Senior School Certificate Examination-2020
Marking Scheme - MATHEMATICS
Subject Code: 041 Paper Code: 65/4/1

***General instructions*:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks 0 - 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

XII MATHEMATICS

QUESTION PAPER CODE 65/4/1

EXPECTED ANSWER/VALUE POINTS

Q. No.	VALUE POINTS	Marks
	SECTION - A	
	Question Numbers 1 to 20 carry 1 mark each.	
	Q. Nos. 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:	
1	The value of $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ is (a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{5}$ (c) $\frac{-\pi}{10}$ (d) $\frac{-3\pi}{5}$ **Answer:** $\boxed{(c)-\frac{\pi}{10}}$	1
2	If $A=\begin{bmatrix}2 & -3 & 4\end{bmatrix}, B=\begin{bmatrix}3\\2\\2\end{bmatrix}, X=\begin{bmatrix}1 & 2 & 3\end{bmatrix}$ and $Y=\begin{bmatrix}2\\3\\4\end{bmatrix}$, then $AB+XY$ equals (a) $[28]$ (b) $[24]$ (c) 28 (d) 24 **Answer:** $\boxed{(a)[28]}$	1
3	If $\begin{vmatrix}2 & 3 & 2\\x & x & x\\4 & 9 & 1\end{vmatrix}+3=0$, then the value of x is (a) 3 (b) 0 (c) -1 (d) 1 **Answer:** $\boxed{(c)-1}$	1
4	$\int_0^{\frac{\pi}{8}}\tan^2(2x)\,dx$ is equal to (a) $\frac{4-\pi}{8}$ (b) $\frac{4+\pi}{8}$ (c) $\frac{4-\pi}{4}$ (d) $\frac{4-\pi}{2}$ **Answer:** $\boxed{(a)\frac{4-\pi}{8}}$	1
5	If $\vec{a}.\vec{b}=\frac{1}{2}\lvert\vec{a}\rvert\lvert\vec{b}\rvert$, then the angle between $\vec{a}$ and $\vec{b}$ is is (a) 0^0 (b) 30^0 (c) 60^0 (d) 90^0 **Answer:** $(c)\,60^0$	1

6	The two lines $x = ay + b, z = cy + d$; and $x = a'y + b', z = c'y + d'$ are perpendicular to each other, if (a) $\frac{a}{a'} + \frac{c}{c'} = 1$ (b) $\frac{a}{a'} + \frac{c}{c'} = -1$ (c) $aa' + cc' = 1$ (d) $aa' + cc' = -1$ **Answer:** $\boxed{(d)\, aa' + cc' = -1}$	1
7	The two planes $x - 2y + 4z = 10$ and $18x + 17y + kz = 50$ are perpendicular, if k is equal to (a) -4 (b) 4 (c) 2 (d) -2 **Answer:** $\boxed{(b)\, 4}$	1
8	In a LPP, if the objective function $z = ax + by$ has the same maximum value on two corner points of a feasible region, then the number of points at which $z_{\max}$ occurs is (a) 0 (b) 2 (c) finite (d) infinite **Answer:** $\boxed{(d)\, \text{infinite}}$	1
9	From the set $\{1, 2, 3, 4, 5\}$, two numbers a and b $(a \neq b)$ are chosen at random. The probability that $\frac{a}{b}$ is an integer is (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$ **Answer:** $\boxed{(b)\, \frac{1}{4}}$	1
10	A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white is (a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{12}$ (d) $\frac{1}{24}$ **Answer:** $\boxed{(c)\, \frac{1}{12}}$	1
In Q. Nos. 11 to 15, fill in the blanks with correct word/sentence:		
11	If $f : R \to R$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then $fof(x) =$ __________. **Answer:** x	1
12	If $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$, then $x.y =$ __________. **Answer: - 3**	1
13	The number of points of discontinuity of f defined by $f(x) = \lvert x \rvert - \lvert x+1 \rvert$ is	

	___________. **Answer:** Zero	1
14	The slope of the tangent to the curve $y = x^3 - x$ at the point $(2,6)$ is __________. **Answer:** 11 **OR** The rate of change of the area of a circle with respect to its radius r , when r=3 cm, is __________. **Answer:** $6\pi\ \text{cm}^2/\text{cm}$	1 1
15	If $\vec{a}$ is a non-zero vector, then $(\vec{a}.\hat{i})i+(\vec{a}.\hat{j})j+(\vec{a}.\hat{k})k$ equals ___________. **Answer:** $\vec{a}$ **OR** The projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$ is _____________. **Answer:** 0	1 1
Q. 16 to 20 are very short answer questions.		
16	Find $\text{adj}A$, if $A=\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$. **Answer:** $\text{adj}A=\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$ ($\frac{1}{2}$ mark for any two correct co-factors)	1
17	Find $\int \frac{2^{x+1}-5^{x-1}}{10^x}dx$ **Answer:** $I=\int\left(2(5^{-x})-\frac{1}{5}(2^{-x})\right)dx=-\frac{2}{5^x \log 5}+\frac{1}{5(2^x)\log 2}+C$	1
18	Evaluate $\int_0^{2\pi}\|\sin x\|dx$ **Answer:** $I=4\int_0^{\frac{\pi}{2}}\sin x\,dx=4$	$\frac{1}{2}+\frac{1}{2}$
19	If $\int_0^a \frac{dx}{1+4x^2}=\frac{\pi}{8}$, then find the value of a . **Answer:**	

