14.5 VELOCITY AND ACCELERATION IN SIMPLE HARMONIC MOTION

The function $x(t) = A\cos\omega t$ is continuous and finite at any time (t). Therefore the velocity, acceleration and force are variable in nature; that also in magnitude and direction within periodic time itself.

Velocity at any time 't' is

$$v(t) = \frac{d}{dt}x(t) = -A\omega\sin\overline{\omega t+\phi} \quad \ldots\ldots\ldots (2)$$

$$= -A\omega\sqrt{1-\cos^2\overline{\omega t+\phi}}$$

$$= -A\omega\sqrt{1-\frac{x^2}{A^2}} = -\omega\sqrt{A^2-x^2}$$

or $v = \omega\sqrt{A^2-x^2}$ (numerically)

When $x = 0$, velocity is maximum, $v_{max} = A\omega$. This is at mean position. When $x = A$, i.e., at extreme position velocity is minimum and it is $v_{min} = 0$.

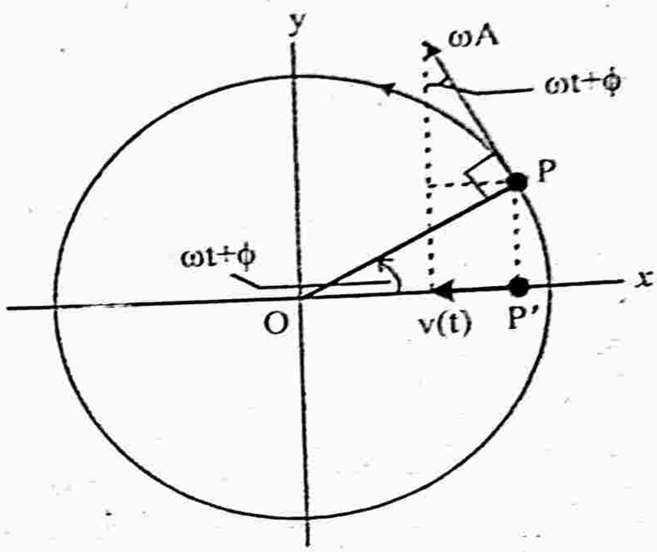


Fig. 10

This figure (Fig. 10), compared with fig. 9, shows the velocity of P, shown by the tangent through P, its value is $v = \omega A$. The projected value is $v(t) = -\omega A\sin\overline{\omega t+\phi}$ .... (3) as is seen from figure. The –ve sign is due to the component directed towards left, in the negative direction of x-axis. (3) is the instantaneous velocity.

Acceleration

Acceleration being derivative of velocity, acceleration $a(t) = \frac{d}{dt}v(t)$ ...... (4)

$$= \frac{d}{dt} - A\omega\sin\overline{\omega t+\phi} =$$

$$-A\omega^2\cos\overline{\omega t+\phi} = -\omega^2 x$$

This acceleration expression shows that

i. acceleration $\propto$ displacement

ii. always against displacement. The two conditions of SHM.

The projection of acceleration of P on x-axis is $a(t) = -\omega^2 A\cos\overline{\omega t+\phi}$

$= -\omega^2 x(t)$

At extreme position, acceleration is maximum $a_{max} = \pm\omega^2 A$

$\because x(t) = \mp A$ and at mean position

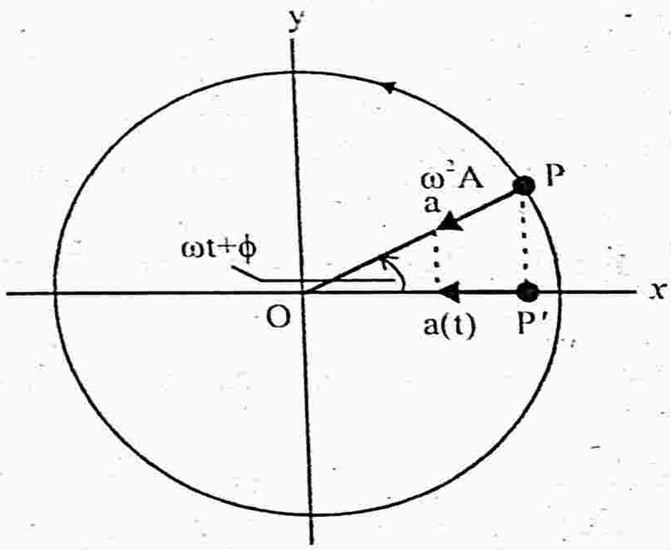
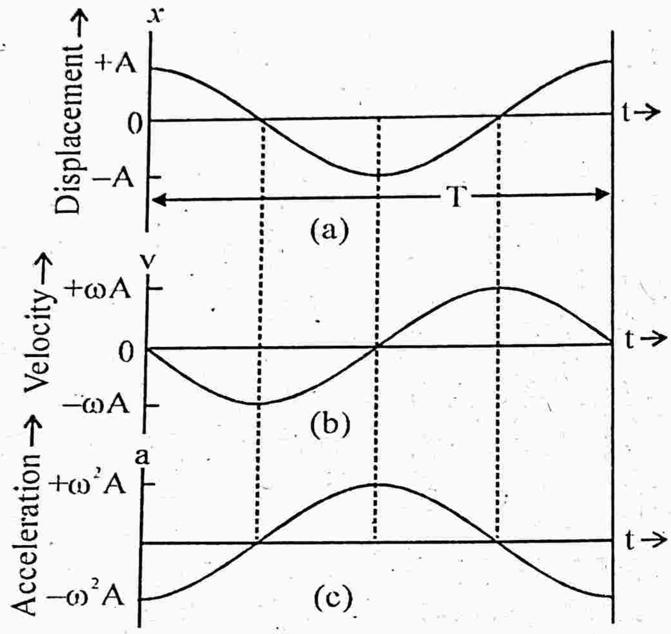


Fig. 11

it is $a_{min} = 0, \because x(t) = 0$



a. Displacement $x(t)$ - time (t) graph

b. Velocity v(t) - time (t) graph

c. Acceleration a(t) - time (t) graph

Fig. 12

Time period (T)

Time period *is defined as the time taken by the particle executing SHM to complete one vibration.*

Acceleration, $a = \omega^2 x$ (neglecting the negative sign)

$$\omega = \sqrt{\frac{a}{x}}$$

Time period, $T = \frac{2\pi}{\omega} = 2\pi \times \frac{1}{\sqrt{(\frac{a}{x})}} = 2\pi\sqrt{\frac{1}{(\frac{a}{x})}}$

$$T = 2\pi\sqrt{\frac{1}{\text{acceleration/unit displacement}}}$$

Frequency, $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$