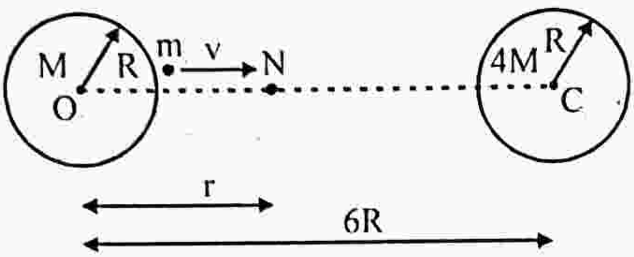
12. Two uniform solid spheres of equal radii R, but mass M and 4 M have a centre to centre separation 6 R, as shown in Figure. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere.



Sol.

The projectile is acted upon by two mutually opposing gravitational forces of the two spheres.

At neutral point

$$\frac{GMm}{r^2} = \frac{4GMm}{(6R-r)^2}$$

$(6R - r)^2 = 4r^2$

$6R - r = \pm 2r$

$r = 2R$ or $-6R$.

The neutral point $r = -6R$ is not possible. So $ON = r = 2R$. It is sufficient to project the particle with a speed which would enable it to reach N. Thereafter, the greater gravitational pull of mass 4M would suffice. The mechanical energy at the surface of M is

$$E_i = \frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{4GMm}{5R}$$

At the neutral point N, the speed approaches zero. The mechanical energy at N is purely potential.

$$E_N = -\frac{GMm}{2R} - \frac{4GMm}{4R}$$

From the principle of conservation of mechanical energy

$$\frac{1}{2}v^2 - \frac{GM}{R} - \frac{4GM}{5R} = -\frac{GM}{2R} - \frac{GM}{R}$$

or

$$v^2 = \frac{2GM}{R}\left(\frac{4}{5} - \frac{1}{2}\right)$$

$$v = \left(\frac{3GM}{5R}\right)^{1/2}$$

8.11 Earth Satellites

A heavenly body revolving around a planet in a fixed orbit is called a ***satellite.***

The natural satellite of earth is moon and artificial (man made) satellites are Sputnik, Aryabatta, INSAT etc.

The gravitational force between the planet and satellite provides necessary centripetal force required to keep the satellite in its orbit.

8.11.1 Orbital Velocity

Orbital Velocity *of a satellite is the velocity with which it revolves round a planet in its fixed orbit.*

8.11.2 Expression for Orbital Velocity

Consider a satellite of mass m, that revolves round the earth in an orbit of radius $R_E + h$, with a velocity v_o (h is the height of the satellite above earth's surface).

The gravitational force between satellite and earth provides necessary centripetal force,

i.e., $\frac{GM_E m}{(R_E+h)^2} = \frac{mv_0^2}{R_E+h}$, $v_o = \sqrt{\frac{GM_E}{(R_E+h)}}$ .... (1)

But $GM_E = gR_E^2$ $\therefore v_o = \sqrt{\frac{gR_E^2}{(R_E+h)}}$ .... (2)

If the satellite is very close to the earth, then

$R_E + h \simeq R_E$

$$\therefore v_o = \sqrt{\frac{gR_E^2}{R_E}} = \sqrt{gR_E} = \sqrt{\frac{GM_E}{R_E}} \quad \text{........ (3)}$$

Substituting the values of G, M_E and R_E we get $v_o = 7.92\ km\ s^{-1}$

The nearest orbit of a satellite is called **minimum orbit** and the corresponding velocity is called first **cosmic velocity.**

You must know

Escape velocity $v_e = \sqrt{2gR_E} = \sqrt{2} \times \sqrt{gR_E} = \sqrt{2} \times v_o$

Escape velocity is called second cosmic velocity also.

8.11.3 The Time Period of a Satellite (T)

The time taken by a satellite to complete one orbital motion around a planet is called ***time period of a*** satellite.

