11.9.10 Energy Distribution in Black Body Radiation and Laws of Black Body Radiation

The quality of radiation from a black body was investigated by Lummer and Pringshem in 1899. The results of the experiments are shown in figure.

The curves show that as the temperature increases the intensity of every wavelength increases. The energy emitted by the body varies at different temperatures. Energy emitted is small for both short and long wavelengths. As temperature increases, the peak of the curve shifts towards the shorter wavelength side. For a particular temperature, there is a particular wavelength (λ_m), for which energy radiated or emitted (E_λ) is maximum.

The energy radiated per unit area in unit time between those wavelengths is proportional to that area under the curve. The total energy emitted per unit area in unit time over all wavelengths is directly proportional to the area under whole curve.

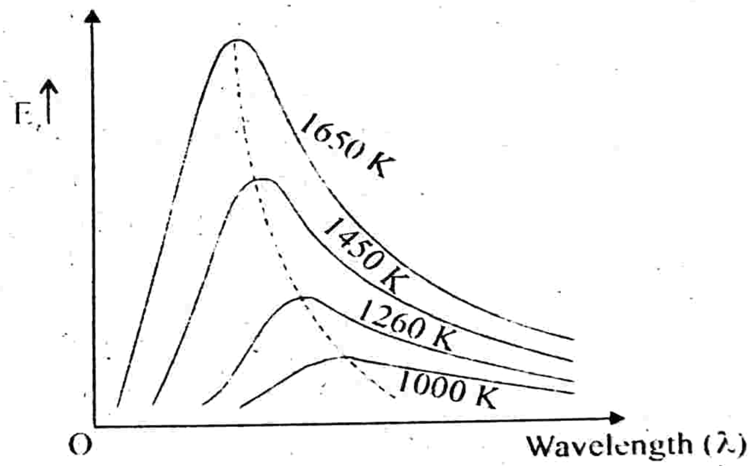

Fig. 11

The curves in figure can be fully explained only by Planck's Quantum theory. Both the theory and experiment lead to three important laws. They are

i. Wien's displacement law   ii. Planck's law and   iii. Stefan's law

i. Wien's Displacement Law

The wavelength corresponding to the maximum intensity of heat radiation is inversely proportional to the absolute temperature of the black body.

ie., $\lambda_m \propto \frac{1}{T}$ or $\lambda_m T = b$, a constant,

known as Wien's constant and its value is 2.898×10^{-3} mK.

ii. Planck's Law

According to Planck, $E_\lambda = \dfrac{C_1}{\lambda^5\left[e^{\frac{C_2}{\lambda T}} - 1\right]}$, where C_1 and C_2 are constants.

iii. Stefan's Law

If E is the total energy radiated from unit area in unit time at a temperature, T from a black body, then $E = \sigma T^4$, where σ is called Stefan's constant.

11.9.11 Solar Constant and Temperature of the Sun

We get heat from the sun. The heat which comes to us travels through millions of kilometres in space. Measurements have been made which give the amount of radiant energy reaching the earth, called solar constant (S_0). The value of solar constant at the upper limit of the earth's atmosphere is about 80000 Jm^{-2} min^{-1} or about 1340 Wm^{-2}. At the surface of earth it is always less than this value. The reason is that due to atmospheric absorption.

***Solar constant** (S_0) is defined as the radiant solar power that a unit area of a black body receives, in the absence of atmosphere, when placed normal to the incident radiation at a distance equal to the mean distance of the earth from the sun.*

If P is the power radiated by the sun and T is the temperature of surface of the sun, then by Stefan's law.

$P = \sigma 4\pi R_S^2 T^4$, where R_S is the radius of the sun.

