7.5.1 Explosion of a Shell in Flight

Consider a shell projected upwards. Its path is a parabola due to gravity. While in flight it explodes at P, and different fragments travel along their own parabolic path as shown in fig. 6.

Now also the centre of mass of all the fragments continue in the same parabolic path APB. This is because the explosion is only due to internal forces.

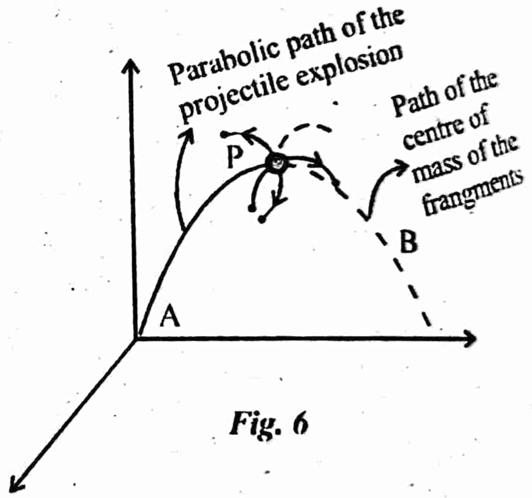

Fig. 6

7.5.2 Decay of Radioactive Nucleus

Consider a radioactive nucleus at rest. Let it decay spontaneously into two fragments. The two fragments will fly apart in opposite directions by obeying the laws of conservation of momentum and energy. Hence the centre of mass of the fragments remain at rest. This is also due to internal forces.

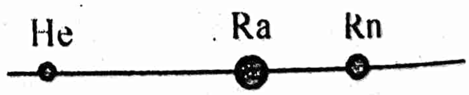

Fig. 7

7.6 Vector Product of Two Vectors

In the earlier chapter, we have discussed the scalar product of two vectors. The scalar quantity work is defined as the scalar product of two vectors, force ($\vec{F}$) and displacement ($\vec{S}$).

Here in this chapter, we have the product of two vectors, which results in a vector quantity itself. The quantities involved in rotational motion, namely torque and angular momentum come under this category, the vector products.

7.6.1 Definition of Vector Product (Cross Product)

A vector product of two vectors **a** and **b** is a vector **c** such that

i. magnitude of **c**, $c = ab\sin\theta$ where a and b are the magnitudes of **a** and **b** and θ is the angle between the two vectors.

ii. **c** is perpendicular to the plane containing **a** and **b**.

iii. If we take a right handed screw with its head lying in the plane of **a** and **b** and the screw perpendicular to this plane and if we turn the screw from the direction of **a** to **b**, then the tip of the screw advances in the direction of **c**.

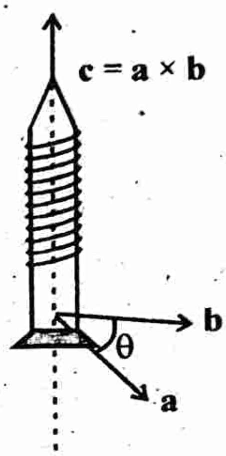

Fig. 8

Right hand thumb rule is also used to determine the direction of the vector product of two vectors. When we curl up the fingers of right hand from the direction of **a** to **b**, then the stretched thumb points in the direction of **c**.

7.6.2 Properties of Vector Products

i. The vector product is not commutative i.e., $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$

ii. Under reflection, the value of vector product does not change [$-\mathbf{a}$ is the reflection of **a**]

i.e., $(-\mathbf{a}) \times (-\mathbf{b}) = \mathbf{a} \times \mathbf{b}$

iii. Vector product is distributive with respect to vector addition i.e., $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c})$

iv. $\mathbf{a} \times \mathbf{a} = aa\sin 0° = 0$ (i.e., the vector product of the same vector results in a null vector). If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the orthogonal unit vectors along X, Y and Z directions respectively, then it results in $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

Similarly, $\hat{i}\times\hat{j} = |\hat{i}||\hat{j}|\sin 90° \times \hat{k} = \hat{k}$, $\quad \hat{j}\times\hat{i} = -\hat{k}$

$$\hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{j} = -\hat{i}$$

$$\hat{k}\times\hat{i} = \hat{j} \qquad \hat{i}\times\hat{k} = -\hat{j}$$

Now $\mathbf{a} \times \mathbf{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) \times (b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$

$$= a_x b_y\hat{k} - a_x b_z\hat{j} - a_y b_x\hat{k} + a_y b_z\hat{i} + a_z b_x\hat{j} - a_z b_y\hat{i}$$

$$= (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

In order to get the same expression, we can use determinant form, which is given as

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Solved Examples

4. Find the scalar and vector products of two vectors $\mathbf{a} = 2\hat{i} + 7\hat{j} - \hat{k}$ and $\mathbf{b} = 4\hat{i} - 3\hat{j} - 4\hat{k}$.

