Q) A 0.150-kg toy is undergoing SHM on the end of a horizontal spring with force constant k = 300 N/m. When the toy is 0.0120 m from its equilibrium position, it is observed to have a speed of 0.400 m/s. What are the toy's (a) total energy at any point of its motion; (b) amplitude of motion; (c) maximum speed during its motion?

A) Here given that the mass of the toy is $m = 0.15\,\text{kg}$.
The force constant of restoring force $k = 300\,\text{N m}^{-1}$.
When the position of the toy from the equilibrium is $x = 0.012\,\text{m}$, then the speed of the toy $v_x = 0.4\,\text{m s}^{-1}$.
**(a)** The total mechanical energy in SHM is given by

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

Where $m$ is the mass of the object, $v_x$ is the velocity, $k$ is the force constant of restoring force and $A$ is the amplitude of SHM.
Hence by substituting the numerical values in equation (1), we get

$$\begin{aligned} E &= \frac{1}{2} \times 0.15\,\text{kg} \times \left(0.4\,\text{m s}^{-1}\right)^2 + \frac{1}{2} \times 300\,\text{N m}^{-1} \times (0.012\,\text{m})^2 \\ &= 0.034\,\text{J} \end{aligned}$$

**(b)** By rearranging the equation (1), WE GET

$$\begin{aligned} A &= \sqrt{\frac{2E}{k}} \\ &= \sqrt{\frac{2 \times 0.034\,\text{J}}{300\,\text{N m}^{-1}}} \\ &= 1.5\,\text{cm} \end{aligned}$$

**(c)** maximum speed of the toy during its motion will be at equlibrium position, where total energy is kinetic energy, and potential energy is zero. Hence, at equilibrium position,

$$E = \frac{1}{2}mv_{\text{max}}^2$$

Hence by rearranging the above equation (4), we get

$$\begin{aligned} v_{\text{max}} &= \sqrt{\frac{2E}{m}} \\ &= \sqrt{\frac{2 \times 0.034\,\text{J}}{0.15\,\text{kg}}} \\ &= 0.67\,\text{m s}^{-1} \end{aligned}$$