	$\int_0^a \frac{dx}{(2x)^2+1}=\frac{\pi}{8}$ $\Rightarrow \frac{1}{2}\left[\tan^{-1}(2x)\right]_0^a=\frac{\pi}{8}$ $\Rightarrow a=\frac{1}{2}$ **OR** Find $\int \frac{dx}{\sqrt{x}+x}$ **Answer:** Put $\sqrt{x}=t$ $\therefore \int \frac{dx}{\sqrt{x}+x}=2\log\left(1+\sqrt{x}\right)+C$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
20	Show that the function $y=ax+2a^2$ is a solution of the differential equation $2\left(\frac{dy}{dx}\right)^2+x\left(\frac{dy}{dx}\right)-y=0$. **Answer:** $y=ax+2a^2 \Rightarrow \frac{dy}{dx}=a$ $\text{LHS}=2\left(\frac{dy}{dx}\right)^2+x\left(\frac{dy}{dx}\right)-y$ $=2(a)^2+x(a)-\left(ax+2a^2\right)=0=\text{RHS}$	$\frac{1}{2}$ $\frac{1}{2}$
	SECTION – B	
Q. Nos. 21 to 26 carry 2 marks each.		
21	Check if the relation R on the set $A=\{1,2,3,4,5,6\}$ defined as $R=\{(x,y): y \text{ is divisible by } x\}$ is (i) symmetric (ii) transitive. **Answer:** (i) As $(2,4)\in R$ but $(4,2)\notin R \Rightarrow R$ is not symmetric. (ii) Let $(a,b)\in R$ and $(b,c)\in R$ $\Rightarrow b=\lambda a$ and $c=\mu b$ Now, $c=\mu b=\mu(\lambda a)\Rightarrow (a,c)\in R$ $\Rightarrow R$ is transitive. **OR** Prove that: $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ **Answer:**	1 1

	$\text{LHS} = \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$ $= \frac{9}{4}\left[\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right] = \frac{9}{4}\cos^{-1}\frac{1}{3}$ $= \frac{9}{4}\sin^{-1}\left(\sqrt{1-\left(\frac{1}{3}\right)^2}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) = \text{RHS}$	1 1
22	Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$, if $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$. **Answer:** $\frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$ $\frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$ $\therefore \frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$ $\therefore \left.\frac{dy}{dx}\right\|_{\theta=\frac{\pi}{3}} = \sqrt{3}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
23	Show that the function f defined by $f(x) = (x-1)e^x + 1$ an increasing function for all $x > 0$. **Answer:** $f'(x) = xe^x$ Now $x > 0$ and $e^x > 0$ for all x $\therefore f'(x) > 0 \Rightarrow f$ is an increasing function.	1 $\frac{1}{2}$ $\frac{1}{2}$
24	Find $\|\vec{a}\|$ and $\|\vec{b}\|$, if $\|\vec{a}\| = 2\|\vec{b}\|$ and $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 12$. **Answer:** $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = 12 \Rightarrow \|\vec{a}\|^2 - \|\vec{b}\|^2 = 12$ $\Rightarrow 3\|\vec{b}\|^2 = 12 \Rightarrow \|\vec{b}\| = 2$ Now, $\|\vec{a}\|^2 = 12 + \|\vec{b}\|^2 = 16 \Rightarrow \|\vec{a}\| = 4$ **OR** Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ **Answer:**	1 1

	$\vec{a}\times\vec{b}=7\hat{i}-6\hat{j}-10\hat{k}$ and $\left\|\vec{a}\times\vec{b}\right\|=\sqrt{185}$ Required unit vector $=\frac{1}{\sqrt{185}}\left(7\hat{i}-6\hat{j}-10\hat{k}\right)$	$1+\frac{1}{2}$ $\frac{1}{2}$
25	Find the equation of the plane with intercept 3 on the y-axis and parallel to xz – plane. **Answer:** Let required plane parallel to xz -plane is $y=k$ Given y -intercept is 3 $\Rightarrow k=3$ $\Rightarrow$ Equation of required plane is $y=3$	1 1
26	Find $\left[P(B\|A)+P(A\|B)\right]$, if $P(A)=\frac{3}{10}, P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{3}{5}$. **Answer:** $P(A\cap B)=\frac{3}{10}+\frac{2}{5}-\frac{3}{5}=\frac{1}{10}$ Now, $P(B\|\mathrm{A})+P(A\|B)$ $=\frac{P(A\cap B)}{P(A)}+\frac{P(A\cap B)}{P(B)}$ $=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$	$\frac{1}{2}$ $\frac{1}{2}$ 1
	SECTION – C	
Q. Nos. 27 to 32 carry 4 marks each.		
27	Prove that the relation R on Z, defined by $R=\{(x,y):(x-y)\text{ is divisible by } 5\}$ is an equivalence relation. **Answer:** For reflexive $x-x=0$, for every $x\in Z$ is divisible by 5 $\Rightarrow(x,x)\in R$ For symmetric $(x,y)\in R\Rightarrow x-y$ is divisible by $5\Rightarrow y-x$ is divisible by 5 $\Rightarrow(y,x)\in R\Rightarrow R$ is symmetric. For transitive Let $(x,y)\in R$ and $(y,z)\in R$ $(x,y)\in R\Rightarrow x-y=5\lambda \quad ...(i)$ $(y,z)\in R\Rightarrow y-z=5\mu \quad ...(ii)$ adding (i) and (ii), $x-z=5(\lambda+\mu)=5k$ $\Rightarrow(x,z)\in R\Rightarrow R$ is transitive. Hence R is an equivalence relation.	1 1 2
28	If $y=\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$, then show that $\frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$	

	Answer: Put $x=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}x$ $\therefore y=\sin^{-1}\left(\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{2}\right)=\sin^{-1}\left(\sin\left(\frac{\pi}{4}+\theta\right)\right)$ $\Rightarrow y=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$ $\Rightarrow \frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$ **OR** Verify the Rolle's Theorem for the function $f(x)=e^x\cos x$ in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. **Answer:** f is continuous in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. f is differentiable in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ with $f'(x)=e^x(\cos x-\sin x)$ Also, $f\left(\frac{-\pi}{2}\right)=f\left(\frac{\pi}{2}\right)=0$ All conditions of Rolle's Theorem are satisfied. *So*, there exist $c\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ such that $f'(c)=0\Rightarrow e^c(\cos c-\sin c)=0$ $\Rightarrow c=\frac{\pi}{4}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	1 2 $\frac{1}{2}$ $\frac{1}{2}$ 2 1 1
29	Evaluate: $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$ **Answer:** $I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx \quad ...(i)$ $\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx \quad ...(ii)$ Adding (i) and (ii) $2I=\int_0^{\pi}\frac{\pi\sin x}{1+\cos^2 x}dx$ Putting $\cos x=t$ gives $I=\frac{\pi}{2}\int_{-1}^{1}\frac{dt}{1+t^2}$ $\Rightarrow I=\frac{\pi}{2}\left[\tan^{-1}t\right]_{-1}^{1}=\frac{\pi^2}{4}$	1 1 1 1
30	For the differential equation given below, find a particular solution satisfying the given condition. $(x+1)\frac{dy}{dx}=2e^{-y}+1\ ;\ y=0$ when $x=0$	