Consider a satellite revolving around the earth in an orbit of radius $R_E + h$ with a velocity v_o.

$$\text{Time period of satellite} = \frac{\text{distance travelled in one revolution}}{\text{orbital velocity}}$$

$$T = \frac{2\pi(R_E+h)}{v_o} \quad \text{....... (1)} \quad \text{But} \quad v_o = \sqrt{\frac{gR_E^2}{R_E+h}}$$

$$\therefore T = 2\pi(R_E+h)\sqrt{\frac{(R_E+h)}{gR_E^2}}$$

$$T = 2\pi\sqrt{\frac{(R_E+h)^3}{gR_E^2}} \quad \text{......... (2)}$$

For minimum orbit,

$$T = 2\pi\sqrt{\frac{R_E^3}{gR_E^2}}, \quad T = 2\pi\sqrt{\frac{R_E}{g}} \quad \text{....... (3)}$$

Substituting the standard values, i.e., $R_E = 6.4 \times 10^6$ m, $g = 9.8\ ms^{-2}$, we get T = 84.6 minutes

i.e., A satellite orbiting close to the surface of the earth has a time period of revolution about 84.6 minutes.

13. What is the minimum velocity of a satellite should have in order to pursue a suitable radius?
Given g = 10 ms^{-2} and radius of earth = 6.4×10^3 km

Sol.

Given, $v_o = ?$ $g = 10\ ms^{-2}$

$R_E = 6.4 \times 10^3\ km = 6.4 \times 10^6\ m$

$$v_o = \sqrt{gR_E} = \sqrt{10 \times 6.4 \times 10^6} = 8 \times 10^3 = 8\ km/s$$

14. A satellite revolves in an orbit close to the surface of a planet of density 6.3 g cm^{-3}. Calculate the time period of the satellite.
(Radius = 6400 km)

Sol.

Given, $G = 6.67 \times 10^{-11}\ Nm^{-2}\ kg^{-2}$,

$R = 6400\ km = 6400 \times 10^3\ m$

$\rho = 6.3\ g\ cm^{-3} = 6.3 \times 10^3\ kgm^{-3}$

$$T = 2\pi\sqrt{\frac{R_E^3}{GM_E}}, \quad M_E = \frac{4}{3}\pi R_E^3\rho$$

$$T = 2\pi\sqrt{\frac{R_E^3}{G\frac{4}{3}\pi R_E^3\rho}} = 2\pi\sqrt{\frac{3}{4\pi G\rho}} = \sqrt{\frac{3\pi}{G\rho}}$$

$$= \sqrt{\frac{3 \times 3.14}{6.67 \times 10^{-11} \times 6.3 \times 10^3}} = 4762\ s$$

15. The planet Mars has two moons, phobos and deimos. (i) Phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4 × 103 km. Calculate the mass of mars. (ii) Assume that earth and mars move in circular orbits around the sun, with the martian orbit being 1.52 times the orbital radius of the earth. What is the length of the martian year in days?

Sol.

$$T^2 = \frac{4\pi^2}{GM_m}R^3 \quad M_m = \frac{4\pi^2}{G}\frac{R^3}{T^2}$$

$$= \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times 10^{-11} \times (459 \times 60)^2}$$

$$M_m = \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times (4.59 \times 6)^2 \times 10^{-5}}$$

$$= 6.48 \times 10^{23}\ kg$$

$$\frac{T_M^2}{T_E^2} = \frac{R_{MS}^3}{R_{ES}^3}$$

where R_{MS} is the mars - sun distance and R_{ES} is the earth - sun distance.

$$\therefore T_M = (1.52)^{3/2} \times 365 = 684\ days$$

16. You are given the following data: $g = 9.81\ ms^{-2}$, $R_E = 6.37 \times 10^6$ m, the distance to the moon $R = 3.84 \times 10^8$ m and the time period of the moon's revolution is 27.3 days. Obtain the mass of the Earth M_E in two different ways.

Sol. $M_E = \dfrac{gR_E^2}{G}$

$$= \frac{9.81 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$= 5.97 \times 10^{24}\ kg.$$

The moon is a satellite of the earth. From the derivation of Kepler's third law

$$T^2 = \frac{4\pi^2R^3}{GM_E}$$

$$M_E = \frac{4\pi^2R^3}{GT^2}$$

$$= \frac{4 \times 3.14 \times 3.14 \times (3.84)^3 \times 10^{24}}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}$$

$$= 6.02 \times 10^{24}\ kg$$

Both methods yield almost the same answer. The difference between them being less than 1%.

