Assignment:

Using PMI, prove that

- $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

- $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \dfrac{n(n+1)(n+2)(n+3)}{4}$

- $x^{2n} - y^{2n}$ is divisible by $(x+y)$

Ans)i) $f(1)=\left(\frac{1\times 2}{2}\right)^2=1=1(true)$

$$f(2)=\left(\frac{2\times 3}{2}\right)^2=9=1+8(true)$$

Let f(n) is true

We now, show for n=n+1

$$f(n+1)=\frac{n^2(n+1)^2}{2}+(n+1)^3=f(n+1)$$

$$=(n+1)^2\left[\frac{n^2}{4}+(n+1)\right]$$

$$=(n+1)^2\left[\frac{n^2+4n+4}{4}\right]$$

$$=(n+1)^2\left[\frac{(n+2)^2}{4}\right]$$

$$=\frac{1}{4}(n+1)^2(n+2)^2$$

$$=\frac{1}{4}[(n+1)(n+2)]^2$$

Hence Proved

Ans)ii)

$$1.2.3 + 2.3.4 + 3.4.5 + \ldots\ldots\ldots\ldots\ldots\text{nterms}$$

$$t_n = n(n+1)(n+2)$$

$$= n\left(n^2 + 3n + 2\right)$$

$$= n^3 + 3n^2 + 2n$$

$$S_n = \sum t_n$$

$$= \sum n^3 + 3\sum n^2 + 2\sum n$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + 3\left(\frac{n(n+1)(2n+1)}{6}\right) +$$
$$\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{2} + \frac{n(n+1)}{1}$$

$$= n(n+1)\left[\frac{n(n+1)}{4} + \frac{2n+1}{2} + 1\right]$$

$$= n(n+1)\left[\frac{n(n+1)+2(2n+1)+4}{4}\right]$$

$$= n(n+1)\left[\frac{n^2+n+4n+2+4}{4}\right]$$

$$= n(n+1)\left[\frac{n^2+5n+6}{4}\right]$$

$$= \frac{n(n+1)(n+2)(n+3)}{4}$$

Ans)iii)

Let $\mathbf{P}(\mathbf{n}) : \mathbf{x}^{2n} - \mathbf{y}^{2n} = (\mathbf{x} + \mathbf{y}) \times \mathbf{d}$ where $\mathbf{d} \epsilon \mathbf{N}$

For $\mathbf{n} = 1$

LHS $= \mathbf{x}^{2\times1} - \mathbf{y}^{2\times1}$

$= \mathbf{x}^2 - \mathbf{y}^2$

$(\mathbf{x} + \mathbf{y})(\mathbf{x} - \mathbf{y})$

=RHS

$\therefore \mathbf{P}(\mathbf{n})$ is true for $\mathbf{n} = 1$

Assume $\mathbf{P}(\mathbf{k})$ is true.

$\mathbf{x}^{2k} - \mathbf{y}^{2k} = (\mathbf{x} + \mathbf{y}) \times \mathbf{m}$ where $\mathbf{m} \epsilon \mathbf{N}$

We will prove that $\mathbf{P}(\mathbf{k} + 1)$ is true.

LHS $= \mathbf{x}^{2\times(k+1)} - \mathbf{y}^{2\times(k+1)}$

$= \mathbf{x}^{2k+2} - \mathbf{y}^{2k+2}$

$= \mathbf{x}^{2k}\mathbf{x}^2 - \mathbf{y}^{2k}\mathbf{y}^2$

$= (\mathbf{x} + \mathbf{y})[\mathbf{m}\mathbf{x}^2 + \mathbf{y}^{2k}(\mathbf{x} - \mathbf{y})]$

$= (\mathbf{x} + \mathbf{y}) \times \mathbf{r}$

where, $\mathbf{r} = [\mathbf{mx}^2 + \mathbf{y}^{2k}(\mathbf{x} - \mathbf{y})]$

$\therefore$ $\mathbf{P}(\mathbf{k}+1)$ is true whenever $\mathbb{P}(\mathbf{k})$ is true.

$\therefore$ By the principle of mathematical induction, $\mathbf{P}(\mathbf{n})$ is true for n, where n is a natual number.