Answer:

Given differential equation can be written as

$$\frac{dy}{2e^{-y}+1}=\frac{dx}{x+1}$$ 1

$$\Rightarrow \int \frac{e^{y}}{2+e^{y}}\,dy=\int \frac{dx}{x+1}$$

$$\Rightarrow \log\left|2+e^{y}\right|=\log\left|x+1\right|+\log C$$ 2

$$\Rightarrow 2+e^{y}=C\left(x+1\right)$$

when $x=0,\ y=0 \Rightarrow C=3$ $\frac{1}{2}$

$\therefore$ Required solution is $2+e^{y}=3\left(x+1\right)$ or $e^{y}=3x+1$ $\frac{1}{2}$

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31 A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day . He produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

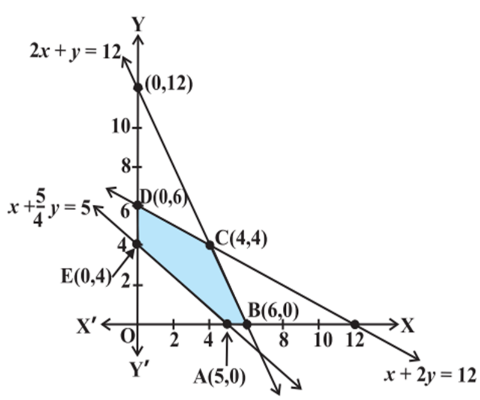
Items	Number of hours required on Machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹ 600 and ₹ 400 on items M and N respectively. How many of each item should he produce so as to maximize his profit assuming that she can sell all the items that he produced? What will be the maximum profit?

Answer:

Let x units of item M and y units of item N are produced.

For correct graph : $1\frac{1}{2}$ marks



Maximize $Z=600x+400y$

subject to

$x+2y\le 12$

$2x+y\le 12$

$x+1.25y\ge 5$

$x\ge 0,\ y\ge 0$ $1\frac{1}{2}$

Corner points values:

$Z_{A(5,0)}=3000,\ Z_{B(6,0)}=3600$

$Z_{C(4,4)}=4000,\ Z_{D(0,6)}=2400$

$Z_{E(0,4)}=1600$ $\frac{1}{2}$

$\therefore$ 4 units each of M and N must be produced to get maximum profit of Rs. 4,000 $\frac{1}{2}$

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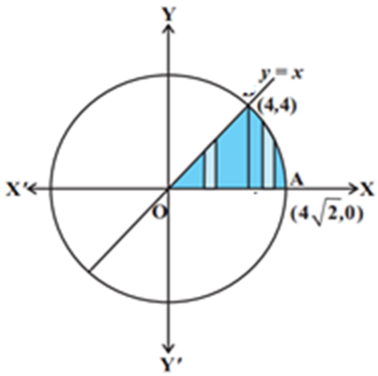
32 A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence find the mean of the number of tails.

	Answer: $P(\text{Head})=\frac{3}{4}, P(\text{Tail})=\frac{1}{4}$ Let X = number of tails. Clearly X can be $0,1,2$ Probability distribution is given by <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$P(X)$</td><td>$\frac{9}{16}$</td><td>$\frac{6}{16}$</td><td>$\frac{1}{16}$</td></tr></table>Mean $=\sum X.P(X)=\frac{1}{2}$ **OR** Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator. **Answer:** Let M be an event of choosing a man and N be an event of choosing a women. A be an event of choosing a good orator. $P(M)=P(W)=\frac{1}{2};$ $P(A\|M)=\frac{5}{100}=\frac{1}{20}, P(A\|W)=\frac{25}{1000}=\frac{1}{40}$ $P(A)=P(A\|M).P(M)+P(A\|W).P(W)$ $=\frac{1}{20}\times\frac{1}{2}+\frac{1}{40}\times\frac{1}{2}=\frac{3}{80}$	1 $\frac{1}{2}$ $1\frac{1}{2}$ 1 $\frac{1}{2}$ 2 $1+\frac{1}{2}$
SECTION – D		
Q. Nos. 33 to 36 carry 6 marks each.		
33	Using properties of determinants prove that: $\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix} = a^3+b^3+c^3-3abc$. **Answer:** $\text{LHS}=\Delta=\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$ $C_3 \to C_3 + C_2$ $\Delta=\begin{vmatrix} a-b & b+c & a+b+c \\ b-c & c+a & a+b+c \\ c-a & a+b & a+b+c \end{vmatrix}$	1

	taking $(a+b+c)$ common from C_3 and applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$ $\Delta = (a+b+c)\begin{vmatrix} a-2b+c & b-a & 0 \\ b-2c+a & c-b & 0 \\ c-a & a+b & 1 \end{vmatrix}$	1+1+1
	expanding along C_3, $\Delta = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$	1
	$= a^3+b^3+c^3-3abc = \text{RHS}$	1
	OR	
	If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = O$. Hence find A^{-1}.	
	Answer: $A^2 = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$	1
	$A^3 = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$	1
	$LHS = A^3 - 4A^2 - 3A + 11I$ $= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$	2
	Now, $A^{-1} = -\frac{1}{11}(A^2 - 4A - 3I)$	1
	$= -\frac{1}{11}\begin{bmatrix} 2 & -5 & -3 \\ -7 & 1 & 5 \\ 4 & 1 & -6 \end{bmatrix}$	1
34	Find the intervals on which the function $f(x) = (x-1)^3(x-2)^2$ is (a) strictly increasing (b) strictly decreasing.	

