8.5.2 Mass and Density of Earth

The acceleration due to gravity at the surface of earth is given by

$$g = \frac{GM_E}{R_E^2} \qquad \therefore \; M_E = \frac{gR_E^2}{G}$$

Knowing the values of g, R_E and G, the mass of earth can be calculated

i.e., $R_E = 6.4 \times 10^6$ m, $\quad G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}, $\quad g = 9.8$ ms^{-2}

$$M_E = \frac{9.8 \times (6.4 \times 10^6)^2}{6.67 \times 10^{-11}} = 6.018 \times 10^{24} \text{ kg}$$

Consider earth to be a spherical body of radius R. If 'ρ' be the mean density of earth, then

$$\rho = \frac{\text{Mass}}{\text{Volume}} \quad \text{........ (1)} \qquad \rho = \frac{M_E}{\frac{4}{3}\pi R_E^3}$$

$$\rho = \frac{3M_E}{4\pi R_E^3} \quad \text{.......... (2)} \qquad \text{But} \quad M_E = \frac{gR_E^2}{G}$$

$$\therefore \rho = \frac{3gR_E^2/G}{4\pi R_E^3} \qquad \therefore \; \rho = \frac{3g}{4\pi R_E G} \quad \text{........ (3)}$$

Substituting standard values, we get

$$\therefore \rho = \frac{3\times9.8}{4\times\pi\times6.4\times10^{6}\times6.67\times10^{-11}} = 5478.4 \text{ kg m}^{-3} \approx 5.5\times10^{3} \text{ kg m}^{-3}$$

Solved Examples

4. If the radius of the earth were increased by three times the actual value, what would be the change in its density to keep 'g' the same?

Sol.

Let ρ be the density of the earth and R is its radius $g=\frac{GM_E}{R_E^2}$.

But $M_E = \frac{4}{3}\pi R_E^3\rho$

$$\therefore g = \frac{G\left(\frac{4}{3}\right)\pi R_E^3\rho}{R_E^2} = \frac{4G\pi R_E^3\rho}{3R_E^2} = \frac{4}{3}G\pi R_E\rho$$

$$\rho = \frac{3g}{4\pi GR_E} \quad \text{......... (1)}$$

When the radius becomes $3R_E$, let ρ' be its density keeping 'g' constant.

$$\rho' = \frac{3g}{4\pi G\times3R_E} = \frac{1}{3}\left(\frac{3g}{4\pi GR_E}\right) = \frac{1}{3}\rho \quad \text{....... (2)}$$

Hence the density of earth becomes 1/3 of its original value.

5. What will be the acceleration due to gravity on the surface of the moon, if its radius were $\frac{1}{4}^{th}$ the radius of earth and its mass $\frac{1}{80}^{th}$ mass of earth?

Sol.

Given, $M_m = \frac{1}{80}M_E$, $R_m = \frac{1}{4}R_E$,

$$g_E = \frac{GM_E}{R_E^2} \quad \text{——— (1)}$$

and $$g_m = \frac{GM_m}{R_m^2} \quad \text{——— (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \frac{g_m}{g_E} = \frac{GM_m}{R_m^2}\times\frac{R_E^2}{GM_E},$$

$$\frac{g_m}{g_E} = \frac{M_m}{M_E}\left(\frac{R_E}{R_m}\right)^2, \quad g_m = g_E\left(\frac{M_m}{M_E}\right)\left(\frac{R_E}{R_m}\right)^2$$

Given $\frac{M_m}{M_E} = \frac{1}{80}$, $\frac{R_E}{R_m} = 4$

$$\therefore g_m = g_E\left(\frac{1}{80}\right)(4)^2 = g_E\left(\frac{1}{80}\right)(16) = g_E\times\frac{1}{5}$$

$$g_m = \frac{g_E}{5}$$

6. If the radius of the earth shrinks by 2%, mass remaining same, then how would the value of acceleration due to gravity change?

Sol.

