**Example 48**

Solve $2x^2 + 3x + 1 = 0$

***Solution***

The equation is $2x^2 + 3x + 1 = 0$

By quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

i.e., $x = \frac{-3 \pm \sqrt{9 - 4(2)(1)}}{4} = \frac{-3 \pm \sqrt{1}}{4}$

$= \frac{-3+1}{4}$ or $\frac{-3-1}{4}$

$= \frac{-2}{4}$ or $\frac{-4}{4}$

$= \frac{-1}{2}$ or $-1$

**Example 49**

Solve the equation $x^2 + 1 = 0$ **(March 2011)**

***Solution***

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm\sqrt{-1} = \pm i$

**Example 50**

Solve $x^2 + 2x + 2 = 0$.

***Solution***

The given equation is $x^2 + 2x + 2 = 0$

By quadratic formula, $x = \dfrac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times 2}}{2 \times 1} = \dfrac{-2 \pm \sqrt{-4}}{2} = \dfrac{-2 \pm 2i}{2} = -1 \pm i$

$\therefore x = -1 + i$ or $x = -1 - i$

**Exampe 51**

Solve $2x^2 - \sqrt{3}\,x + 1 = 0$

***Solution***

$a = 2, \quad b = -\sqrt{3}, \quad c = 1$

$$\therefore x = \frac{\sqrt{3} \pm \sqrt{(-\sqrt{3})^2 - 4(2)(1)}}{4}$$

$$= \frac{\sqrt{3} \pm \sqrt{3-8}}{4} = \frac{\sqrt{3} \pm \sqrt{-5}}{4} = \frac{\sqrt{3} \pm i\sqrt{5}}{4}$$

$\therefore$ The solutions are $\dfrac{\sqrt{3} + i\sqrt{5}}{4}$ or $\dfrac{\sqrt{3} - i\sqrt{5}}{4}$

**Example 52** **(March 2013, NCERT)**

Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$

***Solution***

The equation is $\sqrt{5}x^2 + x + \sqrt{5} = 0$

By quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-1 \pm \sqrt{1 - 4\sqrt{5}.\sqrt{5}}}{2\sqrt{5}} = \frac{-1 \pm \sqrt{1-20}}{2\sqrt{5}} = \frac{-1 \pm \sqrt{-19}}{2\sqrt{5}} = \frac{-1 \pm \sqrt{19}i}{2\sqrt{5}}$$

**Example 53** (August 2009)

Solve the equation $ix^2 - x + 12i = 0$.

***Solution***

The quadratic equation is $ix^2 - x + 12i = 0$

Comparing with the standard form, we get $a = i$, $b = -1$, $c = 12i$

$b^2 - 4ac = (-1)^2 - 4(i)(12i) = 1 - 48(i)^2 = 1 + 48 = 49$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{49}}{2i} = \frac{1 \pm 7}{2i} \qquad x = \frac{1+7}{2i} \text{ or } x = \frac{1-7}{2i}$$

$$\text{i.e., } x = \frac{8}{2i} = \frac{4}{i} = -4i \text{ and } x = \frac{-6}{2i} = \frac{-3}{i} = 3i \qquad (since\ \frac{1}{i} = -i)$$

Thus $x = -4i$ or $x = 3i$

**Example 54**

Consider the equation $z^2 - 2z + 4 = 0$

i. Find two complex numbers $z_1$ and $z_2$ satisfying the above equation. (October 2011)

ii. Simplify $\frac{z_1}{z_2} + \frac{z_2}{z_1}$.

***Solution***

i. The given equation is $z^2 - 2z + 4 = 0$

Comparing with the standard form $a = 1, b = -2, c = 4$.

$$z = \frac{(-b) \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 16}}{2} = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2\sqrt{3}i}{2} = 1 \pm \sqrt{3}i$$

$$\therefore z_1 = 1 + \sqrt{3}i, \qquad z_2 = 1 - \sqrt{3}i$$

ii. $$\frac{z_1}{z_2} + \frac{z_2}{z_1} = \frac{z_1^2 + z_2^2}{z_1 z_2}$$

$$= \frac{(1+\sqrt{3}i)^2 + (1-\sqrt{3}i)^2}{(1+\sqrt{3}i)(1-\sqrt{3}i)}$$

$$= \frac{1 + 2\sqrt{3}i - 3 + 1 - 2\sqrt{3}i - 3}{1 - (-3)}$$

$$= \frac{-4}{4} = -1$$

## SOLUTIONS TO NCERT TEXT BOOK EXERCISE 5.3

**Solve each of the following equations.**

1. $x^2 + 3 = 0$

***Solution***

$x^2 + 3 = 0 \Rightarrow x^2 = -3$

$\therefore x = \pm\sqrt{-3} = \pm\sqrt{3}\,i$

2. $2x^2 + x + 1 = 0$

***Solution***

The quadratic equation is $2x^2 + x + 1 = 0$

Comparing with $ax^2 + bx + c = 0$, we get.

a = 2, b = 1 and c = 1

$b^2 - 4ac = 1^2 - 4(2)(1) = 1 - 8 = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{-7}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{7}\,i}{4}$$