	Answer: $f(x)=(x-1)^3(x-2)^2$ $\Rightarrow f'(x)=(x-1)^2(x-2)(5x-8)$ (a) for strictly increasing, $f'(x)>0$ $\Rightarrow (x-1)^2(x-2)(5x-8)>0$ $\Rightarrow (x-2)(5x-8)>0 \qquad (\text{as } x\neq 1)$ $\Rightarrow x\in\left(-\infty,\frac{8}{5}\right)\cup(2,\infty) \qquad (\text{as } x\neq 1)$ $\therefore x\in(-\infty,1)\cup\left(1,\frac{8}{5}\right)\cup(2,\infty)$ (b) for strictly decreasing, $f'(x)<0$ $\Rightarrow x\in\left(\frac{8}{5},2\right)$	2 1 $1\frac{1}{2}$ $1\frac{1}{2}$
	OR	
	Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its side. Also, find the maximum volume. **Answer:** Let sides of a rectangle are x cm and y cm. Given $2x+2y=36\Rightarrow y=18-x$ Volume, $V=\pi x^2y=\pi x^2(18-x)=\pi\left(18x^2-x^3\right)$ $\Rightarrow \frac{dV}{dx}=\pi\left(36x-3x^2\right)$ For maxima/minima, put $\frac{dV}{dx}=0$ $\Rightarrow x=12\text{ cm} \quad (\because x\neq 0)$ Again, $\frac{d^2V}{dx^2}=\pi(36-6x)$ $\Rightarrow \left.\frac{d^2V}{dx^2}\right\|_{x=12\text{ cm}}=-36\pi<0$ $\therefore$ Volume is maximum when $x=12$ cm. Also, $y=(18-x)\text{ cm}=6\text{ cm}$ Dimension of rectangle are $12\text{ cm}\times 6\text{cm}$ Maximum volume $=\pi x^2y=864\pi\text{ cm}^3$	 1 1 1 1 1 $\frac{1}{2}+\frac{1}{2}$

35	Find the area of the region lying in the first quadrant and enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.	
	Answer: Point of intersection of $y = x$ and $x^2 + y^2 = 32$ in first quadrant is $(4,4)$.	1
	 Correct figure	1
	$\text{Area} = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$	1
	$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}}$	2
	$= 8 + (4\pi - 8) = 4\pi$	1
36	Show that the lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{b} + \mu\vec{a}$ are coplanar and the plane containing them is given by $\vec{r}.(\vec{a} \times \vec{b}) = 0$.	
	Answer: Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are coplanar if $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$ Now, $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)$ $= (\vec{b} - \vec{a}).(\vec{b} \times \vec{a}) \qquad (\because \vec{a}_1 = \vec{a}, \vec{b}_1 = \vec{b}, \vec{a}_2 = \vec{b}, \vec{b}_2 = \vec{a})$ $= \vec{b}.(\vec{b} \times \vec{a}) - \vec{a}.(\vec{b} \times \vec{a})$ $= [\vec{b} \quad \vec{b} \quad \vec{a}] - [\vec{a} \quad \vec{b} \quad \vec{a}] = 0$ Hence the lines are coplanar.	4
	Equation of plane containing them is given by, $(\vec{r} - \vec{a}).(\vec{b} \times \vec{a}) = 0 \qquad [\because (\vec{r} - \vec{a}).\vec{n} = 0]$ $\Rightarrow \vec{r}.(\vec{b} \times \vec{a}) - \vec{a}.(\vec{b} \times \vec{a}) = 0$ $\Rightarrow \vec{r}.(\vec{b} \times \vec{a}) = 0 \qquad (\because [\vec{a} \quad \vec{b} \quad \vec{a}] = 0)$	2

Senior School Certificate Examination-2020
Marking Scheme - MATHEMATICS
Subject Code: 041 Paper Code: 65/4/2

***General instructions*:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks 0 - 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

XII MATHEMATICS

QUESTION PAPER CODE 65/4/2

EXPECTED ANSWER/VALUE POINTS

Q. No.	Value Points	Marks
	SECTION – A	
	Question Numbers 1 to 20 carry 1 mark each.	
	Q. Nos. 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:	
1	The two planes $x-2y+4z=10$ and $18x+17y+kz=50$ are perpendicular, if k is equal to (a) -4 (b) 4 (c) 2 (d) -2 **Answer:** $\boxed{(b)\,4}$	1
2	If $A=\begin{bmatrix}2 & -3 & 4\end{bmatrix}, B=\begin{bmatrix}3\\2\\2\end{bmatrix}, X=\begin{bmatrix}1 & 2 & 3\end{bmatrix}$ and $Y=\begin{bmatrix}2\\3\\4\end{bmatrix}$, then $AB+XY$ equals (a) $[28]$ (b) $[24]$ (c) 28 (d) 24 **Answer:** $\boxed{(a)\,[28]}$	1
3	The value of $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ is (a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{5}$ (c) $\frac{-\pi}{10}$ (d) $\frac{-3\pi}{5}$ **Answer:** $\boxed{(c)-\frac{\pi}{10}}$	1
4	From the set $\{1,2,3,4,5\}$, two numbers a and b $(a\neq b)$ are chosen at random. The probability that $\frac{a}{b}$ is an integer is (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$ **Answer:** $\boxed{(b)\frac{1}{4}}$	1
5	$\int_0^{\frac{\pi}{8}} \tan^2(2x)\,dx$ is equal to (a) $\frac{4-\pi}{8}$ (b) $\frac{4+\pi}{8}$ (c) $\frac{4-\pi}{4}$ (d) $\frac{4-\pi}{2}$ **Answer:** $\boxed{(a)\frac{4-\pi}{8}}$	1
6	The two lines $x=ay+b, z=cy+d$; and $x=a'y+b', z=c'y+d'$ are perpendicular to each other, if (a) $\frac{a}{a'}+\frac{c}{c'}=1$ (b) $\frac{a}{a'}+\frac{c}{c'}=-1$ (c) $aa'+cc'=1$ (d) $aa'+cc'=-1$ **Answer:** $\boxed{(d)\,aa'+cc'=-1}$	1

