SECOND YEAR HIGHER SECONDARY EXAMINATION

MARCH 2021

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Questions carry 3 scores each ANSWER KEY

① $3 - x^2 = 3 - 8$

$x^2 - 8 = 0$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

② i) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

ii) $A \cdot \text{Adj } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

③ $LHL = \lim_{x \to 5^-} f(x)$

$= \lim_{x \to 5^-} kx + 1$

$= 5k + 1$

$RHL = \lim_{x \to 5^+} f(x)$

$= \lim_{x \to 5^+} 3x - 5$

$= 15 - 5$

$= 10.$

But $f(x)$ is continuous at 5.

$\therefore LHL = RHL = f(5)$

$\therefore 5k + 1 = 10$

$5k = 9$

$k = 9/5$

④ $f(x) = x^2 + 2x - 8$

$f(x)$ is a polynomial function which is continuous in $[-4, 2]$ and differentiable in $(-4, 2)$

$f(a) = f(-4)$

$= (-4)^2 + 2(-4) - 8$

$= 16 - 8 - 8$

$= 0$

$f(b) = f(2)$

$= 2^2 + 2(2) - 8$

$= 4 + 4 - 8$

$\therefore f(a) = f(b).$

Then there exist c such that $f'(c) = 0.$

But $f'(x) = 2x + 2$

$f'(c) = 0 \Rightarrow 2c + 2 = 0$

$2c = -2$

$c = -1 \in [-4, 2]$

Hence Roll's theorem is verified.

⑤ Area of a circle, $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$= 2\pi \times 5 \qquad \because r = 5$

$= 10\pi \text{ cm}^2/s$

⑥ Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$\vec{a} \cdot \vec{b} = 1 \times 7 + (3 \times -1) + (7 \times 8)$

$= 7 - 3 + 56$

$= 60$

$|\vec{b}| = \sqrt{7^2 + 1^2 + 8^2} = \sqrt{49 + 1 + 64} = \sqrt{114}.$

$\therefore$ Projection of $\vec{a}$ on $\vec{b} = \frac{60}{\sqrt{114}}$

(7) Position vector of the point is $\vec{i}+4\vec{j}+6\vec{k}$

Normal vector is $\vec{i}-2\vec{j}+\vec{k}$.

Vector equation

$$(\vec{r}-\vec{a})\cdot\vec{N}=0$$

$$[\vec{r}-(\vec{i}+4\vec{j}+6\vec{k})]\cdot(\vec{i}-2\vec{j}+\vec{k})=0$$

Cartesian equation

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$$

$$1(x-1)-2(y-4)+1(z-6)=0$$

$$x-1-2y+8+z-6=0$$

$$x-2y+z+1=0$$

(8)

i) incorrect question

ii) $y=\cos^{-1}(-\frac{1}{2})$

$$\cos y=-\frac{1}{2}$$

$$=\cos(\pi-\frac{\pi}{3})$$

$$=\cos\frac{2\pi}{3} \quad \therefore \cos^{-1}(-\tfrac{1}{2})=\frac{2\pi}{3}$$

Let $y=\sin^{-1}(\frac{1}{2})$

$$\sin y=\frac{1}{2}$$

$$=\sin\frac{\pi}{6}$$

$$\sin^{-1}(\tfrac{1}{2})=\frac{\pi}{6}$$

$$\therefore \cos^{-1}(-\tfrac{1}{2})+2\sin^{-1}(\tfrac{1}{2})$$

$$=\frac{2\pi}{3}+2\times\frac{\pi}{6}$$

$$=\frac{2\pi}{3}+\frac{\pi}{3}$$

$$=\frac{3\pi}{3}$$

$$=\pi$$

(9)

$$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix} -2 & -3 & 1\\ 3 & 2 & 1\\ -1 & -8 & 1\end{vmatrix}$$