3. $x^2 + 3x + 9 = 0$

***Solution***

The quadratic equation is $x^2 + 3x + 9 = 0$

$b^2 - 4ac = 3^2 - 4 \times 1 \times 9 = 9 - 36 = -27$

$$\therefore x = \frac{-3 \pm \sqrt{-27}}{2} = \frac{-3 \pm \sqrt{27}\,i}{2}$$

$$\therefore = \frac{-3 \pm 3\sqrt{3}\,i}{2}$$

4. $-x^2 + x - 2 = 0$ **(August 2014)**

***Solution***

The quadratic equation is $-x^2 + x - 2 = 0$

$b^2 - 4ac = 1^2 - 4(-1)(-2) = 1 - 8 = -7$

$$\therefore x = \frac{-1 \pm \sqrt{-7}}{-2} = \frac{-1 \pm \sqrt{7}\,i}{-2}$$

5. $x^2 + 3x + 5 = 0$

***Solution***

The quadratic equation is $x^2 + 3x + 5 = 0$

$b^2 - 4ac = 3^2 - 4(1)(5) = 9 - 20 = -11$

$$\therefore x = \frac{-3 \pm \sqrt{-11}}{2} = \frac{-3 \pm \sqrt{11}\,i}{2}$$

6. $x^2 - x + 2 = 0$

***Solution***

The quadratic equation is $x^2 - x + 2 = 0$

$b^2 - 4ac = (-1)^2 - 4(1)(2) = 1 - 8 = -7$

$$\therefore x = \frac{1 \pm \sqrt{-7}}{2} = \frac{1 \pm \sqrt{7}\,i}{2}$$

7. $\sqrt{2}x^2 + x + \sqrt{2} = 0$

***Solution***

The quadratic equation is

$\sqrt{2}x^2 + x + \sqrt{2} = 0$

$b^2 - 4ac = 1^2 - 4(\sqrt{2})(\sqrt{2}) = 1 - 8 = -7$

$$\therefore x = \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}} = \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$

8. $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

***Solution***

The quadratic equation is

$\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

$b^2 - 4ac = (-\sqrt{2})^2 - 4\sqrt{3}(3\sqrt{3})$

$= 2 - 36 = -34$

$$\therefore x = \frac{\sqrt{2} \pm \sqrt{-34}}{2\sqrt{3}}$$

$$= \frac{\sqrt{2} \pm \sqrt{34}\,i}{2\sqrt{3}}$$

9. $x^2 + x + \frac{1}{\sqrt{2}} = 0$

***Solution***

The quadratic equation is $x^2 + x + \frac{1}{\sqrt{2}} = 0$

$b^2 - 4ac = 1^2 - 4(1)\left(\frac{1}{\sqrt{2}}\right) = 1 - 2\sqrt{2} < 0$

$$x = \frac{-1 \pm \sqrt{1 - 2\sqrt{2}}}{2}$$

$$= \frac{-1 \pm \sqrt{(2\sqrt{2} - 1)(-1)}}{2}$$

$$= \frac{-1 \pm \sqrt{(2\sqrt{2} - 1)}\, i}{2}$$

10. $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

***Solution***

The quadratic equation is $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

$$b^2 - 4ac = \left(\frac{1}{\sqrt{2}}\right)^2 - 4(1)(1)$$

$$= \frac{1}{2} - 4 = \frac{-7}{2}$$

$$\therefore x = \frac{\frac{-1}{\sqrt{2}} \pm \sqrt{\frac{-7}{2}}}{2} = \frac{\frac{-1}{\sqrt{2}} \pm \sqrt{\frac{7}{2}}\, i}{2}$$

$$= \frac{-1 \pm \sqrt{7}\, i}{2\sqrt{2}}$$

## SOLUTIONS TO NCERT EXERCISE 5.4

**Find the square roots of the following:**

1. $-15 - 8i$

***Solution***

Let $x + iy = \sqrt{-15-8i}$

Squaring, $(x + iy)^2 = -15 - 8i$

i.e., $x^2 - y^2 + 2xyi = -15 - 8i$

Equating real and imaginary parts,

$x^2 - y^2 = -15$ ........(i)

$2xy = -8$ .......(ii)

Now $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$

$= (-15)^2 + (-8)^2$

$= 225 + 64 = 289$

$\therefore x^2 + y^2 = 17$ ......(iii)

(i) + (iii) $\rightarrow 2x^2 = 2 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

(iii) $\rightarrow y^2 = 16 \Rightarrow y = \pm 4$

From (ii) we see the product $xy$ is negative.

i.e., both $x$ and $y$ have opposite sign.

