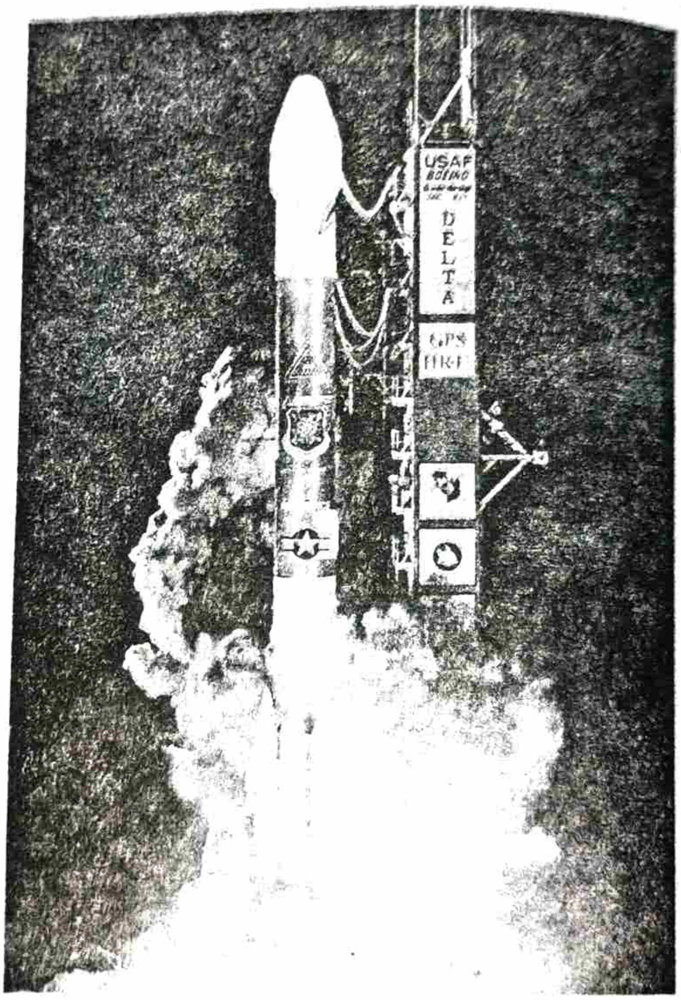
Chapter 8

Gravitation

INSIDE



8.1 Introduction

The force of attraction between any two bodies, on account of their masses is called gravitation and the force of attraction between earth and any body in this Universe is called gravity. Gravitational force is one of the fundamental forces in nature. Galileo, Newton, Kepler, Cavendish, Einstein, etc were the major contributors in the study of gravitation.

Curriculum Objectives

- To develop the concept of universal law of gravitation and the value of G through discussion and solving problems.
- To create the ideas of force of gravity and variation of g with altitude, depth and rotation of earth through discussion.
- To develop the concept of free fall and weightlessness through experiment and discussion.
- To derive the concept of gravitational potential and gravitational PE near the surface of the earth through discussion and drawing figures.
- To get an awareness of artificial satellites, escape velocity, orbital velocity, motion of satellites, geo-stationary satellites, polar satellites through discussion, IT and solving numerical problems.
- To understand the concept of Kepler's laws of planetary motion and derive its mathematical form through discussion.

The effect of force of gravity on falling bodies was established by Galileo (1564 - 1642) through public demonstration. According to him, irrespective of their masses, all bodies, fall towards earth with the same acceleration.

As we observe the positions of stars in the sky, we particularly notice that their positions are unchanged year after year. Ptolemy about 2000 years ago introduced the 'geocentric' model to explain the motion of planets. Later Copernicus (1473 - 1543) put forward the 'heliocentric' theory in which the Sun is at the centre of planetary system.

8.2 Kepler's Laws

The solar system consists of sun, planets, satellites etc. It was Ptolemy - a Greek astronomer, who put forward the first theory on the motion of the planets and sun. According to his geo-centric theory, earth is the centre of all this solar system and planets and sun revolve round the earth in fixed circular orbits.

By the fourteenth century Copernicus introduced another theory, called Heliocentric theory. According to him, sun is the centre of the planetary system and all planets revolve round the sun in circular orbits. But the idea of elliptical orbits for planets were suggested by Kepler. The observations made by Kepler are given in the form of three laws, Kepler's laws of planetary motion.

Johannes Kepler (1571-1630) - *German Scientist and Astronomer. Founder of geometrical optics and laws of planetary motion.*

Kepler's Laws of Planetary Motion

i. Kepler's first law (Law of orbit)

Every planet revolves round the sun in an elliptical orbit with sun at one of its foci.

ii. Kepler's second law (Law of Area)

The radius vector, drawn from the sun to a planet, sweeps out equal areas in equal intervals of time. i.e., areal velocity of the planet around the sun is constant.

