Chapter 14

Oscillations

14.1 Introduction

The motion of a car on a level road is rectilinear (one dimensional) and the motion of a projectile is curvilinear (two dimensional). But both these are non repetitive motions. Now consider uniform circular motion or motion of planets around a star. These are examples of periodic motion. You may consider yet another type of motion - motion of pendulum of a clock, rocking of a cradle, motion of a swing, motion of air particles during the propagation of sound wave through air, motion of excited string of a violin etc. These are also periodic motions and are particularly known as oscillatory motion or sometimes vibrational motion. A study of the subject physics never can go deep without understanding oscillatory or vibrational motion..

14.2 Periodic and Oscillatory Motions

A fly climbs inside a glass vessel. As it reaches the top, it falls quickly down. It then repeats the same. The displacement-time graph (approximate) is as shown, fig. 1(a).

A smart boy walks along a training ground which has evenly made steps of

2 m width and 20 cm height. The height versus time graph will be as shown in fig.1(b) (assuming uniform motion, negligible time of ascent or descent)

An acrobat throws two balls to a certain height repeatedly. The position-time graph of any ball would look like the graph shown in fig.1(c).

All the above said examples show periodic (repeated) motion.

Let us take another example. A small steel ball is placed in a watch glass. It is displaced a few cm sideways and left. The ball makes to and fro motion periodically. At the turning points force is maximum (Is it not interesting?). The force on the ball tries to bring it always back to equilibrium. The resulting motion is oscillatory or vibratory.

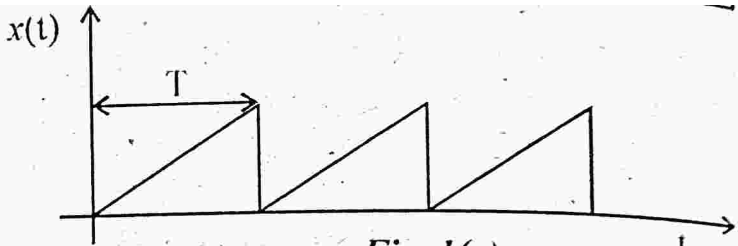


Fig. 1(a)

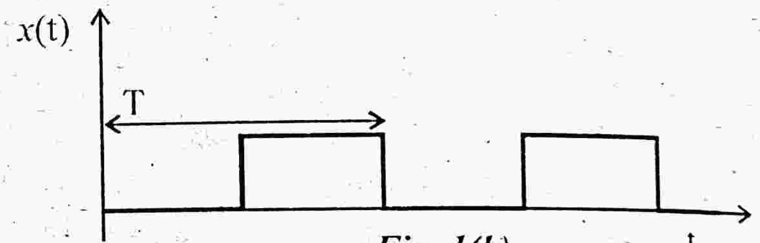


Fig. 1(b)

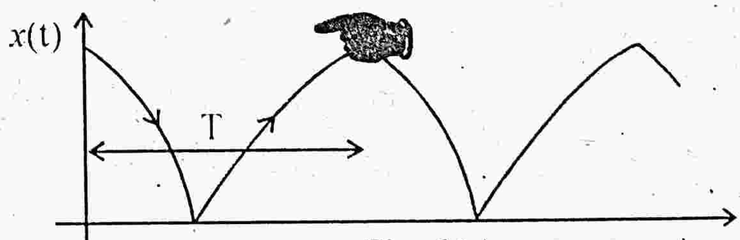


Fig. 1(c)

Examples of periodic motion

e.g. of oscillatory or vibrational motion

i. Motion of earth or planets around sun.

ii. Motion of moon around earth.

iii. Motion of the hands of a clock.

iv. Rotation of earth about its polar axis
v. Motion of Halley's comet around the sun.
vi. Motion of the bob of a simple pendulum.
vii. Heart beat of a healthy person.
viii. Vibrations of atoms in a molecule.

Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.

Let us now have a close look on the motions called oscillations and vibrations. A periodic motion with small frequency (like the oscillations of a branch of a tree under breeze) is called oscillation while the one with high frequency (like the vibrations of an excited string of a violin) is called vibrations.