The acceleration due to gravity on the surface of earth is given by

$$g = \frac{GM_E}{R_E^2} \quad \text{......... (1)}$$

where $M_E \rightarrow$ mass of earth, $R_E \rightarrow$ Radius of earth

Taking log on both side of equation (1) we get, $M_E - 2\log R_E$ .... (2)

log g = log G + log

Differentiating it, $\frac{dg}{g} = 0 + 0 - 2\frac{dR}{R}$

$$\frac{dg}{g} = -2\left(\frac{dR}{R}\right) \quad \text{......... (3)}$$

When radius of earth shrinks by 2%,

then $\frac{dR_E}{R_E} = -\frac{2}{100}$ .... (4)

From (3) and (4), the% increase in g is $\frac{dg}{g}\times100 = -2\times\left(\frac{-2}{100}\right)\times100 = 4\%$

> **Easy way**
>
> $g \propto \frac{1}{R^2}$
>
> If R_E decreases by 2%, g increases by $2\times2\% = 4\%$

8.6 Acceleration Due to Gravity Below and Above the Surface of Earth

Even though the value of 'g' is treated as a constant ($9.8 ms^{-2}$) on the surface of earth, this value may change due to several factors like shape of earth, altitude, depth and rotation of earth.

8.6.1 Variation of 'g' Due to the Shape of Earth

Earth is not a perfect sphere. It is flattened at the poles and bulged out at the equator. Equatorial radius R_e of the earth is about 21 km larger than the polar radius R_p.

We have $g = \frac{GM_E}{R_E^2}$ ...... (1)

i.e., $g \propto \frac{1}{R_E^2}$

$\because$ $R_p < R_e$, $g_p > g_e$

i.e., the acceleration due to gravity at the pole is greater than that at the equator. Thus when a body is taken from equator to the pole, its weight increases.

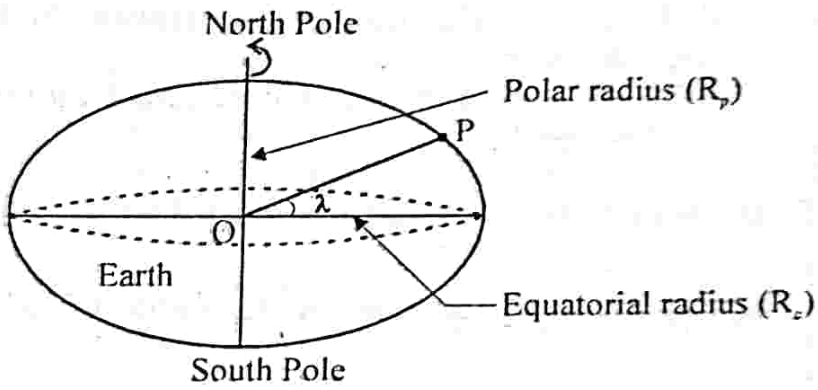


Fig. 7

You must know

At sea level, the value of g at pole is greater than its value at equator by 1.80 cm s^{-2}

8.6.2 Variation of 'g' With Altitude (Height)

Consider earth to be a perfect sphere of mass M_E and radius R_E.

The value of g on the surface of earth is, $g = \frac{GM_E}{R_E^2}$ .... (1)

Suppose the body is taken to a height 'h' above the surface of earth, the value of acceleration due to gravity is, $g_{(h)} = \frac{GM_E}{(R_E + h)^2}$ .... (2)

$$\frac{(2)}{(1)} \Rightarrow \frac{g_{(h)}}{g} = \frac{GM_E/(R_E+h)^2}{GM_E/R_E^2} \quad \therefore \frac{g_{(h)}}{g} = \frac{R_E^2}{(R_E+h)^2} = \frac{R_E^2}{R_E^2\left(1+\frac{h}{R_E}\right)^2}$$

$$= \frac{1}{\left(1+\frac{h}{R_E}\right)^2} = \left(1+\frac{h}{R_E}\right)^{-2} \quad \text{........ (3)}$$

Since h << R, then h/R_E is very small as compared to 1. Expanding the

R.H.S of the above equation by Binomial theorem and neglecting the higher powers of h/R_E, we get

$$\frac{g_{(h)}}{g} = \left(1 - \frac{2h}{R_E}\right) \qquad \therefore g_{(h)} = g\left(1 - \frac{2h}{R_E}\right) \text{......... (4)}$$

Eqn. (4) shows that the value of acceleration due to gravity decreases with height. It is due to this reason that the value of acceleration due to gravity is lesser at mountains than in plains.