By the definition of solar constant, $P = 4\pi R_0^2 S_0$, where R_0 is the mean distance of the earth from the sun.

$$\therefore 4\pi R_0^2 S_0 = \sigma 4\pi R_S^2 T^4, \qquad R_0^2 S_0 = \sigma R_S^2 T^4$$

$$\therefore S_0 = \sigma T^4 \left(\frac{R_S}{R_0}\right)^2; \quad \text{Hence, } T = \left[\left(\frac{R_0}{R_S}\right)^2 \cdot \frac{S_0}{\sigma}\right]^{\frac{1}{4}}$$

$\frac{R_S}{R_0}$ gives the mean angle subtended by the solar radius at the earth. The value of $\frac{R_S}{R_0} = 4.65 \times 10^{-3}$ radians. Knowing the values of σ and S_0, the temperature on the surface of the sun is estimated to be about 5800 K. (= 6000 K)

Also by using Wien's displacement law, the temperature on the surface of the sun can be estimated. The value of λ_m is about 4753 A° (or 475.3nm). The value of Wien's Constant b is 2.898×10^{-3} mK. Hence, $T = \frac{b}{\lambda_m} = 6060$ K.

11.10 Newton's Law of Cooling

It was Newton who first investigated the heat lost by a body to its surroundings. Normally a hot body kept on a table will radiate heat to its surroundings until it attains the thermal equilibrium. According to Newton, the rate of loss of heat is directly proportional to the difference in temperature between the body and its surroundings.

If a hot body at a mean temperature T_2 is kept in a surroundings at a temperature T_1, then according to Newton, the rate of loss of heat,

$$-\frac{dQ}{dt} \propto (T_2 - T_1)$$

$$\text{or } -\frac{dQ}{dt} = k\,(T_2 - T_1) \text{ .... (1)}$$

If the body is of mass m and specific heat capacity s, the quantity of heat lost in a small time dt, when its temperature decreases by dT_2, is,

$dQ = ms\,dT_2$

$\therefore$ Rate of loss of heat,

$$\frac{dQ}{dt} = ms\frac{dT_2}{dt} \text{ .... (2)}$$

On comparing equations (1) and (2),

$$-ms\frac{dT_2}{dt} = k(T_2 - T_1)$$

$$-\frac{dT_2}{T_2 - T_1} = \frac{k}{ms}.dt$$

$$\text{or } -\frac{dT_2}{T_2 - T_1} = K\,dt \text{ .... (3)}$$

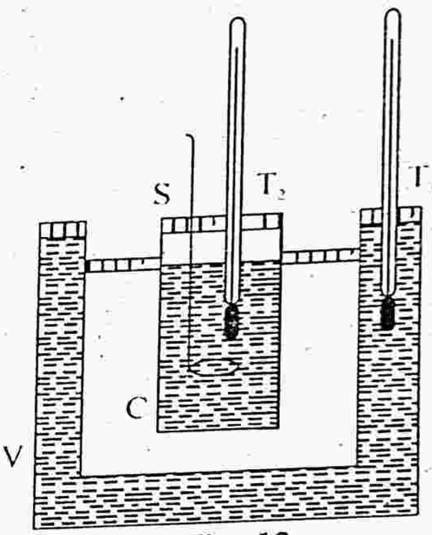

Fig. 12
Verification of Newton's Law of Cooling

where $K = \frac{k}{ms}$, is a constant, depends on the nature of the surface.

Equation (3) on integration,

$\log_e(T_2 - T_1) = -Kt + c$ .... (4)

or $T_2 - T_1 = e^{-Kt+c} = e^c \times e^{-Kt}$

or $T_2 = T_1 + C' e^{-Kt}$ .... (5)

where $C' = e^c$

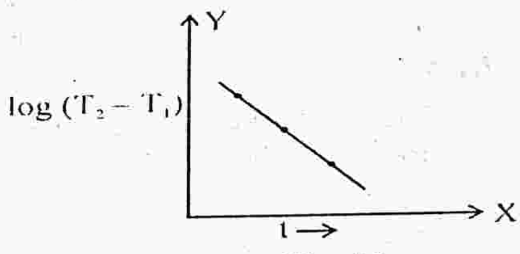

Fig. 13

For the verification of Newton's law of cooling, take a calorimeter and a double walled vessel. Calorimeter is kept inside the vessel as shown in figure 12.