$$=\frac{1}{2}[-2(2+8)+3(3+1)+1(-24+2)]$$

$$=\frac{1}{2}[-20+12+22]$$

$$=\frac{-30}{2}$$

$$=15 \text{ sq. units}$$

10. $\frac{dy}{dx}-y=\cos x$

$$P=-1,\quad Q=\cos x.$$

$$I.F=e^{\int -1\,dx}=e^{-x}$$

Solution is,

$$y\cdot(I.F)=\int(Q\times I.F)\,dx+C.$$

$$y\cdot e^{-x}=\int\cos x\cdot e^{-x}\,dx+C$$

$$I=\cos x\left(\frac{e^{-x}}{-1}\right)-\int(-\sin x)\cdot -e^{-x}\,dx+C$$

$$=-\cos x\, e^{-x}-\int\sin x\, e^{-x}\,dx+C$$

$$=-\cos x\, e^{-x}\left[\sin x\left(\frac{e^{-x}}{-1}\right)-\int\cos x\,(-e^{-x})\,dx\right]+C$$

$$=-\cos x\, e^{-x}+\sin x\, e^{-x}-\int\cos x\, e^{-x}\,dx$$

$$=-e^{-x}\cos x+e^{-x}\sin x-I$$

$$2I=(\sin x-\cos x)e^{-x}$$

$$I = \frac{(\sin x - \cos x)e^{-x}}{2}$$

$\therefore$ Solution is,

$$ye^{-x} = \frac{(\sin x - \cos x)e^{-x}}{2} + C$$

$$y = \frac{(\sin x - \cos x)}{2} + Ce^{x}$$

Questions Carry 4 Scores each.

(11). i) $3A+B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix}$

$$3\begin{bmatrix} 3 & 4 \\ -5 & -1 \end{bmatrix} + B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 12 \\ -15 & -3 \end{bmatrix} + B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} 9 & 12 \\ -15 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -4 \\ 18 & -1 \end{bmatrix}$$

ii) $AB = \begin{bmatrix} 3 & 4 \\ -5 & -1 \end{bmatrix}\begin{bmatrix} -7 & -4 \\ 18 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 51 & -16 \\ 17 & 21 \end{bmatrix}$$

(12) i) 3×3

ii) $AB = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}[1, 3, -6]$

$$= \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -20 \\ 5 & 15 & -30 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -20 & -30 \end{bmatrix} \quad \text{— (1)}$$

$$B' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}$$

$$A' = [-2 \quad 4 \quad 5]$$

$$B'A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}[-2 \quad 4 \quad 5]$$

$$= \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -20 & -30 \end{bmatrix} \quad \text{— (2)}$$

from (1) & (2)

$$(AB)' = B'A'$$

(13) i) c, $\tan^{-1}\frac{x+y}{1-xy}$

ii) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$$= \tan^{-1}\left(\frac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11} \times \frac{7}{24}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{48+77}{264}}{\frac{264-14}{264}}\right)$$

$$= \tan^{-1}\left(\frac{125}{250}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$

(14) i) $x^2+xy+y^2=100$

differentiating w.r.t x.

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0.$$

$$(x+2y)\frac{dy}{dx} = -2x-y$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

$$= \frac{-(2x+y)}{(x+2y)}$$

ii) $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

put $x = \tan\theta$, $\theta = \tan^{-1}x$.

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2\tan^{-1}x$$

differentiating w.r.t x

$$\frac{dy}{dx} = 2\cdot\frac{1}{1+x^2}$$

$$= \frac{2}{1+x^2}$$

(15) $f(x) = 2x^2-3x$

$f'(x) = 4x-3$

$f'(x)=0 \Rightarrow 4x-3=0$

$4x=3$

$x = 3/4$

Intervals are $(-\infty, \frac{3}{4})$, & $(\frac{3}{4}, \infty)$

i) $f'(x)>0$ only if $4x-3>0$

$4x>3$

$x > \frac{3}{4}$.