7	In a LPP, if the objective function $z = ax + by$ has the same maximum value on two corner points of a feasible region, then the number of points at which $z_{\max}$ occurs is (a) 0 (b) 2 (c) finite (d) infinite **Answer:** $\boxed{(d)\text{infinite}}$	1
8	Let $A = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$, then $\lvert AB \rvert$ is equal to (a) 460 (b) 2000 (c) 3000 (d) -7000 **Answer:** $\boxed{(d)-7000}$	1
9	Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, If $\vec{b}$ is a vector such that $\vec{a}.\vec{b} = \lvert\vec{b}\rvert^2$ and $\lvert\vec{a} - \vec{b}\rvert = \sqrt{7}$, then $\lvert\vec{b}\rvert$ equals (a) 7 (b) 14 (c) $\sqrt{7}$ (d) 21 **Answer:** $\boxed{(c)\sqrt{7}}$	1
10	Three dice are thrown simultaneously. The probability of obtaining a total score of 5 is (a) $\frac{5}{216}$ (b) $\frac{1}{6}$ (c) $\frac{1}{36}$ (d) $\frac{1}{49}$ **Answer:** $\boxed{(c)\frac{1}{36}}$	1
In Q. Nos. 11 to 15, fill in the blanks with correct word/sentence:		
11	If $\vec{a}$ is a non-zero vector, then $(\vec{a}.\hat{i})i + (\vec{a}.\hat{j})j + (\vec{a}.\hat{k})k$ equals __________. **Answer:** $\vec{a}$	1
	OR The projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ is ____________. **Answer:** 0	1
12	If $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$, then $x.y =$ __________. **Answer:** -3	1
13	The slope of the tangent to the curve $y = x^3 - x$ at the point $(2,6)$ is _________. **Answer:** 11	1
	OR The rate of change of the area of a circle with respect to its radius r , when r=3 cm, is _________. **Answer:** $6\pi\ \text{cm}^2/\text{cm}$	1
14	If $f : R \to R$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then $fof(x) =$ __________. **Answer:** x	1
15	If $f(x) = 2\lvert x\rvert + 3\lvert\sin x\rvert + 6$, then the right hand derivative of $f(x)$ at $x = 0$ is _______. **Answer:** 5	1

Q. 16 to 20 are very short answer questions.		
16	Evaluate $\int_0^{2\pi} \lvert\sin x\rvert dx$ **Answer:** $I = 4\int_0^{\frac{\pi}{2}} \sin x\,dx = 4$	$\frac{1}{2}+\frac{1}{2}$
17	If $\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$, then find the value of a. **Answer:** $\int_0^a \frac{dx}{(2x)^2+1} = \frac{\pi}{8}$ $\Rightarrow \frac{1}{2}\left[\tan^{-1}(2x)\right]_0^a = \frac{\pi}{8}$ $\Rightarrow a = \frac{1}{2}$ **OR** Find $\int \frac{dx}{\sqrt{x}+x}$ **Answer:** Put $\sqrt{x} = t$ $\therefore \int \frac{dx}{\sqrt{x}+x} = 2\log\left(1+\sqrt{x}\right)+C$	 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
18	Show that the function $y = ax + 2a^2$ is a solution of the differential equation $2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0$. **Answer:** $y = ax + 2a^2 \Rightarrow \frac{dy}{dx} = a$ $\text{LHS} = 2\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y$ $= 2(a)^2 + x(a) - \left(ax + 2a^2\right) = 0 = \text{RHS}$	 $\frac{1}{2}$ $\frac{1}{2}$
19	Find $\int \sin^5\left(\frac{x}{2}\right).\cos\left(\frac{x}{2}\right)dx$ **Answer:** $I = \int \sin^5\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)dx$ Putting $\sin\left(\frac{x}{2}\right) = t$ gives $I = 2\int t^5 dt$ $\therefore I = \frac{t^6}{3} + C = \frac{1}{3}\sin^6\left(\frac{x}{2}\right) + C$	 $\frac{1}{2}$ $\frac{1}{2}$
20	If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then find A^3. **Answer:** $A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$	$\frac{1}{2}+\frac{1}{2}$

SECTION – B		
Q. Nos. 21 to 26 carry 2 marks each.		
21	Check if the relation R on the set $A=\{1,2,3,4,5,6\}$ defined as $R=\{(x,y): y \text{ is divisible by } x\}$ is (i) symmetric (ii) transitive. **Answer:** (i) As $(2,4)\in R$ but $(4,2)\notin R \Rightarrow R$ is not symmetric. (ii) Let $(a,b)\in R$ and $(b,c)\in R$ $\Rightarrow b=\lambda a$ and $c=\mu b$ Now, $c=\mu b=\mu(\lambda a)\Rightarrow (a,c)\in R$ $\Rightarrow R$ is transitive.	1 1
	OR Prove that: $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ **Answer:** $\text{LHS}=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}$ $=\frac{9}{4}\left[\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right]=\frac{9}{4}\cos^{-1}\frac{1}{3}$ $=\frac{9}{4}\sin^{-1}\left(\sqrt{1-\left(\frac{1}{3}\right)^2}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)=\text{RHS}$	1 1
22	Find $\lvert\vec{a}\rvert$ and $\lvert\vec{b}\rvert$, if $\lvert\vec{a}\rvert=2\lvert\vec{b}\rvert$ and $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=12$. **Answer:** $(\vec{a}+\vec{b})(\vec{a}-\vec{b})=12\Rightarrow\lvert\vec{a}\rvert^2-\lvert\vec{b}\rvert^2=12$ $\Rightarrow 3\lvert\vec{b}\rvert^2=12\Rightarrow\lvert\vec{b}\rvert=2$ Now, $\lvert\vec{a}\rvert^2=12+\lvert\vec{b}\rvert^2=16\Rightarrow\lvert\vec{a}\rvert=4$	1 1
	OR Find the unit vector perpendicular to each of the vectors $\vec{a}=4\hat{i}+3\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-\hat{j}+2\hat{k}$. **Answer:** $\vec{a}\times\vec{b}=7\hat{i}-6\hat{j}-10\hat{k}$ and $\lvert\vec{a}\times\vec{b}\rvert=\sqrt{185}$ Required unit vector $=\frac{1}{\sqrt{185}}\left(7\hat{i}-6\hat{j}-10\hat{k}\right)$	$1+\frac{1}{2}$ $\frac{1}{2}$
23	Find $[P(B\mid A)+P(A\mid B)]$, if $P(A)=\frac{3}{10}, P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{3}{5}$. **Answer:** $P(A\cap B)=\frac{3}{10}+\frac{2}{5}-\frac{3}{5}=\frac{1}{10}$ Now, $P(B\mid A)+P(A\mid B)$ $=\frac{P(A\cap B)}{P(A)}+\frac{P(A\cap B)}{P(B)}$ $=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$	$\frac{1}{2}$ $\frac{1}{2}$ 1

