ sq.cm

    a) What is the radius of the cone?

    b) Find the height of the cone?

    c) Calcualte the volume of the cone?

Chapter 9:

# GEOMETRY AND ALGEBRA

1) $ABCD$ is a parallelogram . If $A(1, 1), B(4, 2), C(6, 2)$ then



a) Write the difference between $x$ coordinates of $A$ and $B$

b) Write the difference between $y$ coordinates of $A$ and $B$

c) Write the coordinates of $D$.

**Answers**

a) The difference between $x$ coordinates of $A$ and $B$ is $= 4 - 1 = 3$

b) The difference between $y$ coordinates of $A$ and $C$ is $= 2 - 1 = 1$

c) $D(6 - 3, 2 - 1) = D(3, 1)$

2) In the figure $PQRS$ is a parallelogram .If $P(0,3)$, $PS = 4$, $Q(5.4)$ then
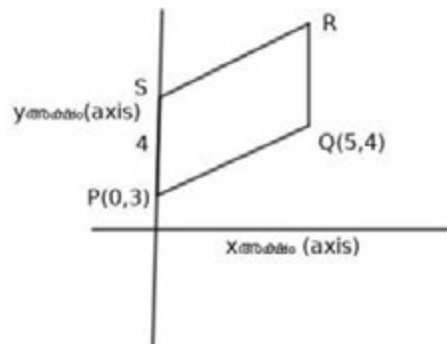


a) Write the coordinates of $S$

b) Write the coordinates of $R$

c) Find the length of the sides.

**Answers**

a) $S(0, 3 + 4) = S(0, 7)$

b) $R(5, 4 + 4) = R(5, 8)$

c) $PQ = \sqrt{(5 - 0)^2 + (4 - 3)^2} = \sqrt{25 + 1} = \sqrt{26}$
$PQ = RS = \sqrt{26}, PS = QR = 4$

3) $P(1, 4)$ the mid point of the side $AB$ ,$Q(2, 3)$ is the mid point of side $BC$ , $R(5, 6)$ is the mid point of side $AC$

a) Draw a suitable diagram representing the position of points

b) Write the coordinates of $B$

c) Write the coordinates of $C$

d) Write the coordinates of $A$

a) Look at the picture
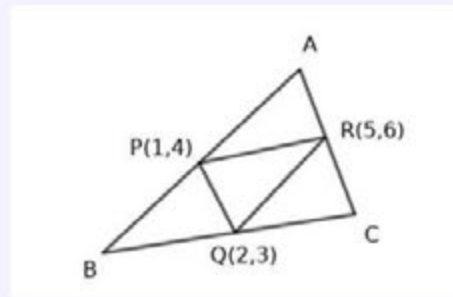


b) $BPRQ$ is a parallelogram .The difference between $x$ coordinates of $P, R$ is $4$
The difference between $y$ coordiantes is $2$
$B(2-4, 3-2) = B(-2, 1)$

c) $PRCQ$ is a parallelogram .The difference between $x$ coordinates of $P, R$ is $4$
The difference of $y$ coordinates of $P, R$ is $2$.
$C(2+4, 3+2) = C(6, 5)$

d) $PQRA$ is a parallelogram. The difference between $x$ coordinates of $Q, R$ is $3$
The difference between $y$ coordinates is $3$
$A(1+3, 4+3) = B(4, 7)$

4) $(1, 5), (4, 1), (4, 5)$ are the mid points of the sides of a triangle.

a) Draw a diagram and mark the mid points

b) Find the coordinates of the vertices of the triangle.

c) What kind of triangle is this ?

d) Calculate the area of this triangle.