Simple harmonic motion is the simplest type of oscillatory or vibratory motion.

e.g.

i. Oscillations of a mass suspended from a spring.
ii. The motion of the pendulum of a wall clock.
iii. Motion of a freely suspended bar magnet.
iv. Vibration of the prongs of an excited tuning fork.
v. Vibration of an atom in a molecule.
vi. The motion of liquid contained in U- tube when it is compressed once in one limb and left to itself.
vii. Vibrations of the wire of a guitar.

An oscillating simple pendulum, a ball rolling on a smooth concave curved surface etc. all come to rest after some oscillations. This is because of the existence of dissipative force, in reality. Hence such motions (i.e., except for ideal case) are actually called damped periodic motions. It may be noted that these motions can be made again regular periodic motions under the action of an impressed periodic force. The resulting motion is then called forced periodic motion.

If a number of periodic oscillations are coupled by an elastic force, all of them oscillate in a beautiful pattern which gives rise to a wave pattern.

14.2.1 Period and Frequency

Time - Period (T)

It is the smallest interval of time after which the periodic motion of a body repeats itself.

Unit : second (s)

If a particle undergoes 'n' oscillations in a time 't' second, then its time - period, $T = \dfrac{\text{Time}}{\text{Number of oscillations}}$, $T = \dfrac{t}{n}$

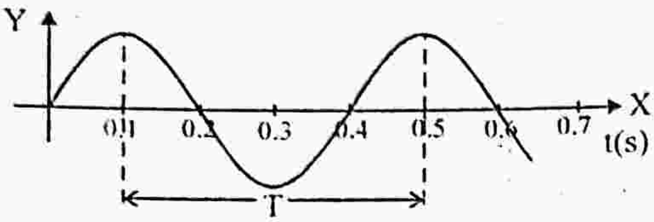
Frequency (ν)

The number of repetitions (of a periodic motion) per second is called ***frequency*** of the periodic motion.

Unit: second^{-1} or Hertz (Hz) or cycles per second (c.p.s).

The relation between ν and T is $\nu = \frac{1}{T}$

Solved Examples

1. 

Find (i) the period and (ii) frequency for the periodic motion whose graph is shown above.

Sol. From figure i. T = 0.4 s

ii. $\nu = \frac{1}{T} = \frac{1}{0.4} = 2.5$ Hz

2. On an average a human heart is found to beat 75 times in a minute. Calculate its frequency and period.

Sol. The beat frequency of heart

= 75/(1 min)

= 75/(60 s) = 1.25 s^{-1}

= 1.25 Hz

The time period T = 1/(1.25 s^{-1})

= 0.8 s

Displacement

In an oscillatory motion any measurable physical quantity or property which varies with time is called a displacement variable. Its value at any instant is called displacement. In mechanics, displacement is the change of position of a body with time in a specified direction. But, oscillatory motion takes place not only in mechanics but also in sound, electricity and magnetism. So displacement may be in physical quantities like position, angle, voltage, pressure, electric and magnetic fields etc.

e.g. i. In a simple pendulum, the displacement variable is the change in angle with respect to the vertical.

The displacement of periodic motion can be represented by a periodic function. A periodic function in time is f(t) = $A\cos\omega t$. Let us increase the argument (i.e., angle $\theta = \omega t$) ωt to $\omega t + 2\pi$ so that t is changed to $\left(t + \frac{2\pi}{\omega}\right)$ or (t + T).