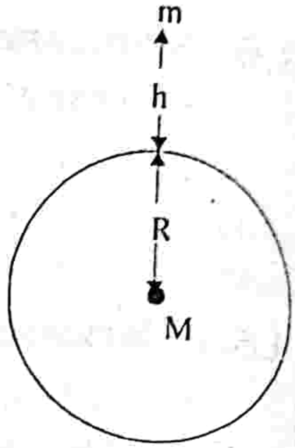


Fig. 8

You must know

i. When $h << R_E$, then the variation of g with height h is given by the equation (4).

ii. If the value of h is not very small as compared to R_E, then equation (3) holds good.

iii. With height h, the decrease in the value of g is $g - g_{(h)} = \frac{2hg}{R_E}$.

iv. Fractional decrease in the value of g = $\frac{g - g_{(h)}}{g} = \frac{2h}{R_E}$.

v. The percentage decrease in the value of $g = \frac{g - g_{(h)}}{g} \times 100 = \frac{2h}{R_E} \times 100$

Loss in weight at height $(h << R_E)$

We have, $g_{(h)} = g\left(1 - \frac{2h}{R_E}\right)$

$$\therefore \quad mg' = mg\left(1 - \frac{2h}{R_E}\right) = mg - \frac{2mgh}{R_E}$$

$$mg' - mg = \frac{-2mgh}{R_E}$$

i.e., $mg - mg' = \frac{2mgh}{R_E}$ $\qquad \therefore$ loss in weight $= \frac{2mgh}{R_E}$

TABLE 1

VARIATION OF g WITH ALTITUDE (AT 45° LATITUDE)

Altitude (in metre)	g (in ms^{-2})	Altitude (in metre)	g (in ms^{-2})
0	9.806	32,000	9.71
1,000	9.803	100,000	9.60
4,000	9.794	500,000	8.53
8,000	9.782	1,000,000	7.41
16,000	9.757	380,000,000	0.00271

8.6.3 Variation of g With Depth

Consider earth to be a homogeneous sphere of radius R_E and mass M_E. When a body of mass m is placed at the surface of earth, it is attracted towards the centre of earth. This gravitational pull is equal to the weight of the body.

i.e., $mg = \frac{GM_E m}{R_E^2}$ .... (1)

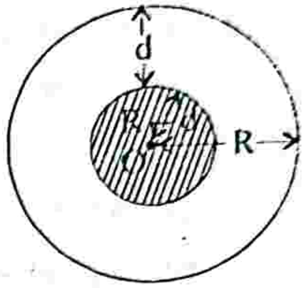


Fig. 9

If ρ is the mean density of earth, then, $M_E = \frac{4}{3}\pi R_E^3 \rho$

i.e., $mg = \frac{4}{3}\frac{\pi R_E^3 \rho m G}{R_E^2}$

$g = \frac{4}{3}\pi G R_E \rho$ .... (2)

If the body is kept at a depth of ' d ' from the surface of earth, then the mass of radius, R_E – d will only be effective for the gravitational pull towards the centre. If $g_{(d)}$ is the acceleration due to gravity at a depth of d,

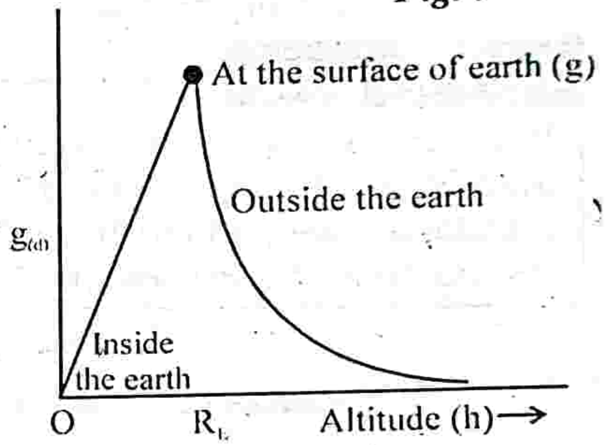


Fig. 10

$g_{(d)} = \frac{4}{3}\pi G(R_E - d)\rho$ .... (3)

$\frac{(3)}{(2)} \Rightarrow \frac{g_{(d)}}{g} = \frac{R_E - d}{R_E} = 1 - \frac{d}{R_E}$

$g_{(d)} = g\left(1 - \frac{d}{R_E}\right)$ .... (4)

Eq. (4) shows that the value of g decreases with depth.
At the centre of earth, d = R_E, $g_{(d)} = g_0$

$g_0 = g\left(1 - \frac{R_E}{R_E}\right) = 0$. Therefore the weight of body of mass 'm' at the centre of earth = mg_0 = Zero.

