7.3.3 Derivation of formula for ^nP_r

In section 7.3.1, we got $^nP_r = n(n-1)(n-2)\ldots\ldots\ldots\ldots(n-r+1) \quad 0 < r \leq n$

Multiplying and dividing the RHS by $(n-r)(n-r-1)\ldots\ldots\ldots\times 3\times 2\times 1$ we get

$$^nP_r = \frac{n(n-1)(n-2)\ldots\ldots\ldots(n-r+1)(n-r)(n-r-1)\ldots\ldots\ldots\times 3\times 2\times 1}{(n-r)(n-r-1)\ldots\ldots\ldots\times 3\times 2\times 1} = \frac{n!}{(n-r)!}, \; 0 \leq r \leq n$$

Particular cases

- In ^nP_r, put $r = n$, we get $^nP_n = \dfrac{n!}{(n-n)!} = \dfrac{n!}{0!} = n!$
- In ^nP_r put $r = 0$, we get $^nP_0 = \dfrac{n!}{(n-0)!} = \dfrac{n!}{n!} = 1$

Theorem

The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r.

Proof

The number of permutations of n different objects taken r at a time is same as the arrangement of n objects in r places in a row.

Let us designate their places of occurrence as 1^{st}, 2^{nd}, 3^{rd}, ..., r^{th} place.

The 1^{st} place can be filled in n ways, following which, the 2^{nd} place can also be filled in n ways [since repetition of objects is allowed] ..., the r^{th} place can be filled in n ways also.

Place	:	1st	2nd	3rd	... r^{th}
No. of ways	:	n	n	n	... n

Therefore by the fundamental principle of counting, the number of permutations of n objects is the product $n \times n \times n \times \ldots \times$ (r times) $= n^r$

Example 17

Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, when the repetition of letters is allowed. **(NCERT)**

Solution

There are 4 letters (n) and 4 places (r).

Number of 4 letter words when repetition of letters is allowed $= n^r = 4^4 = 256$

Example 18

How many different signals can be generated from 6 flags of different colours if each signal makes use of all the flags at a time, placed one below the other?

Solution

Here the generation of signals is equivalent to arranging 6 different flags in 6 positions.

This can be done in ${}^6P_6$ ways $= 6! = 720$

Example 19

In how many ways, can the letters of the word "HEXAGON" be permuted? **(March 2011)**

Solution

There are 7 letters in the word "HEXAGON".

Hence the number of words $= {}^7P_7 = 7! = 5040$

Example 20

Find the number of 3 letter words, with or without meaning, which can be formed by the letters of the word NUMBER, if

i. repetition of letters is not allowed.

ii. repetition of letters is allowed. **(NCERT)**

Solution

i. **repetition is not allowed**

There are 6 letters in the word NUMBER and we are taking 3 letters at a time.

$\therefore$ Number of 3 letter words $= {}^6P_3$

$= 6 \times 5 \times 4 = 120$

ii. **repetition is allowed**

The number of 3 letter words $= 6^3 = 216$