Temperature of the surroundings is read by the thermometer T_1 and that of the calorimeter by T_2. Keep T_2 at a higher temperature and now a stopwatch is switched on. Reading of T_2 is read in a regular interval and the difference in temperature, $T_2 - T_1$ is measured. Now draw a graph connecting logarithmic value of $(T_2 - T_1)$ with time interval. The shape of the graph so obtained is as given in figure 13. It is a straight line graph with a negative slope. This graph is in agreement with the equation (4). Hence verifies Newton's law of cooling.

Newton's law of cooling can also be derived from Stefan's law. According to Stefan's law, the heat energy radiated in unit time, $H = \sigma e A(T_2^4 - T_1^4)$ .... (6)

If T_2 is slightly greater than T_1 by ΔT, then $T_2 - T_1 = \Delta T$

or $T_2 = T_1 + \Delta T$ .... (7)

∴ Equation (6) becomes,

$H = \sigma e A[(T_1 + \Delta T)^4 - T_1^4]$

$= \sigma e A(T_1^4 + 4T_1^3\Delta T + 6T^2(\Delta T)^2 + 4T_1(\Delta T)^3 + (\Delta T)^4 - T_1^4]$

ΔT is being small, higher powers of ΔT can be neglected. Hence

$H = \sigma e A \times 4T_1^3 \Delta T$

Since σ, e, A and T_1 are constants,

$H \propto \Delta T$ or $H \propto (T_2 - T_1)$ or Rate of heat loss $\propto \frac{dQ}{dt}$

$\therefore \frac{dQ}{dt} \propto (T_2 - T_1)$

Hence $\frac{dQ}{dt} = -K(T_2 - T_1)$

where K is a constant and negative sign shows that temperature decreases due to loss of heat.

A dull surface loses heat a little faster than a shining surface. It is because the dull surface is a better radiator. Hence rate of heat loss depends on the exposed surface area also.

Solved Examples

19. A pan filled with hot food cools from 94 °C to 86 °C in 2 minutes when the room temperature is at 20 °C. How long will it take to cool from 71 °C to 69 °C?

Ans.

The average temperature of 94 °C and 86 °C is 90 °C, which is 70 °C above the room temperature. Under these conditions the pan cools 8 °C in 2 minutes.

$\frac{\text{Change in temperature}}{\text{Time}} = K\Delta T$ Where K is a constant.

$\frac{8°C}{2\,min} = K(70°C)$

The average of 69 °C and 71 °C is 70 °C, which is 50 °C above room temperature. K is the same for this situation as for the original.

$\frac{2°C}{Time} = K(50°C)$

When we divide the above two equations, we have

$\frac{8°C/2\,min}{2°C/Time} = \frac{K(70°C)}{K(50°C)}$

Time = 0.7 min = 42 s

20. The tungsten filament of an electric bulb of power 60W has a length 50 cm and diameter 0.6 mm. Assume that the radiation from the

filament is equivalent to 80% that of a perfect black body radiator at the same temperature. Calculate the steady temperature of the filament. Given, $\sigma = 5.7 \times 10^{-8}\ Wm^{-2}K^{-4}$.

Sol.
Given, $e = 1$, $A = 2\pi rh$
$= 2\pi \times 0.3 \times 10^{-3} \times 0.5$
$= 3.14 \times 0.3 \times 10^{-3}\ m^2$

Power radiated per sec, H = 60W
Since rate of energy radiation is 80%, $H = \left(\frac{80}{100}\right)\sigma e A T^4$

$\therefore \left(\frac{80}{100}\right) \times 5.7 \times 10^{-8} \times 3.14 \times 0.3 \times 10^{-3} \times T^4 = 60$

$$T = \left[\frac{60 \times 100}{80 \times 5.7 \times 10^{-8} \times 3.14 \times 0.3 \times 10^{-3}}\right]^{\frac{1}{4}}$$
$= 2536\ K$

21. A star radiates maximum energy at the wavelength 480 nm. Calculate the temperature of the star. Given Wien's constant, $b = 2.898 \times 10^{-3}$ mK.