You must know

i. Decrease in the value of g with depth d, $= g - g_{(d)} = \frac{dg}{R_E}$

ii. Fractional decrease in the value of g with depth $= \frac{g - g_{(d)}}{g} = \frac{d}{R_E}$

iii. Percentage decrease in the value of g $= \frac{g - g_{(d)}}{g} \times 100 = \frac{d}{R_E} \times 100$

iv. The value of acceleration due to gravity at a height h is same as the value of acceleration due to gravity at a depth d (= 2h), provided if h is very small.

8.6.4 Variation of g With Rotation of the Earth

Let 'ω' be the angular velocity of earth. If a body is kept on the surface of earth at a latitude λ, the body also rotates with same angular velocity. The latitude of point P on the surface of earth is defined by angle λ made by radial line OP with equatorial plane.

$$mg' = mg - mR_E\omega^2\cos^2\lambda \quad \therefore \quad g' = g - R_E\omega^2\cos^2\lambda \quad \text{........ (1)}$$

At equator, $\lambda = 0^0$, $\cos\lambda = 1$ $\qquad g' = g - R_E\omega^2$ (minimum)

At pole $\lambda = 90^0, \cos\lambda = 0$ $\qquad g' = g$ (maximum)

Hence when a body is taken from the equator to the pole, the increase in the weight of the body is $mR\omega^2$.

You must know

If the earth stops rotating, the value of g at the equator shall increase by an amount 0.034 ms^{-2}. But no change in the value of g at the poles. If the rotational speed of earth increases, the value of g decreases at all places on the surface of earth except at poles.

TABLE 2

VARIATION OF g WITH LATITUDE (AT SEA LEVEL)

Latitude	g (in ms^{-2})	Latitude	g (in ms^{-2})
0°	9.78039	50°	9.81071
10°	9.78195	60°	9.81918
20°	9.78641	70°	9.82608
30°	9.79329	80°	9.83059
40°	9.80171	90°	9.83217

Solved Examples

7. At what height from the surface of earth will the value of 'g' be reduced by 36% from the value at the surface?. (Radius of earth is 6400 km.)

Sol.

Given, R_E = 6400 km = 6400 × 10^3 m

$$g_{(h)} = 64\% \text{ of } g = \frac{64}{100}\times g \ , \quad g_{(h)} = g\frac{R_E^2}{(R_E+h)^2}$$

$$(R+h)^2 = \frac{gR_E^2}{g_{(h)}} = \frac{gR_E^2}{\frac{64}{100}g} = \frac{100}{64}\times R_E^2$$

$$R_E + h = \frac{10}{8}R_E$$

$$h = \frac{10}{8}R_E - R_E = \left(\frac{10}{8}-1\right)R_E = \frac{2}{8}\times R_E$$

$$= \frac{2}{8}\times 6400\times 10^3 = 1600\times 10^3\,\text{m} = 1600\,\text{km}$$

8. At what height above the earth's surface the value of g is the same as in a mine 100 km deep?

Sol. $$g' = g\left(1-\frac{2H}{R_E}\right) = g\left(1-\frac{h}{R_E}\right)$$

h = 100 km

where H → height above the surface of earth

h → height below the surface of earth

$$\therefore \frac{2H}{R_E} = \frac{h}{R_E} \qquad \therefore 2H = h$$

$$H = \frac{h}{2} = \frac{100}{2} = 50 \text{ km}$$

9. If the earth were a perfect sphere of radius 6.37×10^6 m, rotating about its axis with a period of 1 day ($= 8.64 \times 10^4$ s), how much would differ from the pole to the equator. Take the acceleration due to gravity on the surface of earth as 10 ms^{-2}.