$\therefore f(x)$ strictly increasing on $(\frac{3}{4}, \infty)$

ii) $f'(x)<0$ only if $4x-3<0$

$4x<3$

$x<\frac{3}{4}$

$\therefore f(x)$ strictly decreasing on $(-\infty, \frac{3}{4})$.

(16)

i) Order $= 2$

degree $= 1$

ii) $\frac{dy}{dx} = (1+x^2)(1+y^2)$

$$\frac{dy}{1+y^2} = (1+x^2)\,dx$$

$$\therefore \int\frac{dy}{1+y^2} = \int(1+x^2)\,dx$$

$$\tan^{-1}y = x + \frac{x^3}{3} + C$$

(17) Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ $= \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$

$$= -17\hat{i} + 13\hat{j} + 7\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{289+169+49} = \sqrt{507}.$$

Unit vector $\perp^r$ to both $\vec{a}$ and $\vec{b}$

$$= \frac{-17}{\sqrt{507}}\hat{i} + \frac{13}{\sqrt{507}}\hat{j} + \frac{7}{\sqrt{507}}\hat{k}$$

(18)

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$

$\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$

$\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= -3\hat{i} + 3\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$$

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = -3 + 0 - 6 = -9$$

$$\therefore d = \left| \frac{-9}{3\sqrt{2}} \right|$$

$$= \frac{3}{\sqrt{2}}$$

(19)

i) $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

$$0.4 = \frac{P(A \cap B)}{0.8}$$

$$P(A \cap B) = 0.4 \times 0.8$$

$$= 0.32$$

ii) $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{0.32}{0.5}$$

$$= 0.64$$

iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.8 + 0.5 - 0.32$$

$$= 0.98$$

(20) i) b, (3, 1)

ii) $R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (1,2), (1,3), (1,4), (1,5), (1,6), (2,4), (2,6), (3,6)\}.$

$(a,a) \in R$ for all $a \in A$.

$\therefore$ R is reflexive

$(1,2) \in R$ but $(2,1) \notin R$

$\therefore$ R is not symmetric.

If $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for $a \in A$

$\therefore$ R is transitive.

$\therefore$ R is reflexive, transitive but not symmetric

(21) i) Let $y = x^x$

taking Logarithm

$$\log y = x \log x$$

differentiating w.r.t x.

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$= 1 + \log x.$$

$$\therefore \frac{dy}{dx} = y(1+\log x)$$

$$= x^x(1+\log x)$$

ii) $x = 2at^2,\ y = at^4.$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

$$y = at^4$$

$$\frac{dy}{dt} = 4at^3$$

$$x = 2at^2$$

$$\frac{dx}{dt} = 4at$$

$$\therefore \frac{dy}{dx} = \frac{4at^3}{4at}$$

$$= t^2$$

(22) $\int \frac{x}{(x+1)(x+2)}dx.$

Let $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$$\therefore x = A(x+2) + B(x+1)$$

Putting $x = -1$, $A = -1$

and $x = -2$, $B = 2$

$$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}$$

$$\therefore \int \frac{x}{(x+1)(x+2)}dx$$

$$= \int \frac{-1}{x+1}dx + \int \frac{2}{x+2}dx$$

$$= -\log|x+1| + 2\log|x+2| + C$$

$$= -\log|x+1| + \log|x+2|^2 + C$$

$$= \log\frac{|x+2|^2}{|x+1|} + C$$

$$= \log\left|\frac{(x+2)^2}{(x+1)}\right| + C$$

Questions carry 6 scores each.