Sol.
Given, $\lambda = 480\ nm = 480 \times 10^{-9} m$, $b = 2.898 \times 10^{-3}$ mK.

$\lambda_m T = b$, $\therefore T = \frac{b}{\lambda_m}$

$= \frac{2.898 \times 10^{-3}}{480 \times 10^{-9}} \approx 6038K$

Solutions for NCERT Exercises

1. The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.

Sol. $\frac{t_C}{100} = \frac{t_F - 32}{180}$ and $T = 273.15 + C$

i. Neon: 24.57 K (Triple Point)
In Celsius scale, t_C
$= 24.57 - 273.15 = -248.58°C$

In Fahrenheit scale, $\frac{t_F - 32}{180} = \frac{t_C}{100}$

$t_F = \frac{180 \times t_C}{100} + 32 = \frac{180 \times -248.58}{100} + 32$
$= -447.44 + 32 = -415.44°F$

ii. CO_2: 216.55 K (Triple Point)
In Celsius scale - $t_C = -56.6°C$
In Fahrenheit scale - t_F
$= \frac{180 \times -56.6}{100} + 32 = -69.88°F$

2. Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B?

Sol.

$\frac{T_A}{T_B} = \frac{200}{350} = \frac{4}{7}$ $\quad \therefore T_A = \left(\frac{4}{7}\right)T_B$

3. The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law :
$R = R_0[1 + \alpha(T - T_0)]$
The resistance is 101.6 Ω at the triple-point of water 273.16 K, and 165.5 Ω at the normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4 Ω?

Sol.
$R = R_0[1 + 5 \times 10^{-3}(T - T_0)]$
$R_0 = 101.6\Omega$ at T_0 (say) $T_0 = 273.16$ K
$R_1 = 165.5\Omega$, at T = 600.5 K
$R_1 = R_0[1 + 5 \times 10^{-3}(600.5 - T_0)]$
$R = 123.4\Omega$, $R = R_0[1 + 5 \times 10^{-3}(T - T_0)]$ Where T is the temperature corresponding to resistance R.
$\therefore 165.5 = 101.6[1 + 5 \times 10^{-3}(600.5 - T_0)]$
$\therefore 165.5 = 101.6 + 101.6 \times 5 \times 10^{-3}(600.5 - T_0)$

$\therefore 165.5 - 101.6$

$= 101.6 \times 5 \times 10^{-3} (600.5 - T_0)$

Similarly $123.4 - 101.6$

$= 101.6 \times 10^{-3}(T - T_0)$

Divided by

$$\frac{165.5-101.6}{123.4-101.6} = \frac{101.6\times5\times10^{3}(600.5-273.16)}{101.6\times5\times10^{3}(T-273.16)}$$

$$\therefore \frac{165.5-101.6}{123.4-101.6} = \frac{600.5-273.16}{T-273.16}$$

$$T = 273.16 + \frac{327.34\times21.8}{63.9}$$

$= 384.83K$

4. Answer the following :

a. The triple-point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?

b. There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0 °C and 100 °C respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) scale?

c. The absolute temperature (Kelvin scale) *T* is related to the temperature t_c on the Celsius scale by

$t_c = T - 273.15$

Why do we have 273.15 in this relation, and not 273.16?

d. What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?

Ans.

a. In Celsius scale of temperature, two fixed points for the measurement of temperature are melting point of ice (0°C) and boiling point of water (100°C). But these points may vary due to the presence of impurities in water. If we choose the triple point of water as a reference point, it has a unique temperature and is independent of the nature of the thermometric substance used.

b. The other fixed point is the absolute zero itself.

c. Since the triple point is obtained at 0.01°C and not 0°C

d. Here, $\frac{9}{5}$ of Fahrenheit scale

= 1 division of new Kelvin scale

Triple point = 273.16, which is unique. Hence, on the new scale it is $273.16\times\frac{9}{5}=491.7$

5. Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made :

Temperature	Pressure thermometer A	Pressure thermometer B
Triple-point of water	1.250×10^5 Pa	0.200×10^5 Pa
Normal melting point of sulphur	1.797×10^5 Pa	0.287×10^5 Pa

a. What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B?

b. What do you think is the reason behind the slight difference in answers of thermometers A and B? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings?