24	Show that the function f defined by $f(x)=(x-1)e^{x}+1$ an increasing function for all $x>0$. **Answer:** $f'(x)=xe^{x}$ Now $x>0$ and $e^{x}>0$ for all x $\therefore f'(x)>0 \Rightarrow f$ is an increasing function.	1 $\frac{1}{2}$ $\frac{1}{2}$
25	Find the derivative of $x^{\log x}$ w.r.t. $\log x$. **Answer:** Let $u=x^{\log x}$ and $v=\log x$ Now, $\log u=(\log x)^{2} \Rightarrow \frac{1}{u}\frac{du}{dx}=2\log x.\frac{1}{x}$ $\Rightarrow \frac{du}{dx}=\frac{2\log x}{x}.x^{\log x}$ Again, $v=\log x \Rightarrow \frac{dv}{dx}=\frac{1}{x}$ $\therefore \frac{du}{dv}=2x^{\log x}\log x$	1 $\frac{1}{2}$ $\frac{1}{2}$
26	Find the distance between the parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$. **Answer:** Required distance $=\left\|\frac{-16-5}{\sqrt{16+4+16}}\right\|=\frac{7}{2}$	1+1

SECTION – C

Q. Nos. 27 to 32 carry 4 marks each.

27	A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence find the mean of the number of tails. **Answer:** $P(\text{Head})=\frac{3}{4}, P(\text{Tail})=\frac{1}{4}$ Let $X=$ number of tails. Clearly X can be 0, 1, 2 Probability distribution is given by X: 0, 1, 2; $P(X)$: $\frac{9}{16}$, $\frac{6}{16}$, $\frac{1}{16}$ Mean $=\sum X.P(X)=\frac{1}{2}$	1 $\frac{1}{2}$ $1\frac{1}{2}$ 1
	OR	
	Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator. **Answer:** Let M be an event of choosing a man and N be an event of choosing a women. A be an event of choosing a good orator. $P(M)=P(W)=\frac{1}{2}$; $P(A\|M)=\frac{5}{100}=\frac{1}{20}, P(A\|W)=\frac{25}{1000}=\frac{1}{40}$ $P(A)=P(A\|M).P(M)+P(A\|W).P(W)$ $=\frac{1}{20}\times\frac{1}{2}+\frac{1}{40}\times\frac{1}{2}=\frac{3}{80}$	$\frac{1}{2}$ 2 $1+\frac{1}{2}$

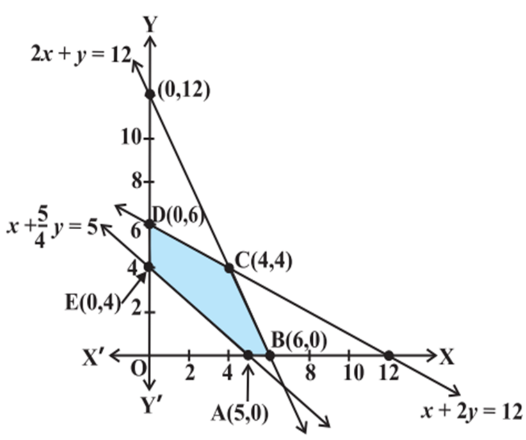
28 A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day . He produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on Machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹ 600 and ₹ 400 on items M and N respectively. How many of each item should he produce so as to maximize his profit assuming that she can sell all the items that he produced? What will be the maximum profit?

Answer:

Let x units of item M and y units of item N are produced.



For correct graph : $1\frac{1}{2}$ marks

Maximize $Z = 600x + 400y$

subject to

$x + 2y \le 12$

$2x + y \le 12$

$x + 1.25y \ge 5$

$x \ge 0, y \ge 0$ $1\frac{1}{2}$

Corner points values:

$Z_{A(5,0)} = 3000, Z_{B(6,0)} = 3600$

$Z_{C(4,4)} = 4000, Z_{D(0,6)} = 2400$

$Z_{E(0,4)} = 1600$ $\frac{1}{2}$

$\therefore 4$ units each of M and N must be produced to get maximum profit of $Rs\ 4,000$ $\frac{1}{2}$

29 If $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$, then show that $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$

Answer:

Put $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1} x$ 1

$\therefore y = \sin^{-1}\left(\frac{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}{2}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{4}+\theta\right)\right)$ 2

$\Rightarrow y = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x$ $\frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$ $\frac{1}{2}$

OR

Verify the Rolle's Theorem for the function $f(x) = e^x \cos x$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

	Answer: f is continuous in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. f is differentiable in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ with $f'(x)=e^x(\cos x-\sin x)$ Also, $f\left(\frac{-\pi}{2}\right)=f\left(\frac{\pi}{2}\right)=0$ All conditions of Rolle's Theorem are satisfied. So, there exist $c\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ such that $f'(c)=0\Rightarrow e^c(\cos c-\sin c)=0$ $\Rightarrow c=\frac{\pi}{4}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	2 1 1
30	For the differential equation given below, find a particular solution satisfying the given condition. $(x+1)\frac{dy}{dx}=2e^{-y}+1$; $y=0$ when $x=0$ **Answer:** Given differential equation can be written as $\frac{dy}{2e^{-y}+1}=\frac{dx}{x+1}$ $\Rightarrow\int\frac{e^y}{2+e^y}dy=\int\frac{dx}{x+1}$ $\Rightarrow\log\left\|2+e^y\right\|=\log\left\|x+1\right\|+\log C$ $\Rightarrow 2+e^y=C(x+1)$ when $x=0, y=0\Rightarrow C=3$ $\therefore$ Required solution is $2+e^y=3(x+1)$ or $e^y=3x+1$	1 2 $\frac{1}{2}$ $\frac{1}{2}$
31	Show that the function $f:R\to R$ defined by $f(x)=\frac{x}{x^2+1}, \forall x\in R$ is neither one-one nor onto. **Answer:** Checking for one-one: here $f(x)=f\left(\frac{1}{x}\right)$. For example $f(2)=f\left(\frac{1}{2}\right)$ $\therefore f$ is not one-one. Checking for onto: Let $y=1\in R$ (co-domain). Then $y=f(x)\Rightarrow\frac{x}{x^2+1}=1$ $\Rightarrow x^2-x+1=0$, which has no real roots. $\therefore R_f\neq$ co-domain $\Rightarrow f$ is not onto.	2 2
32	Evaluate: $\int_{-1}^{2}\left\|x^3-x\right\|dx$ **Answer:** $I=\int_{-1}^{0}(x^3-x)dx+\int_{0}^{1}-(x^3-x)dx+\int_{1}^{2}(x^3-x)dx$ $=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0}+\left[-\frac{x^4}{4}+\frac{x^2}{2}\right]_{0}^{1}+\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{1}^{2}$ $=\frac{1}{4}+\frac{1}{4}+2+\frac{1}{4}=\frac{11}{4}$	$1\frac{1}{2}$ $1\frac{1}{2}$ 1