Sol. Given, $R_E = 6.37 \times 10^6$ m, $T = 8.64 \times 10^4$ s, $g = 10\ ms^{-2}$,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8.64 \times 10^4}$$

At poles, $\lambda = 90^0, g' = g_p$

$$g_p = g - R_E\omega^2 \cos^2 \lambda$$

$$= 10 - R_E\omega^2 \times \cos^2 90 = 10 - 0 = 10\ ms^{-2}$$

$\therefore$ $g_p = g$ ..... (1)

At equator, $\lambda = 0^0$, $g' = g_e$

$$g_e = g - R_E\omega^2\cos^2\lambda = g - R_E\omega^2\cos^2 0$$

$$g_e = g - R_E\omega^2 \text{ .... (2)}$$

$$g_e = g_p - R_E\omega^2 \text{ .... (3) } [\because g_p = g]$$

$$g_p - g_e = R_E\omega^2 = 6.37 \times 10^6 \times \left(\frac{2 \times 3.14}{8.64 \times 10^4}\right)^2$$

$$= 3.37 \times 10^{-2}\ ms^{-2}$$

8.7 Inertial and Gravitational Mass

***Mass** of a body is the amount of matter present in it.*

Mass which determines the inertia of a body is called inertial mass. According to Newton's second law, if a force F acts on a body of mass m_i and produces an acceleration 'a',

then, $F = m_i a$ or $m_i = \frac{F}{a}$ .... (1)

Here the mass m_i is called inertial mass and is defined as the ratio of the magnitude of force applied on a body to the magnitude of acceleration produced it.

Inertial mass is independent on the size, shape and state of the body. It does not change due to the presence of other bodies. If the velocity of the body is comparable with the velocity of light, then the inertial mass changes according to the relativistic equation

$$m_i = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m_0 is the rest mass of the body. i.e., Rest mass is the mass when the body is at rest and c is the velocity of light in vacuum or air ($= 3 \times 10^8\ ms^{-1}$)

The mass of a body which determines the gravitational force of the earth on it is called gravitational mass.

Consider a body of mass m_g placed on the surface of earth. The gravitational force on it is given by

$$F = \frac{GM_E m_g}{R_E^2},$$ where R_E - radius of earth, M_E - mass of earth

$$\therefore F = m_g g \quad \left(\because g = \frac{GM_E}{R_E^2}\right)$$

$m_g = \frac{F}{g}$ .... (2)

Hence gravitational mass is the ratio of the magnitude of gravitational force on a body due to the earth to the magnitude of acceleration due to gravity.

It can be shown that the gravitational mass is equal to inertial mass. Consider two bodies A and B of gravitational masses m_1 and m_2 kept near the surface of earth. The gravitational forces on them are

$F_1 = \frac{GM_E m_1}{R_E^2}$ and $F_2 = \frac{GM_E m_2}{R_E^2}$ respectively

$\frac{F_1}{F_2} = \frac{m_1}{m_2}$ .... (1)

Let m_1' and m_2' be their inertial masses. If they fall freely towards the earth, then the inertial forces acting on them are

$F_1 = m_1' g$ $F_2 = m_2' g$

For a freely falling body its acceleration is due to gravity.

$\frac{F_1}{F_2} = \frac{m_1'}{m_2'}$ .... (2)

From (1) and (2), $\frac{m_1}{m_2} = \frac{m_1'}{m_2'}$

This shows that $m_1 \propto m_1'$ or $m_1 = km_1'$ and $m_2 \propto m_2'$ or $m_2 = km_2$

$\therefore \frac{m_1}{m_2} = \frac{m_1'}{m_2'}$

Thus $m_1 = m_1'$ and $m_2 = m_2'$

$\therefore$ ***Gravitational mass is equal to inertial mass.***

8.7.1 Comparison of Inertial and Gravitational Masses

i. Both are scalar quantities.
ii. Both are measured in the same units.
iii. Both are directly proportional to the quantity of matter contained in a body.
iv. Both are equivalent to each other.
v. Both do not depend on the state or shape of the body.

You must know

i. The gravitational mass is measured by spring balance whereas inertial mass is measured by inertial balance.
ii. Gravitational mass is equal to inertial mass.

8.8 GRAVITATIONAL FIELD

The space around a body, within which its gravitational force of attraction is experienced by other bodies is called **gravitational field.** The presence of other masses can influence the properties of space around a body.