Ans.

a. For the thermometer A,

$T_{tr} = 273.16$ K

$P_{tr} = 1.250 \times 10^5$ Pa

$P = 1.797 \times 10^5$ Pa

Absolute temperature of normal melting point of Sulphur

$$T = \left(\frac{P}{P_{tr}}\right) T_{tr}$$

$$= \frac{1.797 \times 10^5}{1.250 \times 10^5} \times 273.16 = 392.69 \text{ K}$$

For the thermometer B,

$P_{tr} = 0.200 \times 10^5$ Pa

$P = 0.287 \times 10^5$ Pa

∴ Absolute temperature = T

$$= \left(\frac{P}{P_{tr}}\right) T_{tr} = \frac{0.287 \times 10^5}{0.200 \times 10^5} \times 273.16$$

$= 391.98$ K

b. The discrepancy arises because the gases are not perfectly ideal. To reduce the discrepancy, readings should be taken for lower and lower pressures and the plot between temperature measured versus absolute pressure of the gas at triple point should be extrapolated to obtain temperature in the limit pressure tends to zero, when the gases approach ideal gas behaviour.

6. A steel tape 1m long is correctly calibrated for a temperature of 27.0 °C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0°C. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27.0 °C? Coefficient of linear expansion of steel $= 1.20 \times 10^{-5}$ K^{-1}.

Ans.

We have $l = 63$ cm

$\alpha_l = 1.20 \times 10^{-5}$ K^{-1}

$T_1 = 27°C$ $T_2 = 45°C$

$\Delta l = l \alpha_l \Delta T$ $\Delta T = T_2 - T_1 = 180°C$

$= 63 \times 1.2 \times 10^{-5} \times 18 = 0.0136$ cm

∴ Actual length of the rod at 45°C = 63 + 0.0136 = 63.0136 cm

Length of the rod at 27°C = 63 cm

7. A large steel wheel is to be fitted on to a shaft of the same material. At 27 °C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range :

$\alpha_{l(steel)} = 1.20 \times 10^{-5}$ K^{-1}.

Ans. $\alpha_{l(steel)} = 1.20 \times 10^{-5}$ K^{-1}

$\Delta l = -0.01$ cm $l = 8.7$ cm

$$\Delta T = \frac{\Delta l}{l\alpha_{l(steel)}} = \frac{-0.01}{8.7 \times 1.2 \times 10^{-5}} = -96°C$$

∴ The temperature at which the wheel may slip on the shaft $= -96 + 27 = -69°C$

8. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0 °C. What is the change in the diameter of the hole when the sheet is heated to 227 °C? Coefficient of linear expansion of copper $= 1.70 \times 10^{-5}$ K^{-1}.

Ans.

Let $T_1 = 27°C$ $T_2 = 227°C$

∴ $\Delta T = 227 - 27 = 200°C$

$\alpha_l = 1.70 \times 10^{-5}$ K^{-1} $l = 4.24$ cm

$\Delta l = l\,\alpha_l \Delta T$

$= 4.24 \times 1.70 \times 10^{-5} \times 200$

$= 1.44 \times 10^{-2}$ cm

9. A brass wire 1.8 m long at 27 °C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of −39 °C, what is the tension developed in the wire, if its diameter is 2.0 mm? Coefficient of linear expansion of brass = 2.0×10^{-5} K^{-1}; Young's modulus of brass = 0.91×10^{11} Pa.

Ans.