SECTION – D

Q. Nos. 33 to 36 carry 6 marks each.

33 Show that the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ and $\vec{r} = \vec{b} + \mu \vec{a}$ are coplanar and the plane containing them is given by $\vec{r}.\left(\vec{a} \times \vec{b}\right) = 0$.

Answer:

Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are coplanar if $\left(\vec{a}_2 - \vec{a}_1\right).\left(\vec{b}_1 \times \vec{b}_2\right) = 0$

Now, $\left(\vec{a}_2 - \vec{a}_1\right).\left(\vec{b}_1 \times \vec{b}_2\right)$

$$= \left(\vec{b} - \vec{a}\right).\left(\vec{b} \times \vec{a}\right) \qquad \left(\because \vec{a}_1 = \vec{a}, \vec{b}_1 = \vec{b}, \vec{a}_2 = \vec{b}, \vec{b}_2 = \vec{a}\right)$$

$$= \vec{b}.\left(\vec{b} \times \vec{a}\right) - \vec{a}.\left(\vec{b} \times \vec{a}\right)$$

$$= \left[\vec{b} \quad \vec{b} \quad \vec{a}\right] - \left[\vec{a} \quad \vec{b} \quad \vec{a}\right] = 0$$

Hence the lines are coplanar. **4**

Equation of plane containing them is given by,

$$\left(\vec{r} - \vec{a}\right).\left(\vec{b} \times \vec{a}\right) = 0 \qquad \left[\because \left(\vec{r} - \vec{a}\right).\vec{n} = 0\right]$$

$$\Rightarrow \vec{r}.\left(\vec{b} \times \vec{a}\right) - \vec{a}.\left(\vec{b} \times \vec{a}\right) = 0$$

$$\Rightarrow \vec{r}.\left(\vec{b} \times \vec{a}\right) = 0 \qquad \left(\because \left[\vec{a} \quad \vec{b} \quad \vec{a}\right] = 0\right)$$

2

34 Using properties of determinants prove that: $\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$.

Answer:

$$\text{LHS} = \Delta = \begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$$

$C_3 \to C_3 + C_2$

$$\Delta = \begin{vmatrix} a-b & b+c & a+b+c \\ b-c & c+a & a+b+c \\ c-a & a+b & a+b+c \end{vmatrix}$$ **1**

taking $(a+b+c)$ common from C_3 and applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\Delta = (a+b+c)\begin{vmatrix} a-2b+c & b-a & 0 \\ b-2c+a & c-b & 0 \\ c-a & a+b & 1 \end{vmatrix}$$ **1+1+1**

expanding along C_3,

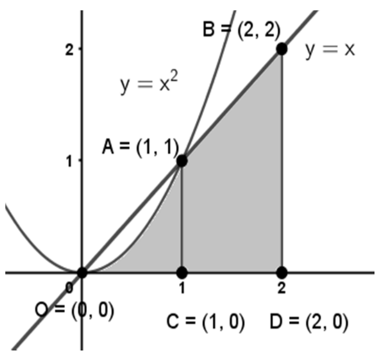
$$\Delta = (a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$$ **1**

$$= a^3 + b^3 + c^3 - 3abc = \text{RHS}$$ **1**

OR

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = O$. Hence find A^{-1}.

	Answer:	
	$A^2 = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$	1
	$A^3 = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$	1
	$LHS = A^3 - 4A^2 - 3A + 11I$	
	$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	2
	$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$	
	Now, $A^{-1} = -\frac{1}{11}\left(A^2 - 4A - 3I\right)$	1
	$= -\frac{1}{11}\begin{bmatrix} 2 & -5 & -3 \\ -7 & 1 & 5 \\ 4 & 1 & -6 \end{bmatrix}$	1
35	Find the intervals on which the function $f(x) = (x-1)^3 (x-2)^2$ is (a) strictly increasing (b) strictly decreasing.	
	Answer:	
	$f(x) = (x-1)^3 (x-2)^2$	
	$\Rightarrow f'(x) = (x-1)^2 (x-2)(5x-8)$	2
	(a) for strictly increasing, $f'(x) > 0$	
	$\Rightarrow (x-1)^2 (x-2)(5x-8) > 0$	1
	$\Rightarrow (x-2)(5x-8) > 0 \qquad (\text{as } x \neq 1)$	
	$\Rightarrow x \in \left(-\infty, \frac{8}{5}\right) \cup (2, \infty) \qquad (\text{as } x \neq 1)$	$1\frac{1}{2}$
	$\therefore x \in (-\infty, 1) \cup \left(1, \frac{8}{5}\right) \cup (2, \infty)$	
	(b) for strictly decreasing, $f'(x) < 0$	
	$\Rightarrow x \in \left(\frac{8}{5}, 2\right)$	$1\frac{1}{2}$
	OR	
	Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its side. Also, find the maximum volume.	
	Answer:	
	Let sides of a rectangle are x cm and y cm.	
	Given $2x + 2y = 36 \Rightarrow y = 18 - x$	1
	Volume, $V = \pi x^2 y = \pi x^2 (18 - x) = \pi\left(18x^2 - x^3\right)$	1
	$\Rightarrow \frac{dV}{dx} = \pi\left(36x - 3x^2\right)$	1
	For maxima/minima, put $\frac{dV}{dx} = 0$	
	$\Rightarrow x = 12$ cm $(\because x \neq 0)$	1