The ***intensity of gravitational field*** *at a point is measured as the force experienced by a unit mass placed at that point.*

The gravitational force between two masses M and m separated at a distance r, according to Newton is, $F = \frac{GMm}{r^2}$ .... (1)

If m = 1 kg, then $F = \frac{GM}{r^2} = E$, the intensity of gravitational field

i.e., $\vec{E} = \frac{\vec{F}}{m}$

$\vec{E}$ has the same direction as that of $\vec{F}$

At the surface of earth, $\vec{E} = \frac{GM_E}{R_E^2} = \vec{g}$ (acceleration due to gravity at the surface of earth)

Unit of intensity of gravitational field

In S.I. - newton kg^{-1} ($N\ kg^{-1}$) or ms^{-2}

In C.G.S - dyne g^{-1} or cms^{-2}

Dimensional Formula

$$E = \frac{F}{m} = \frac{[M^1L^1T^{-2}]}{[M^1]} = [M^0L^1T^{-2}]$$

8.8.1 Gravitational Potential

Gravitational potential *at a point in a gravitational field is defined as the work done in bringing a body of unit mass from infinity to that point.*

Consider a point P at a distance 'r' from a mass M

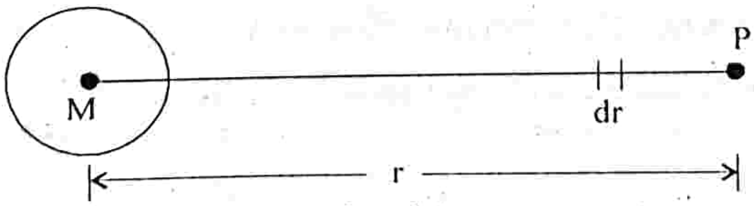


Fig. 11

The gravitational force acting per unit mass kept at P is

$F = \frac{GM \times 1}{r^2} = \frac{GM}{r^2}$ .... (1)

Let the mass be displaced through a distance of 'dr'. For that the work needed is, $dW = F.dr$

$$dW = \frac{GM}{r^2} dr \text{ ...... (2)}$$

∴ Total work done in bringing a unit mass from infinity to P, which is at a distance of r from M, is given by

$$W = \int dW = \int_{\infty}^{r} \frac{GM}{r^2} dr = GM \int_{\infty}^{r} \frac{dr}{r^2}$$

$$= GM\left[\frac{-1}{r}\right]_{\infty}^{r} = -GM\left[\frac{1}{r}\right]_{\infty}^{r} = -GM\left[\frac{1}{r} - \frac{1}{\infty}\right] = -GM\left[\frac{1}{r} - 0\right]$$

$$W = \frac{-GM}{r} \quad \ldots\ldots (3)$$

This work done is equal to the gravitational potential (V)

i.e., $V = -\frac{GM}{r}$ .... (4)

The gravitational potential is negative, since the work is done by the gravitational field and not against the field in bringing a unit mass from infinity to the point under consideration.

Unit of gravitational potential

In S.I. - joule/kg ($J\ kg^{-1}$)

In C.G..S - erg/g ($erg\ g^{-1}$)

Dimensional formula

$$V = \frac{W}{m} = \frac{[M^1L^2T^{-2}]}{[M^1]} = [M^0L^2T^{-2}]$$

You must know

i. Gravitational potential at a point is negative.
ii. It is a scalar quantity.
iii. When $r = \infty$, V = 0, i.e., gravitational potential is maximum (= zero) at infinity.

iv. At the surface of earth, r = R, $V = \frac{-GM_E}{R_E}$

8.9 Gravitational Potential Energy

Gravitational potential energy *is defined as the work done in bringing a mass from infinity to a point in the gravitational field of another body.*