$l = 1.8$ m $T_1 = 27°C$ $T_2 = -39°C$

$2r = 2$ mm $r = 1$ mm $= 10^{-3}$m

$\alpha_l = 2.0 \times 10^{-5}\ K^{-1}$

$Y = 0.91 \times 10^{11}$ Pa

$\Delta T = 66°C$

$\Delta l = l\,\alpha_l \Delta T$

Also Young's modulus, $Y = \dfrac{Fl}{A\Delta l}$

$$Y = \frac{Fl}{A l \alpha_l \Delta T}$$

$$\therefore \text{Tension } F = \frac{YA\alpha_l \Delta T l}{l}$$

$= YA\alpha_l \Delta T = Y\pi r^2 \alpha_l \Delta T$

$= 0.91 \times 10^{11} \times 3.14 \times 10^{-6} \times 2 \times 10^{-5} \times 66 = 3.8 \times 10^2$ N

10. A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250 °C, if the original lengths are at 40.0 °C? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand (Coefficient of linear expansion of brass = $2.0 \times 10^{-5}\ K^{-1}$, steel = $1.2 \times 10^{-5}\ K^{-1}$).

Ans. $l_{brass} = 50$ cm

$\alpha_{brass} = 2.0 \times 10^{-5}\ K^{-1}$ $T_1 = 40°C$

$\Delta l_{brass} = l_{brass} \times \alpha_{brass} \times \Delta T$

$T_2 = 250°C$

$= 50 \times 2 \times 10^{-5} \times 210 = 0.21$ cm

$\Delta T = 210°C$

$l_{steel} = 50$ cm

$\alpha_{Steel} = 1.2 \times 10^{-5}\ K^{-1}$

$\therefore \Delta l_{Steel} = 50 \times 1.2 \times 10^{-5} \times 210 = 0.13$ cm

∴ Change in length of the combined rod, since the ends are free to expand

$= \Delta l_{brass} + \Delta l_{Steel} = 0.21 + 0.13$

$= 0.34$ cm

11. The coefficient of volume expansion of glycerin is $49 \times 10^{-5}\ K^{-1}$. What is the fractional change in its density for a 30 °C rise in temperature?

Ans. $\alpha_V = 49 \times 10^{-5} K^{-1}$

$\Delta T = 30°C$

$$\alpha_V = \left(\frac{\Delta V}{V}\right)\frac{1}{\Delta T}$$

$$\therefore \frac{\Delta V}{V} = \alpha_V \times \Delta T$$

∴ Fractional change in its density

$= \alpha_V \Delta T = 49 \times 10^{-5} \times 30$

$= 0.0147 = 1.5 \times 10^{-2}$

12. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings.
Specific heat of aluminium
$= 0.91\ J\ g^{-1}\ K^{-1}$.

Ans.

Power = P = 10 kW

Since 50% of its power is used for heating, heat lost in 2.5 minutes,

= 50% of P for 2.5 minutes

$= 5 \times 10^3 \times 150 = 750 \times 10^3$ J

Given $s = 0.91\ Jg^{-1}\ K^{-1}$

$= 0.91 \times 10^3\ Jkg^{-1}\ K^{-1}$

m = 8 kg

∴ Rise in heat of the block in 2.5 minutes,

msdT = heat loss $= 750 \times 10^3$

$$\therefore dT = \frac{750 \times 10^3}{8 \times 0.91 \times 10^3} = 103 \text{ K}$$

13. A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper is 0.39 J g^{-1} K^{-1}; Latent heat of fusion of water is 335 J g^{-1}).

Sol.

$msdT = m'L$, $s = 0.39 \times 10^3 J\ kg^{-1} K^{-1}$,

$L = 335 \times 10^3 J.kg^{-1}$

$2.5 \times (0.39 \times 10^3) \times 500$

$= m' \times (335 \times 10^3)$

$$m' = \frac{2.5 \times 0.39 \times 10^3 \times 500}{335 \times 10^3} = 1.5 kg$$

14. In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150cm³ of water at 27°C. The final temperature is 40°C. Compute the specific heat of the metal.