a) see picture
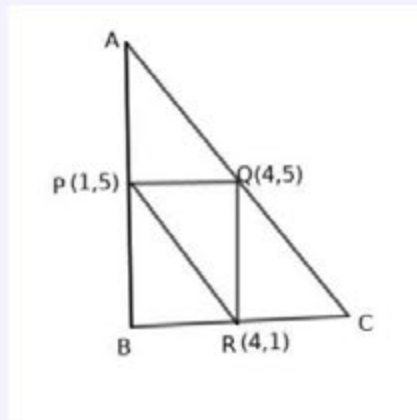


b) In $PQRB$ ,the difference between $x$ coordinates of $P, Q$ is 3.
The difference between $y$ coordinates is $0$.
Therefore $B(4-3,1) = B(1,1)$
In $PQCR$, the difference between $x$ coordinates of $P, Q$ is 3.
The difference between $y$ coordinates is $0$.
Therefore $C(4+3,1) = B(7,1)$
In $APRQ$, the difference between $x$ coordinates of $Q, R$ is $0$ .
The difference of $y$ coordinates is $4$.
Therefore $A(1,5+4) = A(1,9)$

c) $AB = 8, BC = 6, AC = 10.$ $AB^2 + BC^2 = AC^2$. $ABC$ is a right triangle

d) Area $= \frac{1}{2} \times BC \times AB = \frac{1}{2} \times 6 \times 8 = 24$ sq.unit

5) The mid point of the side $AB$ of triangle $ABC$ is $P(1,1)$. The mid point of the side $BC$ is $(5,4)$, mid point of the side $AC$ is $(7,4)$

 a) Find the coordinate of the vertex $A$.

 b) Find the coordinate of the vetex $B$

 c) Find the coordiante of the vertex $C$.

 a) $A(1+7-5, 1+4-4) \rightarrow A(3,1)$

 b) $B(1+5-7, 1+4-4) \rightarrow B(-1,1)$

 c) $C(7+5-1, 4+4-1) \rightarrow C(11,7)$

6) In triangle $ABC$, $A(-3,2), B(1,5), C(3,-4)$ then

 a) Find the coordinates of the mid point of $AB$

 b) Find the coordiantes of the mid point of $BC$

 c) Find the coordinates of the mid point of $AC$

7) Triangle $ABC$ is an equilateral triangle. $A(1,0), B(5,0)$
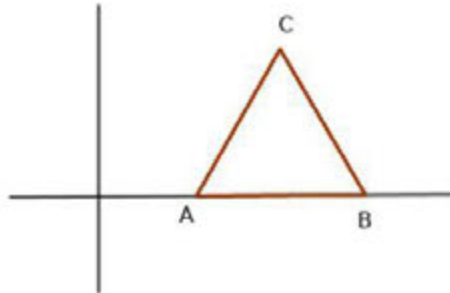


a) Find side of the triangle

b) What is the coordinates of the mid point of $AB$

c) Find the coordinates of $C$

8) $ABCD$ is a parallelogram.If $A(1,2), B(4,y), C(x,6), D(3,5)$ then
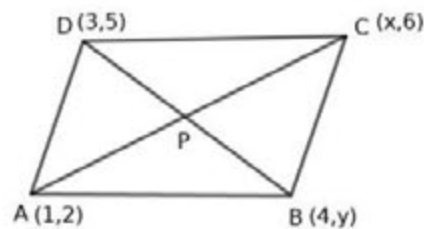


a) What is the $x$ coordinate of the mid point of the diagonal $BD$.

b) Write the coordinates of $C$

c) Write the $y$ coordinate of the diagonal $AC$

d) Write the coordiantes of $B$.

a) Diagonals bisect eachother. $x$ coordinate of the mid point of $BD$ is $\frac{4+3}{2} = \frac{7}{2}$

b) $\frac{1+x}{2} = \frac{7}{2}, 1 + x = 7, x = 6\ C(6,6)$

c) $y$ coordinate of the mid point of $AC$ is $\frac{2+6}{2} = 4$

d) $\frac{5+y}{2} = 4, 5 + y = 8, y = 3\ B(4,3)$

9) The centre of a circle is $(2,-3), AB$ is the diametre of the circle.If $B(4,-3)$ then

    a) What is the radius of the circle?