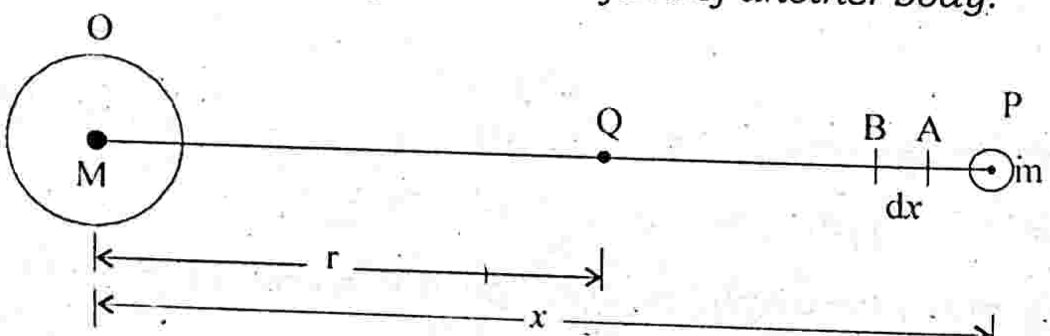


Fig. 12

Consider a body of mass 'M' kept at O. Let another body of mass m be placed at P, at a distance x from M as shown in the above figure.

The gravitational force between them is, $F = \frac{GMm}{x^2}$ .... (1)

In order to displace the body through dx, the work needed is

$$dW = F.dx = \frac{GMm}{x^2}dx \quad .... (2)$$

∴ The total work done in displacing the body of mass m from infinity to Q,

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r} \frac{GMm}{x^2}dx = GMm\int_{\infty}^{r}\frac{1}{x^2}dx = GMm\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$= -GMm\left[\frac{1}{x}\right]_{\infty}^{r} = -GMm\left[\frac{1}{r}-\frac{1}{\infty}\right] = -GMm\left[\frac{1}{r}-0\right]$$

$$W = -\frac{GMm}{r} \quad .... (3)$$

This amount of work done is stored in the body as its gravitational potential energy U.

$$\therefore \quad U = -\frac{GMm}{r} \quad .... (4)$$

Eqn. (4) shows that, Gravitational potential energy = gravitational potential × mass

If the body is displaced from r_1 to r_2 $(r_1 > r_2)$, then

$$U = GMm\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad .... (5)$$

8.9.1 Gravitational Potential Energy Near the Surface of Earth

If a body of mass 'm' is taken from the surface of earth to a height 'h', then in eqn. (5) we can substitute $r_1 = R_E$ and $r_2 = R_E + h$

$$\text{eqn.(5)} \Rightarrow U = GM_E m\left[\frac{1}{R_E}-\frac{1}{(R_E+h)}\right] = GM_E m\left[\frac{R_E+h-R_E}{R_E(R_E+h)}\right]$$

If $h << R_E$, $R_E + h \simeq R_E$

$\therefore U \simeq GM_E m \times \frac{h}{R_E^2}$ But $\frac{Gm}{R_E^2} = g$, the acceleration due to gravity at the surface of earth

$$U = mgh \quad .... (6)$$

Solved Examples

10. Determine the gravitational potential at the height of geostationary satellite. Given mass of earth is 6×10^{24} kg	***Sol.*** Given, $M_E = 6 \times 10^{24}$ kg, $r = R_E + h = 6400 + 36{,}000 = 42{,}400$ km $= 4.24 \times 10^7$m

$$U = \frac{-GM}{r} = \frac{-GM}{R_E + h}$$

$$= \frac{-6.67\times10^{-11}\times6\times10^{24}}{4.24\times10^{7}}$$

$$= -9.44 \times 10^{6}\ J\ kg^{-1}$$

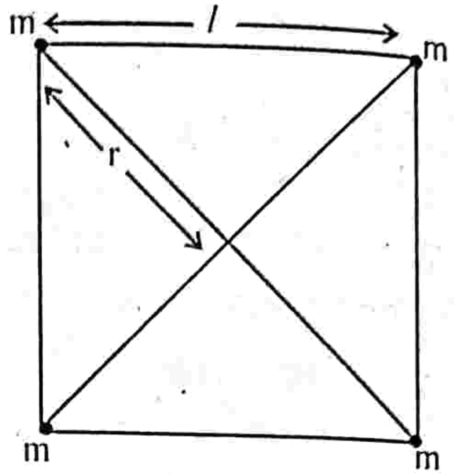
11. Find the potential energy of a system of four particles placed at the vertices of a square of side l. Also obtain the potential at the centre of the square.

Sol.

Consider four masses each of mass m at the corners of a square of side l. We have four mass pairs at distance l and two diagonal pairs at distance $\sqrt{2}\,l$.

