5. A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

Solution

When a coin is tossed, there are two possible outcomes - head or tail. By the Fundamental Principle of Counting, the number of outcomes when three coins are tossed is $2 \times 2 \times 2 = 8$

1st	2nd	3rd
2	2	2

6. Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

Solution

The flag in the upper part of the signal can be chosen in 5 ways and the lower part of the signal can be chosen in 4 ways.

1st	5
2nd	4

By the Fundamental Principle of Counting, the number of different signals formed $= 5 \times 4 = 20$ ways.

7.3 PERMUTATIONS

In this section we will discuss problems involving arrangement of a finite number of objects taking a particular number of objects at a time. We know that AB and BA are different arrangements of the letters A and B. The arrangement of the letters A, B and C taking two at time are AB, BA, AC, CA, BC, CB. An important thing to notice, is the order in which, the letters are arranged. We call each arrangement as a ***permutation***. Thus there are 6 permutations of three objects taking two at a time and is denoted as ${}^3P_2$

We can apply the Fundamental Principle of Counting for the arrangement of A, B and C taking two at a time. This is equivalent to arranging 3 objects in 2 places. We can put any one of the 3 objects at the first place. Thus, we have 3 choices of objects for the first place. For the 2nd place, only $(3 - 1)$ objects remain and hence, we have 2 choices. Therefore, by the Fundamental Principle of Counting, ${}^3P_2$, the permutations of 3 different objects, taking 2 at a time, is the product $3 \times 2 = 6$.

3 ways	2 ways	$3 \times 2 = 6$ ways.
I position	II position	

Definition

A permutation is an arrangement of a number of objects in a definite order taken some or all at a time. It is denoted as ${}^nP_r$ or $P(n, r)$ $0 \le r \le n$. In ${}^nP_r$, n and r are positive integers $n \ge r$

The arrangement of books in a shelf, formation of words with given letters, formation of numbers with given digits, etc are examples of permutations.

7.3.1. Permutations when all the objects are distinct

Now we shall obtain the formula to determine the number of permutations of n different objects, taken some or all at a time.

The number of permutations of n different objects, taken r at a time, is the arrangement of n objects in r places, in a row.

Let us designate their places of occurrence as $1^{st}, 2^{nd}, 3^{rd}, \ldots, r^{th}$ place.

We can place any one of the n objects at the first place. Thus, we have n choices of objects for the first place.

For the 2nd place, only $(n - 1)$ objects remain and hence, we have $(n - 1)$ choices.

Similarly, for the 3rd place, we have $(n - 2)$ choices, and so on.

By the time we reach the r^{th} place, we would have used $(r-1)$ objects and would, therefore, be left with $[n-(r-1)]$ objects. Thus, we have $(n-r+1)$ choice for the r^{th} place.

Place:	1^{st}	2^{nd}	3^{rd}	....	r^{th}
Number of choice:	n	$(n-1)$	$(n-2)$	....	$(n-r+1)$

Therefore, by the Fundamental Principle of Counting, the number of permutations of n different objects, taken r at a time, i.e, ${}^nP_r$, is the product $n(n-1)(n-2)\ ......\ (n-r+1)$.

$${}^nP_r = n(n-1)(n-2) \ldots (n-r+1),\ 0 < r \le n$$

Example 7

a. Find ${}^5P_4$ and ${}^6P_3$.

b. Find the number of 3 letter words which can be formed by the letters of the word NUMBER.

(March 2010)

Solution

a. ${}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$

${}^6P_3 = 6 \times 5 \times 4 = 120$

b. There are 6 different letters in the word NUMBER. Since 3 letters are used at a time, then the number of words formed $= {}^6P_3 = 120$

Example 8

Simplify $\dfrac{{}^nP_4}{{}^{n-1}P_3}$ **(March 2011)**

Solution

$$\frac{{}^nP_4}{{}^{n-1}P_3} = \frac{n(n-1)(n-2)(n-3)}{(n-1)(n-2)(n-3)} = n$$

Example 9

How many 4-digit numbers can be formed, by using the digits 1 to 9 if repetition of digits is not allowed? **(NCERT)**

Solution

To determine four digit numbers using the given nine digits, it is enough to find the number of distinct permutations of 9 things taken 4 at a time.

This is equal to ${}^9P_4$, which is given by ${}^9P_4 = 9.8.7.6 = 3024$

Hence, the total number of 4-digit numbers that can be formed by using the given nine digits is 3024.

Example 10

How many 4-digit numbers are there with no digit repeated? **(August 2009, March 2011)**

Solution

There are 10 digits and out of which zero cannot be placed in thousands' place. So the thousands' place can be filled in 9 ways. The remaining three places can be filled with the remaining nine digits in $^9P_3$ ways.

OR

Th	H	T	U
9	9	8	7

$= 9 \times 9 \times 8 \times 7 = 4536$

$\therefore$ By the Fundamental Principle of Counting, the number of four digit numbers $= 9 \times {}^9P_3 = 4536$

7.3.2 Factorial Notation

In Mathematics we often come across with the products 1.2, 1.2.3, 1.2.3.4, ..., 1.2.3.......n. So inorder to write this product 1.2.3........n, for convenience we use the notation '$n!$' (read as '*n factorial*'). Clearly n is a natural number.

Definition

The factorial of a natural number n is the continued product of the first n natural numbers.

- $n! = 1.2.3.......n.$
- $n! = n(n-1)! = n(n-1)(n-2)!$ etc.
- We define $0! = 1$

For example $5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 5 \times 4!$

$= 5 \times 4 \times 3!$

CAUTION ! $n!$ is defined for whole numbers only.

Example 11

Evaluate

i. 7! ii. 5! iii. $7! - 5!$ iv. $\frac{7!}{5!}$ v. $2 \times 6!$ vi. $\frac{12!}{10! \times 2!}$

vii. $\frac{5!}{3!}$

(NCERT, March 2015)

Solution

i. $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

ii. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

iii. $7! - 5! = 5040 - 120 = 4920 = 5!(42 - 1) = 120 \times 41 = 4920$

OR $7! - 5! = 7 \times 6 \times 5! - 5!$

iv. $\frac{7!}{5!} = \frac{5040}{120} = 42$ OR $\frac{7!}{5!} = \frac{7 \times 6 \times 5!}{5!} = 7 \times 6 = 42$

v. $2 \times 6! = 2 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 1440$

vi. $\frac{12!}{10!\times 2!}=\frac{12\times 11\times 10!}{10!\times 2!}=\frac{12\times 11}{2}=66$

vii. $\frac{5!}{3!}=\frac{5\times 4\times 3!}{3!}=20$

Example 12

Choose the correct answer: $\frac{0!}{1!}$

a. 0 b. 1 c. 2 d. 3 **(March 2015)**

Solution

$\frac{0!}{1!}=\frac{1}{1}=1$

$\therefore$ Option (b) is the correct answer.

Example 13

Compute i. 2! + 3! ii. Is 2! + 3! = 5!

Solution

i. 2! + 3! = 2.1 + 3.2.1 = 2 + 6 = 8

ii. 5! = 5.4.3.2.1 = 120. Therefore $2! + 3! \neq 5!$