8.12 Energy of an Orbiting Satellite

The kinetic energy of the satellite in a circular orbit with orbital velocity v is

$$K.E = \frac{1}{2}mv^2 \quad \text{But } v = \sqrt{\frac{GM_E}{(R_E + h)}}$$

$$K.E = \frac{GM_E m}{2(R_E + h)} \quad .... (1)$$

The gravitational potential energy at a distance $(R_E + h)$ from the centre of earth is $P.E = \frac{-GM_E m}{(R_E + h)}$ .... (2)

From (1) and (2), it is clear that, K.E is positive whereas the P.E is negative. The magnitude of K.E is half the P.E.

Thus total energy, E = K.E + P.E

$$= \frac{GM_E m}{2(R_E + h)} - \frac{GM_E m}{(R_E + h)} = -\frac{GM_E m}{2(R_E + h)} \quad .... (3)$$

You must know

- When a body is projected from the surface of earth with a velocity less than the escape velocity, the sum of P.E and K.E is negative.
- If the total energy of the satellite becomes positive, the satellite will escape from the gravitational pull of the earth.

Solved Examples

17. A 400 kg satellite is in a circular orbit of radius $2R_E$ about the Earth. How much energy is required to transfer it to a circular orbit of radius $4R_E$? What are the changes in the kinetic and potential energies?

Sol.

$$E_i = \frac{GM_E m}{4R_E} \qquad E_f = \frac{GM_E m}{8R_E}$$

The change in total energy is

$$\Delta E = E_f - E_i$$

$$= \frac{GM_E m}{8R_E} = \left(\frac{GM_E}{R^2_E}\right)\frac{mR_E}{8}$$

$$\Delta E = \frac{gmR_E}{8} = \frac{9.81 \times 400 \times 6.37 \times 10^6}{8}$$

$$= 3.13 \times 10^9 J$$

The kinetic energy is reduced,

$\Delta K = K_f - K_i = -3.13 \times 10^9$ J.

The change in potential energy is twice the change in the total energy.

$\Delta U = -6.25 \times 10^9$ J

8.13 Geostationary and Polar Satellites

8.13.1 Geostationary Satellite

A satellite which appears stationary with respect to the earth is called ***geostationary satellite.*** The orbit of such a satellite is called geosynchronous orbit. Synchronous satellites always revolve around the earth in the equatorial plane with a time period of 24 hour. The direction of motion of the satellite is same as that of the earth i.e., from west to east.

The height of a geostationary satellite is obtained from the equation,

$$h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$$

A geostationary satellite always chooses a circular orbit of radius about **36,000 km**. These satellites are mainly used for communication purpose, TV and radio broadcasting and weather forecasting.

The INSAT group of satellites sent up by India are one such group of Geostationary satellites widely used for telecommunication in India.

8.13.2 Polar Satellite

A satellite which revolves in polar orbit is called a polar satellite. The polar orbit is in north - south direction, while earth spins below it in east - west direction. As a result, a polar satellite can eventually scan the entire surface of the earth.

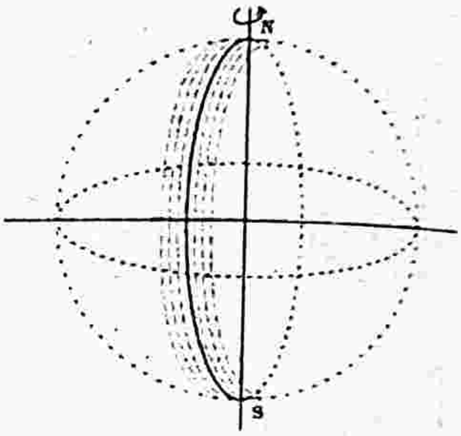


Fig. 14

The satellites in low lying polar orbits (500 - 800 km) are used for monitoring the weather, environment and spying. These satellites can view polar and equatorial regions at close distances with good resolution. Information gathered from such satellites is extremely useful for remote sensing, meteorology as well as for environmental studies of the earth.