$\therefore x = 1$ and $y = -4$ or $x = -1$ and $y = 4$

$\therefore$ The square roots of $-15 - 8i$ are $1 - 4i$ or $-1 + 4i$ or $\pm(1 - 4i)$

2. $-8-6i$

***Solution***

Let $x+iy=\sqrt{-8-6i}$

Squaring, $(x+iy)^2=-8-6i$

$x^2-y^2+2xyi=-8-6i$

Equating real and imaginary parts,

$x^2-y^2=-8$ .......(i)

$2xy=-6$ .......(ii)

Now $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$

$=(-8)^2+(-6)^2$

$=64+36=100$

$\therefore x^2+y^2=10$ .......(iii)

(i) + (iii) $\rightarrow 2x^2=2 \Rightarrow x^2=1 \Rightarrow x=\pm 1$

(iii) $\rightarrow y^2=9 \Rightarrow y=\pm 3$

From (ii) we see the product $xy$ is negative.

i.e., $x$ and $y$ have opposite sign.

$\therefore x=1$ and $y=-3$ or $x=-1$ and $y=3$

The square root of $-8-6i$ are $1-3i$ and $-1+3i$ or $\pm(1-3i)$

3. $1-i$

***Solution***

Let $x+iy=\sqrt{1-i}$

Squaring, $(x+iy)^2=1-i$

$x^2-y^2+2xyi=1-i$

Equating real and imaginary parts,

$x^2-y^2=1$ .......(i)

$2xy=-1$ ......(ii)

Now $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$

$=(1)^2+(-1)^2=2$

$\therefore x^2+y^2=\sqrt{2}$ .....(iii)

(i) + (iii) $\rightarrow 2x^2=\sqrt{2}+1 \Rightarrow x^2=\dfrac{\sqrt{2}+1}{2} \Rightarrow x=\pm\sqrt{\dfrac{\sqrt{2}+1}{2}}$

(iii) $\rightarrow y^2=\dfrac{\sqrt{2}-1}{2} \Rightarrow y=\pm\sqrt{\dfrac{\sqrt{2}-1}{2}}$

From (ii) we see the product $xy$ is negative.

i.e., $x$ and $y$ have opposite sign.

$$\therefore x=\sqrt{\frac{\sqrt{2}+1}{2}} \text{ and } y=-\sqrt{\frac{\sqrt{2}-1}{2}} \text{ or } x=-\sqrt{\frac{\sqrt{2}+1}{2}} \text{ and } y=\sqrt{\frac{\sqrt{2}-1}{2}}$$

$\therefore$ The square root of $1-i$ are

$$\sqrt{\frac{\sqrt{2}+1}{2}}-\sqrt{\frac{\sqrt{2}-1}{2}}i \text{ and } -\sqrt{\frac{\sqrt{2}+1}{2}}+\sqrt{\frac{\sqrt{2}-1}{2}}i \text{ or } \pm\left(\sqrt{\frac{\sqrt{2}+1}{2}}-\sqrt{\frac{\sqrt{2}-1}{2}}i\right)$$

4. $-i$

Let $x+iy=\sqrt{-i}$

Squaring $(x+iy)^2=-i$

i.e., $x^2-y^2+2xyi=-1$

Equating real and imaginary parts

$x^2-y^2=0$ .......(i)

$2xy=-1$ ......(ii)

Now $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$

$=0^2+(-1)^2=1$

$x^2+y^2=1$ .....(iii)

(i) + (iii) $\rightarrow 2x^2=1 \Rightarrow x^2=\frac{1}{2} \Rightarrow x=\pm\frac{1}{\sqrt{2}}$

(iii) $\rightarrow y^2=\frac{1}{2} \Rightarrow y=\pm\frac{1}{\sqrt{2}}$

From (ii) we see the product $xy$ is negative.

i.e., both $x$ and $y$ have opposite sign.

$$\therefore x=\frac{1}{\sqrt{2}} \text{ and } y=\frac{-1}{\sqrt{2}} \text{ or } x=\frac{-1}{\sqrt{2}} \text{ and } y=\frac{1}{\sqrt{2}}$$

$\therefore$ The square root of $-i$ are $\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i$ and $\frac{-1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$ or $\frac{1-i}{\sqrt{2}}$ and $\frac{-1+i}{\sqrt{2}}$ or $\pm\left(\frac{1-i}{\sqrt{2}}\right)$

5. $i$

Let $x+iy=\sqrt{i}$

Squaring, $(x+iy)^2=i$

$x^2-y^2+2xyi=i$

Equating real and imaginary parts,

$$x^2 - y^2 = 0 \qquad \text{.......(i)}$$

$$2xy = 1 \qquad \text{......(ii)}$$

Now $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$

$$= 0^2 + 1^2 = 1$$

$\therefore x^2 + y^2 = 1 \qquad$ .....(iii)

(i) + (iii) $\rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

(iii) $\rightarrow y^2 = \frac{1}{2} \Rightarrow y = \pm\frac{1}{\sqrt{2}}$

From (ii) we see the product $xy$ is positive.

i.e., both $x$ and $y$ have same sign.

$\therefore x = \frac{1}{\sqrt{2}}$ and $y = \frac{1}{\sqrt{2}}$ or $x = \frac{-1}{\sqrt{2}}$ and $y = \frac{-1}{\sqrt{2}}$

$\therefore$ The square roots of $i$ are $\frac{1+i}{\sqrt{2}}$ and $\frac{-1-i}{\sqrt{2}}$ or $\pm\left(\frac{1+i}{\sqrt{2}}\right)$