    b) Write the coordinates of the end $A$ of the diametre

    c) $CD$ is another diametre perpendicular to $AB$. Write the coordiantes of $C$ and $D$

    d) What is the area of $ACBD$?

a) $AB$ is parallel to $x$ axis. Radius $OB = |\ 4 - 2\ | = 2$

b) $A(2 - 2, -3) \rightarrow A(0, -3)$

c) $CD$ is parallel to $y$ axis . $C(2, -3 + 2) \rightarrow C(2, -1)$
$D(2, -3 - 2) \rightarrow D(2, -5)$

d) Area of $ABCD = \frac{d^2}{2} = \frac{4^2}{2} = 8$ sq.unit

10) Two vertices of $\triangle ABC$ are on $x$ axis .If $A(1,3)$ then
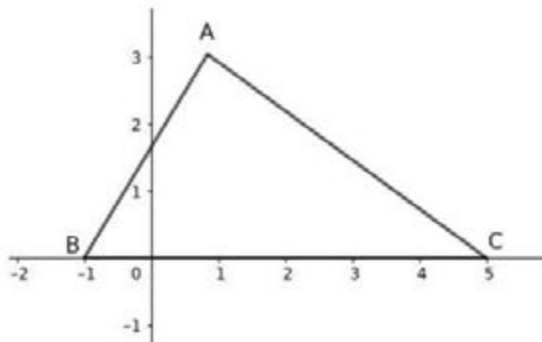


    a) Write the coordiantes of $B$ and $C$ from the figure

    b) Find the point at which the median from $A$ intersect $BC$

a) $B(-1,0), C(5,0)$

b) Mid point of $BC$ is the point where the median intersect the side $BC$ .$M(\frac{-1+5}{2}, 0) = M(2,0)$

11) Draw $x$ axis and $y$ axis (rough diagram), mark the points $A(4,3)$ and $B(12,7)$

    a) What is the slope of this line?

    b) Write the coordinates of another point on this line?

    c) How many lines are there having the same slope?

12) Consider the points $A(2,3), B(3,4), C(4,5)$

    a) Find the slope of the line passing through $A(2,3)$ and $B(3,4)$

    b) Find the slope of the line passing through $B(3,4)$ and $C(4,5)$

    c) Are these points on a line? How can we realize it.

    d) Write the coordinates of one more point on the line?

**Answers**

    a) Slope of $AB$ is $= \frac{4-3}{3-2} = 1$

    b) Slope of $BC$ is $= \frac{5-4}{4-3} = 1$

    c) Slope of $AB$ and slope of $BC$ are equal. $B$ is the common point . So $A, B, C$ are on a line.

    d) Find the linear relation between the coordinates of points. Using this relation we can write points .

13) Consider the points $A(2,0), B(-6,-2), C(-4,-4), D(4,-2)$

    a) Find the slope pf the lines $AB$ and $CD$

    b) Find the slope of the line $AD$ and $BC$

    c) Is $ABCD$ a parallelogram ?Explain

**Answers**

    a) Slope of $AB$ is $\frac{-2-0}{-6-2} = \frac{-2}{-8} = \frac{1}{4}$
Slope of $CD$ is $= \frac{-2-^-4}{4-^-4} = \frac{2}{8} = \frac{1}{4}$
$AB$ is parallel to $CD$

    b) Slope of $AD$ is $= \frac{-2-0}{4-2} = \frac{-2}{2} = -1$
Slope of $BC$ is $\frac{-4-^-2}{-4-^-6} = \frac{-2}{2} = -1$
$AD$ is parallel to $BC$

    c) Since opposite sides are parallel $ABCD$ is a parallelogram.

