Lab 31

Application of integrals

Aim

- To find the area enclosed by curves using definite integrals

Concepts

- Definite integrals

Discussion

We discuss different methods of finding area enclosed by curves with the help of GeoGebra

Activity 31.1 Area bounded by x axis

If the curve $y = f(x)$ in $[a, b]$ is above the x axis area of the region bounded by the curve, x axis $x = a$ and $x = b$ is given by

$A = \int_a^b f(x)dx$. We can find this using the Input Command `Integral(f,a,b)`

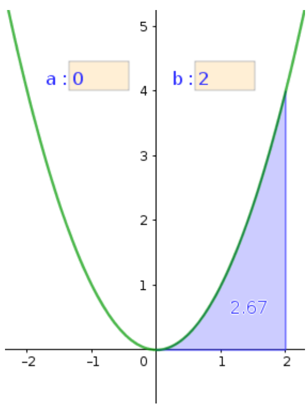
Procedure

- Create two number sliders **a** and **b** and Input boxes for them
- Draw the graph of the function $f(x) = x^2$.
- Find the area enclosed by the curve $y = f(x)$, x axis, $x = 0$ and $x = 2$. [For this set $a = 0$, $b = 2$ and give the Input Command `Integral(f,a,b)`]

  Find the area enclosed by the curve $y = x^2$, x axis, between

  1) $x = -1$ and $x = 3$

  2) $x = -5$ and $x = 0$



- Create a Input Box for the function $f(x)$.

  Find the area enclosed by the curve $y = x^2 - 1, x$ axis between $x = -1$ and $x = 1$.

  Find the required area and complete the following table.

Sl.No.	Enclosed Region	Area
1	$y^2 = x, x$ axis, $x = 1, x = 4$	
2	$\frac{x^2}{16} + \frac{y^2}{9} = 1$	
3	$x^2 + y^2 = 4$	
4	$y = \sin x, x$ axis, $x = \pi, x = 2\pi$	

- save as Activity33.1

Activity 31.2 Area of a region which is above and below x axis

If a curve lies on both sides of the x axis it is not possible to find its area by direct integration. We discuss different methods of solving such problems.

Procedure

- Find the area enclosed by the curve $y = x^2 - 3, x$ axis $x = -3$ and $x = 3$
- Open the applet Activity33.1. Set $f(x) = x^2 - 3, a = -3$ and $b = 3$.

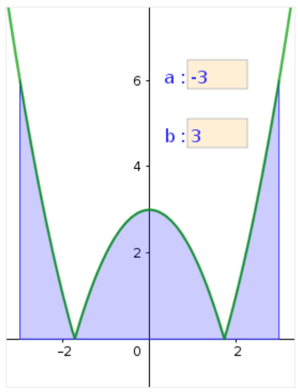
What is the value of $\int_{-3}^{3} x^2 - 3dx$?. Give the reason.

What is the area of the portion below the x axis?

[To find this find the points of intersection A and B of the curve with the x axis and give the input command `Integral(f,x(A),x(B))`]

What is the area of the portion above the x axis?

Find the total area of the region.



Alternate Method

- Edit the function as $f(x) = |x^2 - 3|$. Set $a = -3$ and $b = 3$

Find the area using the input command **Integral**(f,a,b)

Find the required area and complete the following table.

Sl.No.	Enclosed Region	Area
1	$y = \sin x, x$ axis, $x = 0, x = 2\pi$	
2	$y = x^3, x$ axis, $x = -2, x = 4$	
3	$y = \|x\| - 2, x$ axis, $x = -1, x = 3$	
4	$y = 3x + 2, x$ axis, $x = -1, x = 1$	

If the area bounded by the curve $y = \sqrt{3x}$ and the line $x = 12$ is divided in the ratio 1:7 by the line $x = a$, then find the value of a. (If necessary change the maximum value of the slider **b**)

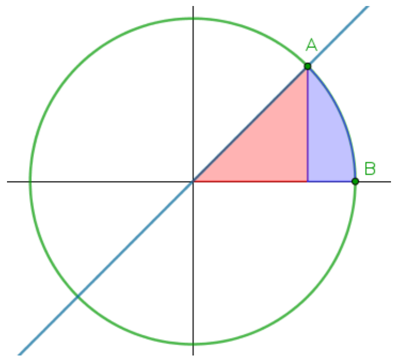
Activity 31.3 Combination of curves

We discuss the methods of finding the area bounded by a combination of curves.

Find the area of the region bounded by the curve $x^2 + y^2 = 32$ and the line $y = x$.

Procedure

- Draw the circle $x^2 + y^2 = 32$ and the line $y = x$.
- Identify the enclosed region.
- Mark the point of intersection A of the line with the circle. Also mark the point of intersection B of the circle with the x axis.
- Find the area under the line $y = x$ using the input command `a=Integral(x,0,x(A))`.
- Find the area under the circle using the input command `b=Integral(sqrt(32-x^2),x(A),x(B))`.



Find the required area.

Alternate method

- Define the function as $f(x) = \begin{cases} x & \text{if } 0 < x < x(A) \\ \sqrt{32 - x^2} & \text{if } x(A) < x < x(B) \end{cases}$

For this give the input command `f(x)=If(0<x<x(A),x,x(A)<x<x(B),sqrt(32-x^2)`

Find the area using the input command `Integral(f,0,x(B))`

Find the area of the following regions.

1) Region in the first quadrant bounded by x axis, the line $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 4$
2) Region lying above x axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.
3) Region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.
4) The Region $\{(x, y) : 0 \le y \le x^2, 0 \le y \le x + 1, 0 \le x \le 2$

Activity 31.A Availability of medicine in the blood

Medicine can be administered to patients in different ways. For a given method let c(t) be the concentration of the medicine in the blood t hours after the dose is given.Over the time interval $0 \le t \le a$ the area between the grapf of $c(t)$ in the interval $[0, a]$ indicates the availability of the medicine for the patients body over the time.

Since GeoGebra does not have the time axis, we will change the independent variable to x.

Two methods Method 1 and Method II for administrating the medicine is governed by the functions $f(x)$ and $g(x)$ represented as follows

$$f(x) = 5(e^{-0.2x} - e^{-x})$$

$$f(x) = 4(e^{-0.2x} - e^{-3x})$$

In this activity we try to explore the availability of medicine to the patients body

Procedure

- Create a slider a with minimum value 0 and maximum value 24 , corresponding to 24 hour period
- Create the function $f(x)$
- Create the function $g(x)$
- Evaluate the integral of f from 0 to a using integral function .This can be achieved by inputting $Integral(f, 0, a)$
- Evaluate the integral of g from 0 to a as above .In the algebra view we can see that two variables are created for the area of the two figures
- Move the slider a and observe the change in the areas

If 4 is the value of a , which area is more?Which method provides better availability of the medicine?

If 24 is the value of a, which area is more? Which method provides better availability of the medicine?

Move the slider in such a way that the areas of both the functions are same.What does this condition signify in medical perspective?