Hence, $W_r = -4\frac{Gm^2}{l} - 2\frac{Gm^2}{\sqrt{2}\,l}$

$$= -\frac{2Gm^2}{l}\left(2+\frac{1}{\sqrt{2}}\right) = -5.41\frac{Gm^2}{l}$$



The gravitational potential at the centre of the square $\left(r = \frac{\sqrt{2}\,l}{2}\right)$ is

$$U_r = -4\sqrt{2}\frac{Gm}{l}$$

8.10 Escape Velocity

If a body is thrown upwards, its velocity decreases due to gravity. The height reached by the body can be increased by throwing it with a larger velocity.

Escape velocity *is the minimum velocity with which a body may be projected such that it escapes from the gravitational attraction of earth permanently.*

8.10.1 Expression for Escape Velocity

Consider a body of mass m kept at a height r from the centre of the earth.

The gravitational force acting on it,

$$F = \frac{GM_E m}{r^2} \quad \ldots\ldots (1)$$

Let the body be displaced through a small distance dr. Then the work done,

$$dW = \underline{F}.\underline{dr} = \frac{GM_E m}{r^2}dr$$

$\therefore$ The total work done to move the body from the surface of earth ($r = R_E$) to infinity is given by

$$W = \int_{R_E}^{\infty} \frac{GM_E m}{r^2}dr = GM_E m\int_{R_E}^{\infty}\frac{1}{r^2}dr = GM_E m\left[-\frac{1}{r}\right]_{R_E}^{\infty} = GM_E m\left[\frac{1}{R_E} - \frac{1}{\infty}\right]$$

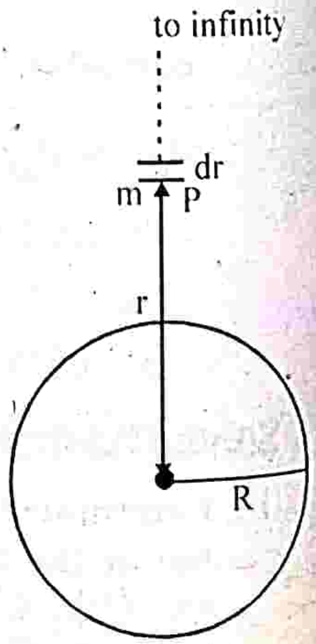


Fig. 13

$$W = \frac{GM_E m}{R_E} \quad ...... (2)$$

Let v_e be the escape velocity of the body, then KE of the body when projected is $= \frac{1}{2}mv_e^2$ ...... (3)

The body can escape from the gravitational pull of earth only if the KE is equal to the work done in overcoming the gravity

i.e., $\frac{1}{2}mv_e^2 = \frac{GM_E m}{R_E}$

$$v_e = \sqrt{\frac{2GM_E}{R_E}}$$

But $g = \frac{GM_E}{R_E^2}$ or $GM_E = gR_E^2$

$$v_e = \sqrt{2gR_E} \quad .... (4)$$

The escape velocity of a body from any planet,

$v_e = \sqrt{\frac{2GM_{Ep}}{R_{Ep}}}$ where M_{Ep} = mass of the planet, R_{Ep} = Radius of the planet

From the above equation it is clear that escape velocity is independent of the mass of the body thrown upwards, but depends on mass and size of the planet. Substituting the standard values in eqn (4),

i.e., $R_E = 6.4 \times 10^6$ m, $g = 9.8$ ms^{-2}, we get $v_e = 11.2$ km/s.

Note

$v_e = \sqrt{\frac{2GM_E}{R_E^2}}$ But $M_E = \frac{4}{3}\pi R_E^3 \rho$ where ρ is the mean density

$v_e = \sqrt{\frac{2G\frac{4}{3}\pi R_E^3 \rho}{R_E^2}}$, i.e., $v_e = \sqrt{\frac{8\pi\rho G R_E}{3}}$

You must know

The escape velocity of a body from the moon is 2.38 kms^{-1}. The root mean square velocity of gas molecule is ($\approx 2.5\,\text{kms}^{-1}$) more than escape velocity from the moon. Therefore gas molecules escape from the moon and hence there is no atmosphere on moon.