6.2 INEQUALITIES

Let us consider the following statements in Table 1 and Table 2

Table 1	Table 2
$2x + 5 = 0$	$5x + 7 \leq 0$
$5x + 6 = -2$	$-2x + 12 > 0$
$3x - 4y = 6$	$4x - 3y < -6$
$4x + 7y = 9$	$5x + 2 \geq 0$

Comparing Tables 1 and 2, we can see that all the statements in Table 1 contain the equality sign (=), and the statements in Table 2 contain inequality sign, called **inequalities**.

Definition

A statement involving variables and the sign of inequality ($<$, $\leq$, $>$, $\geq$, $\neq$) is called an inequality. If each term of an inequality is of first degree, then it is a **linear inequlality.**

- **Numerical inequalities:** $-3 < 3$, $4 > 2$, $5 > 0$
- **Literal inequalities:** $x < 4$, $y > -5$, $x \leq 3$, $y \geq 6$
  - *Strict inequalities*

$2x + 5 < 1$ $x - 6 > 7$	linear inequalities in one variable
$6x + 2y < -5$ $3x - y > 8$	linear inequalities in two variables
$2x^2 + 5x - 3 > 0$ $x^2 - 2x + 5 < -3$	quadratic inequalities in one variable

  - *Slack inequalities*

$2x + 5 \leq 1$ $x - 6 \geq 7$	linear inequalities in one variable
$6x + 2y \leq -5$ $3x - y \geq 8$	linear inequalities in two varbiables
$2x^2 + 5x - 3 \geq 0$ $x^2 - 2x + 5 \leq -3$	quadratic inequalities in one variable

6.3 ALGEBRAIC SOLUTIONS OF LINEAR INEQUALITIES IN ONE VARIABLE AND THEIR GRAPHICAL REPRESENTATION

Illustration 1

Consider the inequality $x < 5$, x is a real number.

$x = 1$,	$1 < 5$ is a true statement.	$x = 5$,	$5 < 5$ is a false statement.
$x = -1$,	$-1 < 5$ is a true statement.	$x = 7$,	$7 < 5$ is a false statement..
$x = 2.4$,	$2.4 < 5$ is a true statement.	$x = 5.2$,	$5.2 < 5$ is a false statement.
$x = 4$,	$4 < 5$ is a true statement.	$x = 10$,	$10 < 5$ is a false statement.

We observe that the values of x less than 5 make the inequality a true statement and we call them as the solutions of the inequality. Hence the solution of the inequality is the interval $(-\infty, 5)$.

Definition:

The solution of an inequality in one variable is the value of the variable(s) that makes the inequality a true statement.

- An inequality may or may not have a solution.
- If the inequality has a solution, then it may have infinitely many solutions.

REMEMBER THE FOLLOWING RULES

Rule 1 : When equal numbers are added to or subtracted from both sides of an inequality, the sign of inequality is not changed.

For example, $-2 < 5$ then, $-2 + 3 < 5 + 3$

Rule 2 : When both sides of an inequality are multiplied or divided by same positive number, the sign inequality is not changed.

For example, $-5 < 9$, then $-5(3) < 9(3)$

Rule 3 : When both sides of an inequality are multiplied or divided by the same negative number, the sign of the inequality is reversed.

For example, $-5 < 9$, then $-5(-3) > 9(-3)$

Illustration 2

Let us solve the inequality $5x - 3 < 3x + 1$, when x is an integer.

The inequality is $5x - 3 < 3x + 1$

Adding 3 to both sides, we get $5x - 3 + 3 < 3x + 1 + 3$

$$5x < 3x + 4$$

Subtracting $3x$ from both sides, we get

$5x - 3x < 3x + 4 - 3x \quad \Rightarrow 2x < 4$

Collect the variable to the left side, the constants to the right side

Dividing both sides by 2, we get $x < 2$

Divide by the coefficient of the variable

The inequality becomes a true statement when x takes integer values less than 2. Hence the solution is $1, 0, -1, -2, \ldots$

The solution can be represented on a number line as shown in Fig 6.1

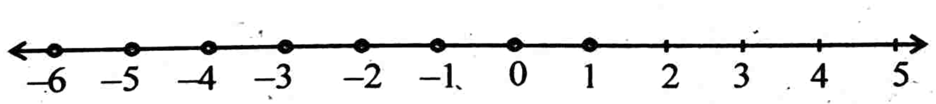

Fig. 6.1