

Q) A transverse harmonic wave on a string is described by

$$y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$$

where  $x$  and  $y$  are in cm and  $t$  in s. The positive direction of  $x$  is from left to right.

(a) Is this a travelling wave or a stationary wave?

If it is travelling what are the speed and direction of its propagation?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) What is the least distance between two successive crests in the wave?

Ans) (a) The equation of progressive wave travelling from right to left is given by the displacement function:

$$y(x, t) = a \sin(\omega t + kx + \phi) \dots$$

(i)

The given equation is:

$$y(x, t) = 3.0 \sin(36t + 0.018x + \frac{\pi}{4})$$

...(ii)

On comparing both the equations, we find that equation (ii) represents a travelling wave, propagating from right to left.

Now using equations (i) and (ii), we can write:

$$\omega = 36 \text{ rad/s and } k = 0.018 \text{ m}^{-1}$$

We know that:

$$v = \omega/2\pi \text{ and } \lambda = 2\pi/k$$

Also,

$$v = f\lambda$$

$$\therefore v = (\omega/2\pi) \times (2\pi/k) = \omega/k$$

$$= 36/0.018 = 2000 \text{ cm/s} = 20 \text{ m/s}$$

Hence, the speed of the given travelling wave is 20 m/s.

(b) Amplitude of the given wave,

$$a = 3 \text{ cm}$$

Frequency of the given wave:

$$f = \omega/2\pi = 36/2 \times 3.14 = 573 \text{ Hz}$$

(c) On comparing equations (i) and (ii), we find that the initial phase angle,  $\phi = \pi/4$

(d) The distance between two successive crests (or troughs) is equal to the wavelength of the wave.

Wavelength is given by the relation:  $k = 2\pi/\lambda$

$$\therefore \lambda = 2\pi/k = 2 \times 3.14/0.018 = 348.89 \text{ cm} = 3.49 \text{ m}$$