Sol. Mass of metal = m = 0.2 kg

Specific heat of metal = s

Initial temperature of metal $= T_0$

$= 150°C$

Mass of water $= m_1 = 150 \times 10^{-6} \times 10^3$

= 0.15 kg

(Mass = volume × density)

Water equivalent of calorimeter

= W = 0.025 kg

Specific heat of water $= s_w$

$= 4.2 \times 10^3 J\ kg^{-1} K^{-1}$

Initial temperature of calorimeter and water $= T_1 = 27°C$

Final temperature of the mixture

= T = 40°C

Heat lost by metal = Heat gained by (calorimeter + water)

$ms\ (T_0 - T) = (m_1 + W)\ s_w (T - T_1)$

$$s = \frac{(m_1 + W)s_w(T - T_1)}{m(T_0 - T)}$$

$$= \frac{(0.15 + 0.025) \times 4.2 \times 10^3 (40 - 27)}{0.2 \times (150 - 40)}$$

$= 434 J.kg^{-1}.K^{-1}$

15. Given below are observations on molar specific heats at room temperature of some common gases.

Gas	Molar specific heat (C_v) (cal mol^{-1} K^{-1})
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon - monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monatomic gases. Typically, molar specific heat of a monatomic gas is 2.92 cal/mol K. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine?

Ans. The gases are diatomic, and have other degrees of freedom (i.e., have other modes of motion) possible besides the translational degrees of freedom. To raise the temperature of the gas by a certain amount, heat is to be supplied to increase the average energy of all the modes. Consequently, molar specific heat of diatomic gases is more than that of monatomic gases. It can be shown that, if only rotational modes of motion are considered, the molar specific heat of diatomic gases is nearly $\frac{5}{2}R$ which agrees with the observations for all the gases listed in the table.

The higher value of molar specific heat of chlorine indicates that besides rotational modes, vibrational modes are also present in chlorine at room temperature.

16. Answer the following questions based on the P-T phase diagram of carbon dioxide:

a. At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium?

b. What is the effect of decrease of pressure on the fusion and boiling point of CO_2?

c. What are the critical temperature and pressure for CO_2? What is their significance?

d. Is CO_2 solid, liquid or gas at a. –70 °C under 1 atm, b. – 60 °C under 10 atm, c. 15 °C under 56 atm?

Ans.

a. At the triple point temperature = –56.6°C and pressure = 5.11 atm

b. If pressure decreases, both boiling point and freezing point of CO_2 decrease.

c. The critical temperature and pressure of CO_2 are 31.1°C and 73 atm respectively. Above this temperature, CO_2 will not liquefy even if compressed to high pressures.

d. a. Vapour b. Solid c. Liquid

17. Answer the following questions based on the P - T phase diagram of CO_2:

a. CO_2 at 1 atm pressure and temperature – 60 °C is compressed isothermally. Does it go through a liquid phase?

b. What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure?

c. Describe qualitatively the changes in a given mass of solid CO_2 at 10 atm pressure and temperature – 65 °C as it is heated upto room temperature at constant pressure.

d. CO_2 is heated to a temperature 70 °C and compressed isothermally. What changes in its properties do you expect to observe?

Ans.

a. No, vapour condenses to solid directly.

b. It condenses to solid directly without passing through the liquid phase.

c. It turns to liquid phase and then to vapour phase. The fusion and boiling points are where the horizontal line on P - T diagram at the constant pressure of 10 atm intersects the fusion and vapourisation curves.

d. It will not exhibit any clear transition to the liquid phase, but will depart more and and more from ideal gas behaviour as its pressure increases.

18. A child running a temperature of 101°F is given an antipyrin (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98°F in 20 min, what is the average rate of extra evaporation caused, by the drug. Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal g^{-1}.

Sol. Amount of heat gained by sweat = heat lost from body

i.e., $mL = Ms_wT$, where m is mass of sweat in 20 min., L - latent heat, M - mass of the body, s_w - specific heat, T - fall in temperature.

$$T = 3\times\frac{5}{9} = \frac{5}{3}°C$$

$$\therefore m = \frac{Ms_w T}{L} \text{ and rate is}$$

$$\frac{m}{t} = \frac{Ms_w T}{L\times t} = \frac{30\times10^3\times1\times5}{3\times20\times580}$$

= 4.2 g/min.