14) $A(1,-2), B(x,4)$ are the points on a line of slope $3$.

    a) Find $x$

    b) Write the coordinates of another point on this line

    c) Find the point at which the line cut $x$ axis

    d) Find the point at which the line cut $y$ axis

**Answers**

    a) Slope : $\frac{y_2-y_1}{x_2-x_1} = 3, \frac{4-^-2}{x-1} = 3, \frac{6}{x-1} = 3, 3x-3 = 6, 3x = 9, x = 3$ , $B(3,4)$

    b) Since $3$ is the slope another point is $C(3+1, 4+3) \rightarrow C(4,7)$

    c) $y$ coordinate of the point on $x$ axis is $0$ The point is $P(x,0)$ Consider $A(1,-2)$ ,and $(x,0)$
$\frac{-2-0}{1-x} = 3, x = \frac{5}{3}, P(\frac{5}{3}, 0)$

    d) $x$ coordinate of the point on $y$ axis is $0$. Point is $Q(0,y)$. $\frac{y-^-2}{0-1} = 3, y = -5$ $Q(0,-5)$

15) $A(-4,2), B(2,6), C(8,5), D(9,-7)$ are the vertices of a quadrilateral

    a) Find the coordinates of the mid point of the sides.

b) Prove that the quadrilateral formed by the mid points is a parallelogram

c) Find the coordinaters of the point where the diagonals intersect.

**Answers**

a) Mid point of $AB$ is $P(\frac{-4+2}{2}, \frac{2+6}{2}) \rightarrow P(-1, 4)$
   Mid point of $BC$ is $Q(\frac{2+8}{2}, \frac{6+5}{2}) \rightarrow Q(5, \frac{11}{2})$
   Mid point of $CD$ is $R(\frac{8+9}{2}, \frac{5+-7}{2}) \rightarrow R(\frac{17}{2}, -1)$
   Mid point of $AD$ is $S(\frac{-4+9}{2}, \frac{2+-7}{2}) \rightarrow S(\frac{5}{2}, -\frac{5}{2})$

b) Find slope of $PQ$ and $RS$. These are found to be equal. Both are $\frac{1}{4}$ . $PQ$ is parallel to $RS$.
   Slopes of $PS$ and $QR$ are equal . Slopes are equal. It is a parallelogram

c) Try yourself

16) $A(-4, 3), B(7, 3), C(5, 1), D(-2, 1)$ are the vertices of a quadrilateral.

   a) Find the slope of the sides $AB$ and $CD$

   b) Prove that $ABCD$ is an isosceles trapezium

   c) Find the area of the trapezium

   d) Calculate the perimetre of the trapezium

**Answers**

a) $y$ coordinates of $A, B$ are equal . $AB$ is parallel to $x$ axis. Slope of $AB$ is 0
   $y$ coordinates of $C, D$ are equal . $CD$ is parallel to $x$ axis. slope 0

b) Since $AB$ is parallel to $CD$ , $ABCD$ is a parallelogram.
   $AD = \sqrt{(-2 -^- 4))^2 + (1 - 3)^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$
   $BC = \sqrt{(5 - 7))^2 + (1 - 3)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$
   $AD = BC$, $AB$ is parallel to $CD$. It is isosceles trapezium

c) $AB =| 7 -^- 4 |= 11$, $CD =| 5 -^- 2 |= 7$, Distance between the parallel sides $=| 3 - 1 |= 2$
   Area $\frac{1}{2} \times 2 \times (11 + 7) = 18$

d) Perimetre $= 11 + 7 + 2\sqrt{2} + 2\sqrt{2} = 18 + 4\sqrt{2}$

# Chapter 10:

# POLYNOMIALS

1) Consider the polynomial $p(x) = x^2 - 7x + 12$

    a) Write $p(x) = (x - a)(x - b)$.

    b) Write $p(x)$ as the product of two first degree polynomials.

    c) Find the solution of the equation $p(x) = 0$

**Answers**

    a) $x^2 - 7x + 12 = (x - a)(x - b) = x^2 - (a + b)x + ab$
       $a + b = 7, ab = 12$

$$(a - b)^2 = (a + b)^2 - 4ab$$
$$(a - b)^2 = (7)^2 - 4 \times 12 \rightarrow a - b = \pm 1$$

