Assignment:

$2^n > n$, for every natural number n.

Ans) **Consider a function**

$$\mathbf{f(n) = 2^n - n}$$

Therefore

$$\mathbf{f'(n) = 2^n \log 2 - 1}$$

Now

$\mathbf{f'(n) > 0}$ **implies**

$$\mathbf{2^n \log(2) > 1}$$

$$\mathbf{2^n > \frac{\log e}{\log 2}}$$

$$\mathbf{2^n > \log_2(e)}$$

$$\mathbf{n\log 2 > \log(\log_2(e))}$$

$$\mathbf{n > \log_2(\log_2(e))}$$

Now

$\mathbf{2 < e < 3}$

$\mathbf{1 < \log_2(e) < \log_2(3)}$

$\mathbf{1 < \log_2(e) < 1.58}$

$\mathbf{\log_2(1) < \log_2(e) < \log_2(1.58)}$

$\mathbf{0 < \log_2(\log_2(e)) < 0.7}$

Hence

$\mathbf{n > \log_2(\log_2(e))}$

$\mathbf{n > 0.7}$

Thus

$\mathbf{f(n)}$ is increasing for all $\mathbf{n > 0.7}$.

But $\mathbf{n}$ is a natural number.

Hence

$\mathbf{f(n)}$ is increasing for all natural numbers

$\mathbf{n \in N}$

Hence

$\mathbf{2^n - n > 0}$

$\mathbf{2^n > n}$ for all $\mathbf{n \in N}$