19. A 'thermacole' icebox is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45°C, and coefficient of thermal conductivity of thermacole is $0.01 Js^{-1}m^{-1}K^{-1}$. (Heat of fusion of water = $335\times10^3 J\ kg^{-1}$)

Ans. $A = 6a^2 = 6\times(30\times10^{-2})m^2$

$= 0.54 m^2$

$dx = 5\times10^{-2} m$,

$T_1 - T_2 = 45 - 0 = 45°C$

$K = 0.01\ Wm^{-1}K^{-1}$;

$t = 6$ hours $= 6\times3600$ sec

$$Q = KA\frac{(T_1-T_2)}{dx}t$$

$$Q = \frac{0.01\times0.54\times54\times6\times3600}{5\times10^{-2}} = 1{,}04{,}976\ J$$

$L = 335\times10^3 J/kg$

$$Q = mL \qquad \therefore m = \frac{Q}{L} = \frac{104976}{335000}$$

$= 0.313$ kg

ie., Amount of ice melted = m

= 0.313 kg

Total amount of ice = M = 4kg

Amount of ice left = M – m

= 4 – 0.313 = 3.687 kg

20. A brass boiler has a base area of $0.15\ m^2$ and thickness 1 cm. It boils water at the rate 6 kg/minute when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. TC of brass = $109\ Wm^{-1}K^{-1}$ and L.H. of steam = 2256 kJ/kg.

Ans. Let T be the temperature at the point of contact and $T_i = 100°C$

$$\frac{dQ}{dt} = KA\left(\frac{dT}{dx}\right) = mL_{steam}$$

$$dT = \frac{m.L_{steam}.dx}{KA}; \qquad dT = T - T_i$$

$m = 6$ kg/min $= \frac{6}{60}$ kg/sec,

$L_{steam} = 2256\times10^3 J/kg$

$dx = 1\times10^{-2}$ m, $K = 109\ Wm^{-1}K^{-1}$,

$A = 0.15\ m^2$

$$dT = \frac{6}{60}\times\frac{2256\times10^3\times1\times10^{-2}}{109\times0.15}$$

$$= \frac{2256}{109\times0.15}$$

$\therefore dT = T - T_i = 137.9 = 138°C$

$T - 100 = 138°C$

$\therefore T = 238°C$

21. Explain why

a. a body with large reflectivity is a poor emitter

b. a brass tumbler feels much colder than a wooden tray on a chilly day

c. an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives a too low value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace

d. the earth without its atmosphere would be inhospitably cold

e. heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

Sol. a. A body with large reflectivity is a poor absorber of heat. Poor absorbers are poor emitters.

b. Brass has got high thermal conductivity. Brass absorbs heat from human body and so the brass tumbler feels colder. Wood is a bad conductor of heat. So heat does not flow from human body to wooden tray. Hence the reason.

c. Temperature of the hot iron = T
Heat radiated per sec per $m^2 = E = \sigma T^4$
When the body is placed in the open at a temperature T_0, heat radiated per sec per m^2 is $E' = \sigma(T^4 - T_0^4)$. $E' < E$ and hence the reason.

d. The heat radiation received by the earth from the sun during day- time is kept trapped by the atmosphere. Hence the reason.

e. Steam has more heat than hot water at the same temperature.

22. A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C. The temperature of the surroundings is 20°C.

Ans. $\frac{dT}{dt} \propto \left[\frac{T_1 + T_2}{2} - T_S\right]$; $dT = T_1 + T_2$

$$\frac{(80-50)}{5} \propto \left[\frac{80+50}{2} - 20\right]$$

$$\frac{(60-30)}{t} \propto \left[\frac{60+30}{2} - 20\right]$$

Then, $\frac{30}{5} \times \frac{t}{30} = \frac{65-20}{45-20} = \frac{45}{25}$

$\therefore t = \frac{45 \times 5}{25} = 9$ minutes