      If $a - b = 1$ then, $a - b = 1, a + b = 7 \rightarrow 2a = 8, a = 4, b = 3$
      (What change occur in the answer corresponding to $a - b = -1$. Try yourself )

    b) $p(x) = (x - 4)(x - 3)$

    c) $p(x) = 0 \rightarrow (x - 4)(x - 3) = 0$
      $x = 3, 4$

2) Consider the polynomial $p(x) = x^3 - 4x^2 + 2x + k$

    a) If $x$ is a factor then find k.

    b) $x - 1$ is a first degree factor of $p(x)$ then what is $k$?

    c) Write the polinomial giving the value og k for x-1 in a factor.

    d) Is $(x + 1$ a factor of this polynomial .

a) $k = 0$

b) If $x - 1 = 0$ then $p(1) = 0$
$1^3 - 4 \times 1^2 + 2 \times 1 + k = 0, k = 1$

c) $p(x) = x^3 - 4x^2 + 2x + 1$

d) $p(-1) = (-1)^3 - 4(-1)^2 + 2(-1) + 1 = -1 - 4 - 2 + 1 \neq 0$
$x + 1$ is not a factor

3) Consider the polynomial $p(x) = x^2 - 8x + 12$

a) If $p(x) = (x - a)(x - b)$ then what is $a + b$ and $ab$

b) Find $a, b$ and write $p(x)$ as the product of two first degree factors

c) Find the solution of the equation $p(x) = 0$

a) $x^2 - 8x + 12 = (x - a)(x - b) = x^2 - (a + b)x + ab, a + b = 8, ab = 12$

b) $(a - b)^2 = (a + b)^2 - 4ab$
$(a - b)^2 = 8^2 - 4 \times 12 = 16, a - b = 4.$
$a + b = 8, a - b = 4 \rightarrow 2a = 12, a = 6, b = 2$
$p(x) = (x - 6)(x - 2)$

c) $p(x) = 0 \rightarrow (x - 6)(x - 2) = 0, x = 6, x = 2$

4) If $p(x) = x^3 - 4x^2 + 6x - k$ then

a) Find $k$ such that $x - 1$ a factor of $p(x)$

b) Write the polynomial . Is $(x + 1)$ a factor of $p(x)$

c) What is the speciality of the coefficients of $p(x)$ having $x - 1$ a factor

d) Write three polynomials having $x - 1$ a factor

a) Since $(x - 1)$ a factor $p(1) = 0$.
$1^3 - 4 \times 1^2 + 6 \times 1 - k = 0, 1 - 4 + 6 - k = 0, k = 3$

b) $p(x) = x^3 - 4x^2 + 6x - 3$
$p(-1) = (-1)^3 - 4 \times (-1)^2 + 6 \times (-1) - 3 = -1 - 4 - 6 - 3 = -14 \neq 0$
$p(-1) \neq 0$. Therefore $(x + 1)$ not a factor.

c) Sum of the coefficients will be zero $(x - 1)$

d) It can be ant polynomial with sum of the coefficients is zero .
$x^3 - x^2 + x - 1, 2x^3 - 4x^2 + 5x - 3, x^3 - 4x^2 + 2x + 1$

5) Consider the polynomials $p(x) = x^3 + 1$ , $q(x) = x^3 + x^2 + x + 1$

a) Find $p(-1)$ and $q(-1)$

b) What is the factor common to both the polynomials

a) $p(-1) = (-1)^3 + 1 = -1 + 1 = 0$
$q(-1) = (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$

b) $p(-1) = 0, q(-1) = 0$ implies $(x + 1)$ is a factor of both .$(x + 1)$ is the common factor

