Lab 19

Invertible Functions

Aim:

- To explore invertible functions

Concepts:

- Bijective functions
- Inverse of a function
- Composition of functions

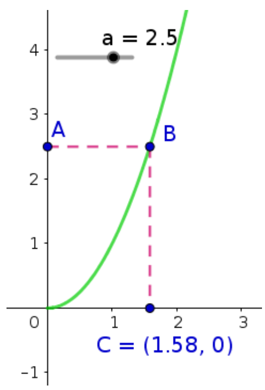
Discussion :
We discuss the method of finding the value of $f^{-1}(a)$,for a given real number a,from the graph of $f(x)$.Methods of drawing graph of inverse of a function is also discussed.

Activity 19.1 Inverse of a Function 1

If f is an invertible function and (x, y) is a point on the graph of f then $f(x) = y$ and $f^{-1}(y) = x$. We use this idea to find the value of $f^{-1}(a)$, for given values of a, from the graph of $f(x)$.

Procedure :

- Draw the graph of $f(x) = x^2, x \geq 0$
- Crate a slider a
- Plot the point $A(0, a)$
- Draw the line passing through A and parallel to the x axis
- Plot the point of intersection B of the above line with the curve
- Draw the line through B, perpendicular to the x axis
- Plot the point of intersection C with the x axis
- Hide the lines and draw the line segments AB and BC
- Show the coordinates of C



Using this applet, how can we find an approximate value of $\sqrt{a}$ for a given value of a ?

Find approximate values of the following

i) $\sqrt{3}$ ii) $\sqrt{4.5}$

- Note : if we need more accuracy we can increase the number of decimal places. (Options $\rightarrow$ Rounding)
- Save the applet as Activity 19.1

Activity 19.2 Inverse of a Function 2

Procedure :

Use the applet ML19.2

About the applet:

This applet is similar to that we created in the above activity.In addition there are two input boxes to edit the function f and the slider a

Select suitable functions and find approximate values of the following

		Function	Value
i	$\sqrt[3]{4}$		
ii	$\log_2 10$		
iii	$\frac{1}{\sqrt{5}}$		

Activity 19.3 Inverse of a Function 3

Procedure

- Find the inverse of the function $f(x) = x^3$
- Draw the graphs of $f(x)$ and $f^{-1}(x)$
- Draw the line $y = x$

  Compare the graphs of f and f^{-1} with respect to the above line. What do you observe ?

  If we know the graph of function f how can we draw a rough sketch of the graph of f^{-1} ?
- Using the tool *Reflect about line* we can draw the reflection of a graph of a function about a line (Click on the graph of the function and then on the line)
- Draw the graph of function as $f(x) = x^2$ and find its reflection on the line $y = x$.Does it represent the graph of a function?Why?
- To get the graph of inverse of a function ,restrict the domain of the function -if needed-in order to make it one to one,draw the graph of the function in the restricted domain and find the reflection on the line $y = x$

  Find the inverse of the following functions algebraically and draw its graph. Draw the graph of the given function and find its reflection on the line $y = x$. Verify whether it coincides with the graph of the given function. Find the domain and range of the given function.

	Function	Inverse	Domain	Range
i	$\sqrt{x+2}$			
ii	$9x^2 + 6x - 5$			
iii	$\frac{x}{x+2}$			

In algebra view ,the equation of a reflected curve is shown only in its parametric form. The command Invert(f) gives the graph of inverse of a function and its equation in explicit form.It is not necessary to restrict the domain to get the inverse.But this command also has some drawbacks
It works only if the function contains just one x.If f is a quadratic function ,then the command :Invert(CompleteSquare(f)) gives the inverse
If f is a rational function with first degree polynomials in numerator ,then the command Invert(PartialFractions(f)) gives the inverse
In CAS view 'invert' command directly works even if the function contains more than one x

Activity 19.4 Composition and Inverse

Procedure :

- Use the applet Activity 18.4
- Give $f(x) = 3x - 2$ and $g(x) = \dfrac{x+2}{3}$

  Observe the graphs of $f \circ g$ and $g \circ f$. What do you see ? Why it happens so ?