8.14 WEIGHTLESSNESS

The weight of a body is simply the force of gravity acting on it.

If m is the mass of a body and g is the acceleration due to gravity, then the weight of the body

$W = mg$

Consider an astronaut (or space - man) of mass m is present in the artificial satellite. When the satellite is orbiting around earth, the man in the satellite experiences a centrifugal force whose direction is away from the centre. Therefore the two forces acting are

i. The gravitational force, $F_g = \frac{GM_Em}{r^2}$, directed towards the centre of earth

ii. Centrifugal force, $F_c = ma = \frac{mv_0^2}{r}$, directed opposite to the force of gravity.

$\therefore$ Net force on astronaut towards centre of earth

$$F = F_g - F_c = \frac{GM_Em}{r^2} - \frac{mv_0^2}{r}$$

$$= \frac{GM_Em}{r^2} - m\frac{GM_E}{r^2} = 0$$

Hence an astronaut feels weightlessness in an artificial satellite.

You must know

- If an astronaut inside an artificial satellite of earth stands on a spring balance, the reading of balance will be zero.
- A body experiences weightlessness only in an artificial satellite and not in a natural satellite.

Solutions for NCERT Exercises

1. Answer the following.
   a. You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?
   b. An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?
   c. If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the sun's pull is greater than the moon's pull. However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why?

Sol.

a. No. b. Yes.

c. Tidal effect depends inversely on the cube of distance.

2. Choose the correct alternative.
   a. Acceleration due to gravity increases/decreases with increasing altitude.
   b. Acceleration due to gravity increases/decreases with increasing depth. (Assume the earth to be a sphere of uniform density).
   c. Acceleration due to gravity is independent of mass of the earth/mass of the body.
   d. The formula $-GM_Em\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth.

Sol.

a. Decreases b. Decreases

c. Mass of the body

d. More

3. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth ?

Sol.

$T^2 \propto R^3$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3, \; R_1 = R$$

$$\frac{R_1}{R_2} = \left(\frac{T_1}{T_2}\right)^{\frac{2}{3}}$$

$$R_2 = R_1\left(\frac{T_2}{T_1}\right)^{\frac{2}{3}} = R\left(\frac{2T}{T}\right)^{\frac{2}{3}} = R(2)^{\frac{2}{3}}$$

$\therefore$ Size, $\frac{R_2}{R_1} = R(2)^{\frac{2}{3}}$ ie., smaller by a factor of 5.028

4. Io, one of the satellites of Jupiter has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

Sol.

$T = 1.769 \text{ days} = 1.769 \times 24 \times 60 \times 60$

$R + h = 4.22 \times 10^8 m$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$T^2 = \frac{4\pi^2(R+h)^3}{GM}$$

$$M = \frac{4\pi^2(R+h)^3}{GT^2}$$

$$= \frac{4 \times (3.14)^2 \times (4.22 \times 10^8)^3}{6.67 \times 10^{-11} \times (1.769 \times 24 \times 60 \times 60)^2}$$

$$= \frac{2963 \times 10^{24}}{1.558} = 1.9 \times 10^{27} \text{ kg} \; ..... \; (1)$$

Mass of Sun = 2×10^{30}kg .... (2)

Hence the mass of Jupiter is about one thousandth that of the Sun.

5. Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 *ly* from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 10^5 *ly*.

Sol.

$$T = 2\pi\sqrt{\frac{R^3}{Gm}}$$

$M = 2.5 \times 10^{11} \times 2 \times 10^{30}$ kg

$R = 50{,}000 \times 9.46 \times 10^{15}$m

$$T = 2\pi\sqrt{\frac{(50000 \times 9.46 \times 10^{15})^3}{6.67 \times 10^{-11} \times 2.5 \times 10^{11} \times 2 \times 10^{30}}}$$

$= 3.54 \times 10^8$ years

6. Choose the correct alternative:
   a. If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.
   b. The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

Sol.

a. Kinetic energy   b. Less

7. Does the escape speed of a body from the earth depend on (a) the mass of the body (b) the location from where it is projected (c) the direction of projection (d) the height of the location from where the body is launched?