6) If $p(x) = x^3 - 8$ then

    a) Check whether $x - 2$ a factor of $p(x)$

    b) Write a first degree factor of $x^3 - 27$

**Answers**

    a) $p(2) = 2^3 - 8 = 8 - 8 = 0$
       $x - 2$ is a factor of $p(x)$

    b) $q(x) = x^3 - 27$ implies $q(3) = 3^3 - 27 = 27 - 27 = 0$
       $x - 3$ is a factor of $x^3 - 27$

7) Consider the equation $p(x) = x^3 + 4x^2 + x - 7$

    a) Check whether $x - 1$ a factor of this polynomial or not

    b) If not, which number should be subtracted from $p(x)$ to get another polynomial
       $q(x)$ in which $x - 1$ is a factor

**Answers**

    a) $p(1) = 1^3 + 4 \times 1^2 + 1 - 7 = 6 - 7 = -1 \neq 0$
       $x - 1$ is not a factor

    b) Since $p(1) = -1$ ,on subtracting $-1$ from $p(x)$ we get $(x - 1)$ a factor .
       $q(x) = x^3 + 4x^2 + x - 6$

8) Consider the equation $p(x) = x^2 + 4x + k$

    a) If $k = 0$ write the first degree factors of $p(x)$

    b) If $k = 4$ what are the facors of this polynomial ?

    c) What is the **maximum value** for $k$ to get two first degree factors of $p(x)$?

**Answers**

    a) If $k = 0$ , $p(x) = x^2 + 4x$. $x$ is a factor. $x + 4$ is also a factor.
            $= $ x $($x $+ 4$ $)$

    b) If $x = 4$ then $x^2 + 4x + k = x^2 + 4x + 4 = (x + 2)(x + 2)$.Both factors are $x + 2$.

    c) $p(x) = x^2 + 4x + k$ .
       $(x - a), (x - b)$ are the factors $x^2 + 4x + k = (x - a)(x - b) = x^2 - (a + b)x + ab$
       $a + b = -4, ab = k$ . $(a - b)^2 = 4^2 - 4k$. If $k$ is greater than $4$ , $(a - b)^2$ become a
       negative number. It is meaningless. $k$ should be less than or equal to $4$.

9) Consider the polynomial $p(x) = x^2 + 6x + k$

    a) If $k = 0$ then what are the first degree factors of $p(x)$?

    b) What is the value of $k$ to get two equal first degree factors ?

    c) If $k = 8$ what are the first degree factors of $p(x)$?

**Answers**

    a) If $k = 0$ , $p(x) = x^2 + 6x \rightarrow x(x + 6)$. First degree factors are $x, x + 6$

    b) $x^2 + 6x + k = x^2 + 2 \times 3 \times x + 3^2 \rightarrow (x + 3)^2$ .For this $k = 9$ . $x + 3, x + 3$ are equal
       the factors

    c) If $k = 8$ then $p(x) = x^2 + 6x + 8 = x^2 + 4x + 2x + 8 = x(x + 4) + 2(x + 4) = (x + 4)(x + 2)$
       First degree factors are $(x + 4), (x + 2)$

10) Consider the polynomial $x^2 + kx + 6$

    a) If $x - 1$ is a factor what is $k$?

    b) Write other first degree factor of this polynomial.

    c) Find the solution of $x^2 - 7x + 6 = 0$

### Answers

    a) If $x - 1$ is a factor sum of the coefficients will be $0$. $k + 7 = 0 \longrightarrow k = -7$

    b) $x^2 - 7x + 6 = x^2 - x - 6x + 6 = x(x-1) - 6(x-1) = (x-1)(x-6)$. Other factor is $x - 6$

    c) $x = 1, x = 6$

# CHAPTER 11. STATISTICS

### Focus Points

a) Statistics is the study of numerical data collected from a group , its analysis and interpretation .

b) Mean is an average calculated by dividing the sum of the numerical data by the number of observations.It considers each and every data collected from the group.

c) When the numbers are arranged either in increasing or decreasing order, the number that comes in the middle in the median

1) The marks obtained in ten class tests are given below

$$14, 17, 11, 19, 15, 17, 13, 10, 14, 18$$

    a) Calculate the mean of the marks .

    b) What are the marks comes in the middle if the marks are arranged in the increasing order?

    c) What is the median mark ?

    d) How many class tests are there with mark above median mark?