(23) i) $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

$$a_{ij} = 3i - j$$

$$a_{11} = 3-1 = 2$$

$$a_{12} = 3-2 = 1$$

$a_{21} = 3\times2-1 = 5$

$a_{22} = 3\times2-2 = 4$

$a_{31} = 3\times3-1 = 8$

$a_{32} = 3\times3-2 = 7.$

$$\therefore A = \begin{bmatrix} 2 & 1 \\ 5 & 4 \\ 8 & 7 \end{bmatrix}$$

ii) $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$

$$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$A' = \begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}$$

$$A+A' = \begin{bmatrix} 4 & 4 \\ 4 & -8 \end{bmatrix}$$

$$\frac{1}{2}(A+A') = \begin{bmatrix} 2 & 2 \\ 2 & -4 \end{bmatrix}$$

$$A-A' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A-A') = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A+A') + \frac{1}{2}(A-A') = A$$

ie $\begin{bmatrix} 2 & 2 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$

(24) $AX = B$

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$|A| = -17 \neq 0$

$$\text{adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{-1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{-1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$= \frac{-1}{17}\begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1, \; y = 2, \; z = 3$

(25)

i) $gof(1) = g(f(1)) = g(2) = 3$

$gof(3) = g(f(3)) = g(5) = 1$

$gof(4) = g(f(4)) = g(1) = 3.$

ii) Let $y = 2x+1$

$2x = y-1$

$x = \frac{y-1}{2}$

g is the inverse of f if

$fog = gof$.

Let $g(x) = \frac{x-1}{2}$

$$fog(x) = f(g(x))$$
$$= f\left(\frac{x-1}{2}\right)$$
$$= 2\left(\frac{x-1}{2}\right)+1$$
$$= x-1+1$$
$$= x$$

$$gof(x) = g(f(x))$$
$$= g(2x+1)$$
$$= \frac{2x+1-1}{2}$$
$$= \frac{2x}{2}$$
$$= x.$$

$gof(x) = fog(x) = x$.

$\therefore f$ is invertible.

$\therefore f^{-1}(x) = \frac{x-1}{2}$

(26)

i) $\frac{dy}{dx} = 3x^2-1$

$\frac{dy}{dx}$ at $x=2$ $= 3\times 2^2-1$

$= 3\times 4 - 1$

$= 11$

ii) $x = 2$

$y = 2^3-2 = 8-2 = 6.$

$\therefore$ Equation of the tangent with slope 11 and passing through $(2,6)$ is given by

$$y-y_0 = m(x-x_0)$$
$$y-6) = 11(x-2)$$
$$y-6 = 11x-22$$
$$11x-y-22+6=0$$
$$11x-y-16=0$$

iii) $(\sin x+\cos x)^2 = \sin^2 x+\cos^2 x+2\sin x\cos x$

$= 1+\sin 2x$

Maximum value of $\sin 2x$ is 1.

$\therefore (\sin x+\cos x)^2 = 1+1$

$= 2$

$\therefore \sin x+\cos x = \sqrt{2}$

Maximum value of $\sin x+\cos x$ is $\sqrt{2}$

(27)

i) $\int \sin x \sin(\cos x)\,dx$.

put $t = \cos x$

$dt = -\sin x\,dx$.

$$\int \sin(\cos x)\sin x\,dx$$
$$= \int \sin t\,(-dt)$$
$$= -\int \sin t\,dt.$$

$= -^{-}\cos t\ \cancel{d}t + C$

$= \cos t\ \cancel{d}t + C$

$= \cos(\cos x) + C$

ii) $\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx$.

$\int \frac{\tan^{-1}x}{1+x^2}\,dx$.

Put $t = \tan^{-1}x$

$dt = \frac{1}{1+x^2}\,dx$

$= \int t\,dt$

$= \frac{t^2}{2} + C. \quad = \frac{(\tan^{-1}x)^2}{2} + C$

$\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx = \left[\frac{(\tan^{-1}x)^2}{2}\right]_0^1$