Sol. Scalar product of two vectors is given by **a** . **b**

$= (2\hat{i} + 7\hat{j} - \hat{k}).(4\hat{i} - 3\hat{j} - 4\hat{k})$

$= 8 - 21 + 4 = -9$

Vector product of **a** and **b** is given by

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 7 & -1 \\ 4 & -3 & -4 \end{vmatrix}$$

$= \hat{i}(-28-3) - \hat{j}(-8+4) + \hat{k}(-6-28)$

$= -31\hat{i} + 4\hat{j} - 34\hat{k}$

7.7 Angular Velocity and its Relation with Linear Velocity

In the earlier discussions on the rotational motion of a rigid body about a fixed axis, all the particles of the body move in a circle with the fixed axis as its centre.

Consider a particle P of a rigid body rotating about a fixed Z-axis suffers an angular displacement of $\Delta\theta$ in a small interval of time Δt as in fig. 9. Its instantaneous angular velocity is given by

$$\omega = \underset{\Delta t \to 0}{\text{Lt}} \left(\frac{\Delta\theta}{\Delta t}\right) = \frac{d\theta}{dt} \quad \text{............ (1)}$$

If Δx is the arc traced by a rotating particle in a time Δt, at a distance of 'r' from the axis of rotation, then

$$\Delta x = r\,\Delta\theta$$

$$\frac{\Delta x}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \quad \text{or} \quad \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta x}{\Delta t} = r.\ \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta\theta}{\Delta t}$$

i.e., $\dfrac{dx}{dt} = r.\dfrac{d\theta}{dt}$

$$\boxed{v = r\omega} \quad \text{..................(2)}$$

In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$

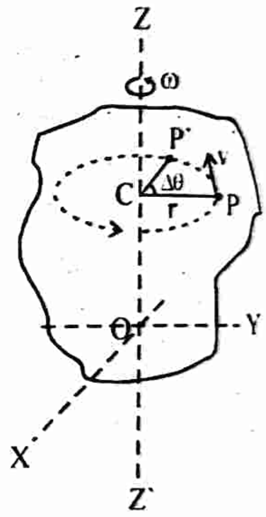

Fig. 9

Rotation about a fixed axis

For a particle on the axis, r = 0 and hence v = 0. Thus, particles on the axis are stationary and the axis is fixed.

Since in the mean time Δt, all the particles of the rigid body move through the same angle, all the particles possess the same angular velocity. Hence ω is considered as the uniform angular velocity of the whole body.

Angular velocity is a *vector quantity* whose direction is along the axis of rotation of the body and points out in the direction in which a right handed screw would advance, when its head is rotated with the body (fig. 10).

Unit of angular velocity is rad s^{-1} and its dimension is $M^0L^0T^1$.

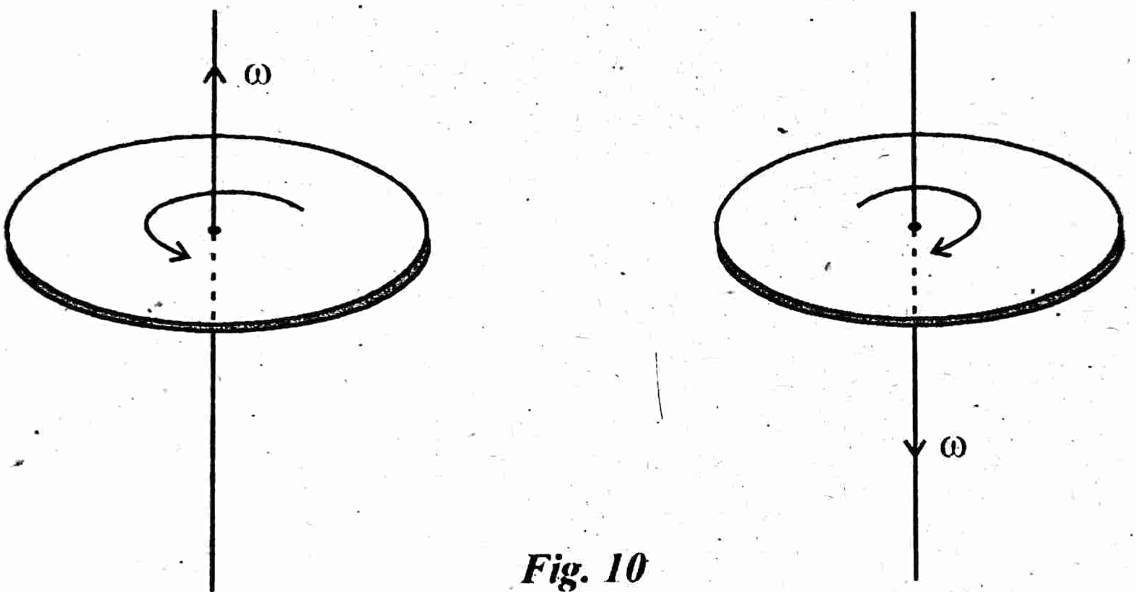

Fig. 10

If the head of a right handed screw rotates with the body, the screw advances in the direction of the angular velocity ω

If the sense (clockwise or anticlockwise) of rotation of the body changes, so does the direction of ω