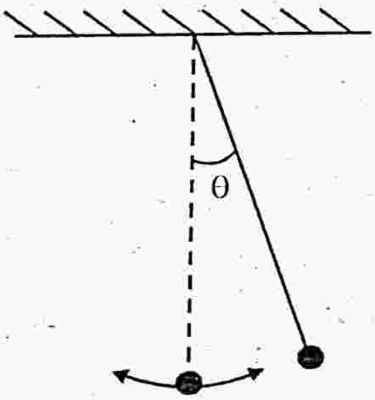


Fig. 2(a)

A simple pendulum. Angular displacement is θ

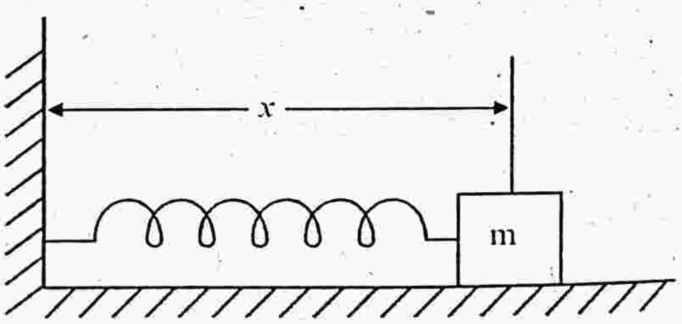


Fig. 2(b)

A loaded spring. The position of oscillating mass is x, measured from fixed wall

$\therefore f(t + T) = A\cos\overline{\omega t + 2\pi} = A\cos\omega t$ since $\cos\overline{2\pi + \theta} = \cos\theta$. Thus the function $f(t)$ is periodic with period $T = \left(\frac{2\pi}{\omega}\right)$ i.e., A function is periodic if, $f(t) = f(t + T)$

Let us try with another function $f(t) = A\sin\omega t + B\cos\omega t$

To bring this to the form sin A cos B + cos A sin B, let us put $A = D\cos\phi$ and $B = D\sin\phi$

$\therefore f(t) = D\sin\omega t\cos\phi + D\cos\omega t\sin\phi = D\sin\overline{\omega t + \phi}$ where D and ϕ are constants given by $A^2 = D^2\cos^2\phi$, $B^2 = D^2\sin^2\phi$ so that $D^2 = A^2 + B^2$ or $D = \sqrt{A^2 + B^2}$ and the argument modifier ϕ, is given by $\tan\phi = \frac{B}{A}$ so that $\phi = \tan^{-1}\left(\frac{B}{A}\right)$.

According to French Mathematician Joseph Fourier (1768-1830), ***any periodic function can be expressed as a combination (superposition) of sine and cosine functions of different time periods and with suitable coefficients.***

Solved Examples

3. Which of the following functions of time represents (a) periodic and (b) non-periodic motion? Give the period for each case of periodic motion [ω is any positive constant].

   i. $\sin\omega t + \cos\omega t$

   ii. $\sin\omega t + \cos 2\omega t + \sin 4\omega t$

   iii. $e^{-\omega t}$ iv. $\log(\omega t)$

Sol.

i. $\sin\omega t + \cos\omega t$ is a periodic function, it can also be written as $\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$.

Now $\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$

$$= \sqrt{2}\sin\left(\omega t + \frac{\pi}{4} + 2\pi\right)$$

$$= \sqrt{2}\sin\left[\omega\left(t + \frac{2\pi}{\omega}\right) + \frac{\pi}{4}\right]$$

$\therefore$ The periodic time of the function is $\frac{2\pi}{\omega}$.

ii. By Fourier concept, this is an example for periodic motion. For $\cos 2\omega t$, we may write $\cos\left(t + \frac{2\pi}{2\omega}\right)$ or $\cos\left(t + \frac{T_0}{2}\right)$, where $\frac{T_0}{2} = \frac{2\pi}{2\omega} = T$ and for $\sin 4\omega t$, $T = \frac{T_0}{4}$.

The period of first is a multiple of period of second or third. The smallest interval of time after which the given function repeats is T_0 and hence the function has period $T_0 = \frac{2\pi}{\omega}$.

iii. The function $e^{-\omega t}$ is not periodic, it decreases monotonically with increasing time and tends to zero as $t \to \infty$ and thus, never repeats its value.

iv. The function $\log(\omega t)$ increases monotonically with time t. It, therefore, never repeats its value and is a non-periodic function. It may be noted that as $t \to \infty$, $\log(\omega t)$ diverges to ∞. It, therefore, cannot represent any kind of physical displacement.