$= \frac{(\tan^{-1}1)^2}{2} - \frac{(\tan^{-1}0)^2}{2}$

$= \frac{(\frac{\pi}{4})^2}{2} - 0$

$= \frac{\pi^2}{32}$

(28) $x + 2y = 10$

x	y
0	5
10	0

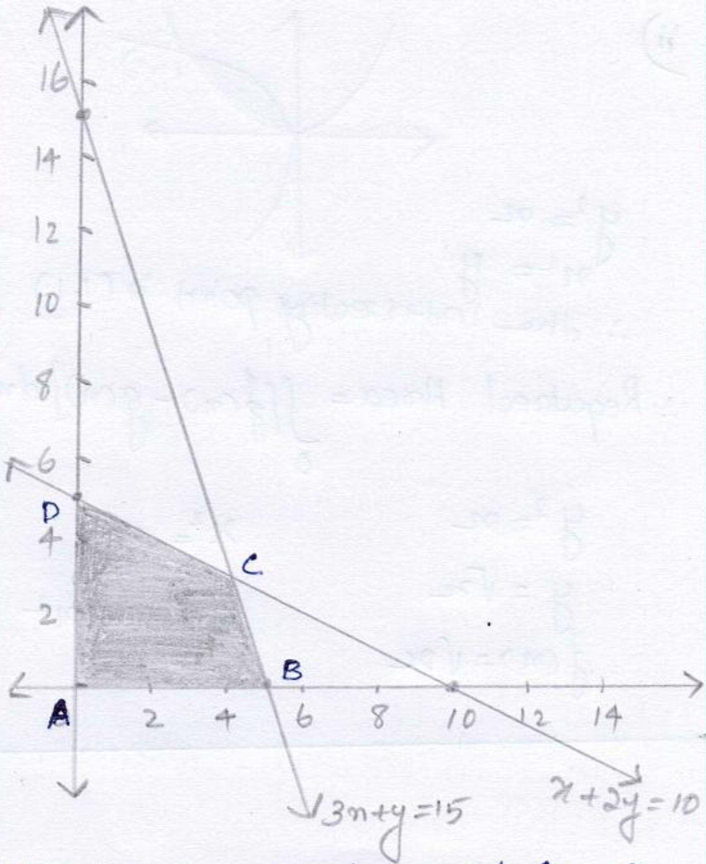
$3x + y = 15$

x	y
0	15
5	0

Corner Points	$z = 3x + 2y$
A(0,0)	0
B(5,0)	15
C(4,3)	18 Maximum.
D(0,5)	10

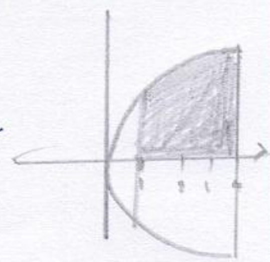
$3x + 6y = 30$
$3x + y = 15$
$5y = 15$
$y = 3$

$\therefore x = 10 - 2y$
$= 10 - 6$
$= 4$
$(4,3)$



Maximum value is 18 at (4,3).

(29) i)



Required Area $= \int_1^4 y\,dx$

$= \int_1^4 \sqrt{x}\,dx$

$= \frac{2}{3}\left[x^{3/2}\right]_1^4$

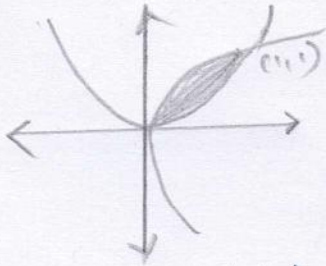
$= \frac{2}{3}\left[4^{3/2} - 1^{3/2}\right]$

$= \frac{2}{3}\left[2^3 - 1\right]$

$= \frac{2}{3} \times 7$

$= \frac{14}{3}$

ii)



$y^2 = x$

$x^2 = y$

$\therefore$ the intersecting point is $(1,1)$

$\therefore$ Required Area $= \int_0^1 [f(x) - g(x)]\, dx$

$y^2 = x$

$y = \sqrt{x}$

$f(x) = \sqrt{x}$

$x^2 = y$

$\therefore g(x) = x^2$

$\therefore$ Area $= \int_0^1 (\sqrt{x} - x^2)\, dx$

$= \left[\frac{2}{3} x^{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1$

$= \frac{2}{3} - \frac{1}{3}$

$= \frac{1}{3}$