### Answers

    a) Mean $= \frac{14+17+11+19+15+17+13+10+14+18}{10} = 14.8$

    b) Marks in the ascending order $10, 11, 13, 14, 14, 15, 17, 17, 18, 19$
        $n = 10$ (even number ). 5 th and 6 th comes in the middle. These are 14 and 15

    c) median $= \frac{14+15}{2} = 14.5$

    d) There are 5 marks above median $14.5$

2) Atmospheric tempereature of seven days in a week are given below .

$$26°C, 28°C, 25°C, 30°C, 27°C, 26°C, 25°C$$

    a) Write the numbers in the ascending order.

    b) Calculate the mean of the temperatures.

    c) What is the median temperature?

    d) How many days are having temperature less than median temperature?

    e) How many days are having temperatures are there above median temperature?

### Answers

    a) $25°C, 25°C, 26°C, 26°C, 27°C, 28°C, 30°C$

    b) Mean temperature $= \frac{25+25+26+26+27+28+30}{7} = 26.7°C$

    c) 26 comes in the middle. Median is $26°C$

    d) 1 day

    e) 3 days

3) Consider the arithmetic sequence $7, 10, 13 \cdots$

    a) How many terms are there below 100?

    b) Which term comes in the middle?

    c) Calculate the mean of the numbers in the sequence below 100

    d) CXalculate the median of numbers in the sequence below100

    e) What is the relation between mean and median?

### Answers

    a) $x_n = 3n + 4$. $3n + 4 < 100 \to 3n < 96, n < 32$. There are 11 numbers below 100 in this sequence .

    b) Since 31 is odd $\frac{31+1}{2} = 16$th comes in the middle. $x_{16} = 3 \times 16 + 4 = 52$

    c) $x_{31} = 3 \times 31 + 4 = 97$. Sum $= (7 + 97) \times \frac{31}{2} = \frac{104 \times 31}{2} = 1612$
       Mean $= \frac{1612}{31} = 52$

    d) Middle term is the median .It is the 16 th term. $x_{16} = 3 \times 16 + 4 = 52$

    e) Mean and median are equal.

4) Consider th counting numbers from 1 to 100.

    a) How many multiples of 7are there below 100?

    b) Calculate the mean of the multiples of 7 below 100.

    c) What is the median of the multiples of 7 belw 100?

    d) How many multiples are there more are median in this collection ?

### Answers

    a) $7, 14, 21 \cdots 98$ are the numbers . $7n = 98 \to n = \frac{98}{7} = 14$.
       There are 14 numbers .

    b) Sum of multiples of 7 below $a00$ is $7 + 14 + 21 \cdots + 98 = \frac{14}{2} (7 + 98) = 105 \times 7 = 735$
       Mean $= \frac{735}{14} = 52.5$

    c) 7 th and 8 th comes in the middle . $x_7 = 7 \times 7 = 49, x_8 = 7 \times 8 = 56$. ഓധ്യാനം$= \frac{49+56}{2} = 52.5$

    d) $x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}$ are more than $52.5$
       There are 7numbers above median

5) The algebraic form of an arithmetic sequence is $3n + 2$

    a) Write the sequence

    b) Calculate the mean of first 20 terms.

    c) Calculate the median of the first 20 numbers of this sequence number of this sequence .

    d) What is the relation between mean and median.

6) Answer the following questions.

   a) Find the mean of 100 odd numbers .

   b) Find the median of first 100 odd numbers

   c) What is the mean of first $n$ even numbers ?

   d) What is the median of first $n$ even numbers ?