Sol.

a. No   b. No   c. No

d. Yes. It depends on gravitational potential, which depends on latitude and height.

8. A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

Sol.

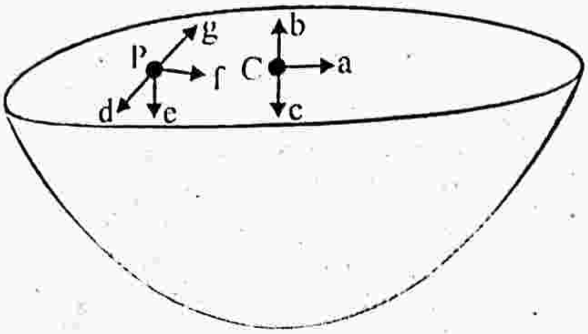
All quantities vary over an orbit except angular momentum and total energy.

9. Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.

Sol.

(b) (c) and (d)

10. In the following two exercises, choose the correct answer from among the given ones.
    The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Figure)
    (i) a, (ii) b, (iii) c, (iv) 0.



11. For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow.
(i) d, (ii) e, (iii) f, (iv) g.

Sol.

For these 10 and 11 problems, complete the hemisphere to a sphere. At both P and C, potential is constant and hence intensity = 0. Therefore for the hemisphere c and e are correct.

12. A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth is 6×10^{24}kg. Neglect the effect of other planets etc. (Orbital radius = 1.5×10^{11} m)

Sol.

Mass of the sun, M = 2×10^{30} kg,
Mass of the earth, m = 6×10^{24} kg
Orbital radius, r = 1.5×10^{11} m
Let 'P' be a point between the earth and sun, at a distance 'x' from the earth where the gravitational force is zero. Then

$$\frac{Gm}{x^2} = \frac{GM}{(r-x)^2}, \quad \frac{(r-x)^2}{x^2} = \frac{M}{m} = \frac{2\times10^{30}}{6\times10^{24}} = \frac{10^6}{3}$$

$$\frac{r-x}{x} = \frac{10^3}{\sqrt{3}} \text{ i.e., } \frac{r}{x} = \frac{10^3}{\sqrt{3}} + 1 \approx \frac{10^3}{\sqrt{3}}$$

$$\therefore x = \frac{r.\sqrt{3}}{10^3} = \frac{1.732\times1.5\times10^{11}}{10^3}$$

$= 2.6 \times 10^8$ m

13. How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.

Sol.

$R = 1.5 \times 10^8$ km

$$M_E R\omega^2 = \frac{GM_E M_S}{R^2}$$

$$M_s = \frac{R^3\omega^2}{G} = \frac{R^3 4\pi^2}{GT^2}$$

$$= \frac{(1.5\times10^8)^3 \times 4 \times (3.14)^2}{6.67\times10^{-11}\times(365\times86400)^2}$$

$= 2 \times 10^{30}$kg

14. A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50×10^8 km away from the sun?

Sol.

$$T^2 \propto R^3$$

$$R_s = 29.5\, R_E; \quad \frac{T_S}{T_E} = \left(\frac{R_S}{R_E}\right)^{3/2} = 29.5$$

$$R_s = (29.5)^{2/3} \times R_E = (29.5)^{2/3} \times 1.5 \times 10^8$$

$= 1.5 \times 10^{12}$ m

15. A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth? (Take g = 10 ms^{-2}).

Sol.

W = mg = 63 N .... (1)
h = R/2

$$g' = g\frac{R_E^2}{(R_E+h)^2},$$

$$\frac{g'}{g} = \left(\frac{R_E}{R_E+h}\right)^2 = \left(\frac{R_E}{R_E+R_E/2}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$ .... (2)

Let $W' = mg'$ ...... (3)

$$\frac{(2)}{(1)} \Rightarrow \frac{W'}{W} = \frac{mg'}{mg} = \frac{g'}{g} = \frac{4}{9}$$

$$W' = \frac{4}{9} \times W = \frac{4}{9} \times 63 = 28\text{N}$$