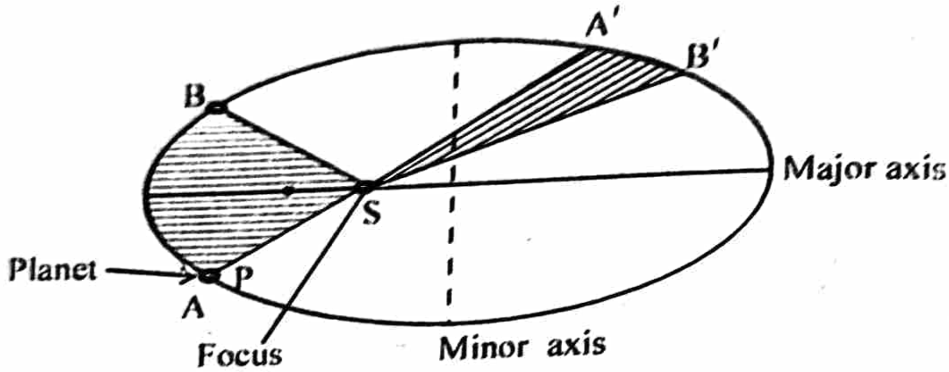


Fig. 1

Proof

A planet P is at B′ at $t = 0$ and travels to A′ in a time interval Δt. Let ΔA be the area described.

From figure, $\Delta A = \frac{1}{2} \times r \times (r\Delta\theta) = \frac{1}{2} r^2 \Delta\theta$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta\theta}{\Delta t}$$

$$\underset{\Delta t \to 0}{Lt} \frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \underset{\Delta t \to 0}{Lt} \left(\frac{\Delta\theta}{\Delta t}\right)$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega \quad \text{..... (1)}$$

Angular momentum, $L = mr^2\omega$ ...... (2)

$$\therefore \frac{dA}{dt} = \frac{L}{2m}$$

Torque exerted on the planet P about the sun S is zero (The line of action of the gravitational force passes through the axis).

$\therefore \tau = 0$ i.e., L = constant $\quad \therefore \frac{dA}{dt} = \text{a constant}$.

i.e., areal velocity of the planet is constant

iii. Kepler's third law (law of period or law of harmonic)

The square of the time period of a planet round the sun (or star) is proportional to the cube of the semi-major axis of its elliptical orbit. i.e., $T^2 \propto a^3$

Solved Examples

1. The distance of planet Jupiter from the sun is 5.2 times that of the earth. Find the period of revolution of Jupiter around the sun.

Sol. Given, $R_J = 5.2\ R_E$, $T_J = ?$, $T_E = 1$ year

$$\left(\frac{T_J}{T_E}\right)^2 = \left(\frac{R_J}{R_E}\right)^3$$

$$T_J = T_E\left(\frac{R_J}{R_E}\right)^{3/2} = 1\left(\frac{5.2R_E}{R_E}\right)^{3/2}$$

$= 11.86$ years

8.3 Universal Law of Gravitation

The event that inspired Newton to arrive at universal law of gravity was the fall of an apple. The explanation of terrestrial gravitation and a mathematical key to arrive at Kepler's laws can be obtained from universal law of gravitation.

If 'v' is the velocity of moon in its orbit of radius R_m, the centripetal acceleration is $a_m = \frac{v^2}{R_m}$. But $v = \frac{2\pi R_m}{T}$. $\therefore a_m = \frac{4\pi^2 R_m}{T^2}$, the substitution of $R_m = 3.84 \times 10^8$m and period T = 27.3 days gives us a value of a_m much less than usual value g = 9.8. From this Newton understood that earth's gravity decreases with distance. In agreement with central force conservation Newton concluded that gravitational force varies inversely as the square of the distance. i.e., $a_m \propto R_m^2$ and $g \propto R_E^{-2}$, so that $\frac{g}{a_m} = \frac{R_m^2}{R_E^2} = 3600$, $\therefore g = a_m \times 3600 = 0.002761 \times 3600 \approx 9.9$ in close agreement.

According to Newton's law of gravitation, every body in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Consider two bodies of masses m_1 and m_2 placed at locations A and B respectively. Let 'r' be the distance of separation between A and B. Then the gravitational force between them is

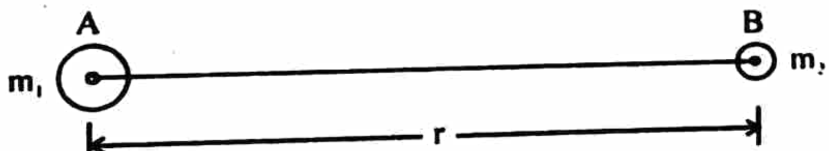


Fig. 2

i. $F \propto m_1 m_2$ .... (1)

ii. $F \propto \frac{1}{r^2}$ .... (2)

$F \propto \frac{m_1 m_2}{r^2}$ Hence, $F = \frac{G m_1 m_2}{r^2}$

where G is a constant called Universal gravitational constant.

The value of G is $6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$ in SI

Dimensional formula for G

$$G = \frac{Fr^2}{m_1 m_2} \qquad [G] = \frac{[ML^1T^{-2}][L^2]}{[M][M]} \qquad \therefore [G] = [M^{-1}L^3T^{-2}]$$