- Find the inverse of the following functions and verify your answer using above applet.[Without using the applet, one can verify this by drawing the graphs of $g \circ f$ and $f \circ g$]

	$f(x)$	$f^{-1}(x)$
i	$\dfrac{x-2}{3}$	
ii	$(3-x^3)^{\frac{1}{3}}$	

Additional Activities

Activity 19.A Self Invertible Functions

We know that the inverse of $f(x) = \dfrac{1}{x}$ is the function itself. We may call it as a self invertible function. Here we discuss a method to generate self invertible functions from existing one. The logic behind the construction is that the graphs of self invertible functions are symmetric with respect to the line $y = x$ (why ?)

Procedure

- draw the graph of the function $f(x) = \dfrac{1}{x}$ and the line $y = x$
- The graph is symmetric with respect to the line. If we shift it along the line, it keeps its symmetry
- Create a slider a
- Create the function $g(x) = f(x - a) + a$. Find the simplified form of g using the command `Simplify(g)`.

- Create an input box for f and save the applet as Activity 19.A

We get different self invertible functions for different values of a. Find some functions and verify it using the applet Activity 18.4

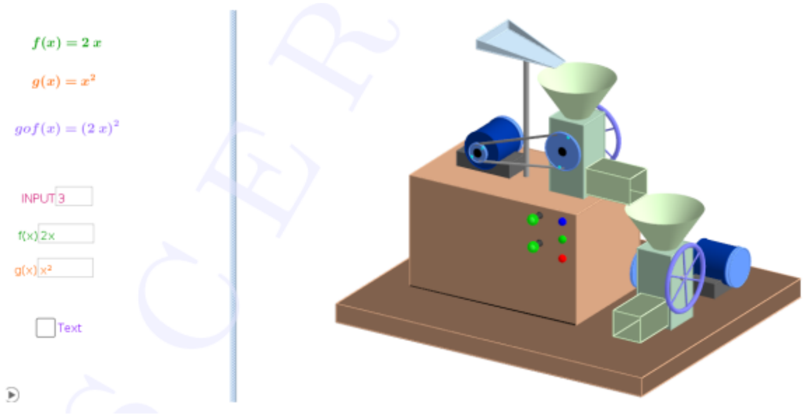
Starting from the functions $\frac{2}{x}$, $\frac{-3}{2x}$, $\frac{2x-5}{2x-2}$ generate more self invertible functions.

Activity 19.B Composition Machine

Last year we have studied functions as input -output machines,which gives outputs corresponding to the given inputs.Here we compare composition of functions with combination of machines in which output of one of the machine is the input of the other.

Use Applet ML19.A

About the applet;



In 3D graphics we can see two machines combined together.Output of the first machine is the input of the second.There are three buttons on the machine, Blue,Green ,and Red,in order to refresh,start or stop the machine.Two indicator lights are provided which turns red if the corresponding input is not suitable for the machines.From the Graphics view we can edit the functions as well as the input number

Procedure

- Set $f(x) = 2x$ and $g(x) = x^2$

Find the values of $g \circ f(2)$ and $g \circ f(3)$ and$g \circ f(-3)$

- Set $f(x) = 2x$ and $g(x) = \sqrt{x}$

Find the values of $g \circ f(2)$ and $g \circ f(3)$

What happens to the indicators if we give -3 as the input? why?

What can you say about the domain of the function g in order to define $g \circ f$?

- Set $f(x) = \sqrt{}$ and $g(x) = x^2$

- Find the values of $g \circ f(2)$ and $g \circ f(1)$ and$g \circ f(0)$
- What happens to the indicators if we give -2 as the input? why?
- Find the domain in which $g \circ f(x) = x$
- Find the domain in which $f \circ g(x) = x$.