	Again, $\frac{d^2V}{dx^2} = \pi(36-6x)$ $\Rightarrow \left.\frac{d^2V}{dx^2}\right\vert_{x=12\,\text{cm}} = -36\pi < 0$ $\therefore$ Volume is maximum when $x = 12$ cm. Also, $y = (18-x)$ cm $= 6$ cm Dimension of rectangle are $12\text{ cm} \times 6\text{cm}$ Maximum volume $= \pi x^2 y = 864\pi\text{ cm}^3$	1 $\frac{1}{2}+\frac{1}{2}$
36	Using integration, find the area of the region $\{(x,y): 0 \le y \le x^2, 0 \le y \le x, 0 \le x \le 2\}$. **Answer:**  Parabola $y = x^2$ and line $y = x$ intersect at $(0,0)$ and $(1,1)$. Correct Figure Required Area $= \int_0^1 x^2 dx + \int_1^2 x\,dx$ $= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2$ $= \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$ sq. units	1 1 2 1 1

Senior School Certificate Examination-2020
Marking Scheme - MATHEMATICS
Subject Code: 041 Paper Code: 65/4/3

***General instructions*:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**

2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**

3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.

5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

9. A full scale of marks 0 - 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.

10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).

11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
- Leaving answer or part thereof unassessed in an answer book.
- Giving more marks for an answer than assigned to it.
- Wrong totaling of marks awarded on a reply
- Wrong transfer of marks from the inside pages of the answer book to the title page.
- Wrong question wise totaling on the title page.
- Wrong totaling of marks of the two columns on the title page.
- Wrong grand total.
- Marks in words and figures not tallying.
- Wrong transfer of marks from the answer book to online award list.
- Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
- Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.

13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

XII MATHEMATICS

QUESTION PAPER CODE 65/4/3

EXPECTED ANSWER/VALUE POINTS

Q. No.	Value Points	Marks
	SECTION – A	
	Question Numbers 1 to 20 carry 1 mark each.	
	Q. Nos. 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:	
1	The two lines $x = ay + b, z = cy + d$; and $x = a'y + b', z = c'y + d'$ are perpendicular to each other, if (a) $\frac{a}{a'} + \frac{c}{c'} = 1$ (b) $\frac{a}{a'} + \frac{c}{c'} = -1$ (c) $aa' + cc' = 1$ (d) $aa' + cc' = -1$ **Answer:** $\boxed{(d)\, aa' + cc' = -1}$	1
2	If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is (a) 3 (b) 0 (c) -1 (d) 1 **Answer:** $\boxed{(c) -1}$	1
3	In a LPP, if the objective function $z = ax + by$ has the same maximum value on two corner points of a feasible region, then the number of points at which $z_{\max}$ occurs is (a) 0 (b) 2 (c) finite (d) infinite **Answer:** $\boxed{(d)\text{ infinite}}$	1
4	From the set $\{1, 2, 3, 4, 5\}$, two numbers a and b $(a \neq b)$ are chosen at random. The probability that $\frac{a}{b}$ is an integer is (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$ **Answer:** $\boxed{(b)\frac{1}{4}}$	1
5	$\int_0^{\frac{\pi}{8}} \tan^2(2x)\, dx$ is equal to (a) $\frac{4-\pi}{8}$ (b) $\frac{4+\pi}{8}$ (c) $\frac{4-\pi}{4}$ (d) $\frac{4-\pi}{2}$ **Answer:** $\boxed{(a)\frac{4-\pi}{8}}$	1
6	If $\vec{a}.\vec{b} = \frac{1}{2}\lvert\vec{a}\rvert\lvert\vec{b}\rvert$, then the angle between $\vec{a}$ and $\vec{b}$ is is (a) 0^0 (b) 30^0 (c) 60^0 (d) 90^0 **Answer:** $\boxed{(c)\, 60^0}$	1

7	A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white is (a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{12}$ (d) $\frac{1}{24}$ **Answer:** $\boxed{(c)\frac{1}{12}}$	1
8	The value of $\tan^{-1}\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ is (a) $\frac{3+\sqrt{5}}{2}$ (b) $\frac{3-\sqrt{5}}{2}$ (c) $\frac{-3+\sqrt{5}}{2}$ (d) $\frac{-3-\sqrt{5}}{2}$ **Answer:** Since there is a mistake in the question, so full marks may be awarded.	1
9	If $A=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then $\det(\text{adj}A)$ equals (a) a^{27} (b) a^{9} (c) a^{6} (d) a^{2} **Answer:** $\boxed{(c)\, a^6}$	1
10	The line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane (a) $2x+3y+4z=0$ (b) $3x+4y-5z=7$ (c) $2x+y-2z=0$ (d) $x-y+z=2$ **Answer:** $\boxed{(b)\,3x+4y-5z=7 \text{ or } (c)\,2x+y-2z=0}$	1
In Q. Nos. 11 to 15, fill in the blanks with correct word/sentence:		
11	The slope of the tangent to the curve $y=x^3-x$ at the point $(2,6)$ is ________. **Answer:** 11 **OR** The rate of change of the area of a circle with respect to its radius r , when r=3 cm, is ________. **Answer:** $\boxed{6\pi\ \text{cm}^2/\text{cm}}$	1 1
12	If $f: R \to R$ be given by $f(x)=(3-x^3)^{\frac{1}{3}}$, then $fof(x)=$ __________. **Answer:** $\boxed{x}$	1
13	If $\vec{a}$ is a non-zero vector, then $(\vec{a}.\hat{i})i+(\vec{a}.\hat{j})j+(\vec{a}.\hat{k})k$ equals __________. **Answer:** $\boxed{\vec{a}}$ **OR** The projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$ is ____________. **Answer:** $\boxed{0}$	1 1
14	If $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix}=\begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$, then $x.y=$ __________. **Answer:** $\boxed{-3}$	1

15	If $f(x)=x\|x\|$, then $f'(x)=$ ________. **Answer:** $\boxed{f'(x)=\begin{cases}2x & ,x\geq 0\\ -2x & ,x<0\end{cases}}$	1
Q. 16 to 20 are very short answer questions.		
16	Show that the function $y=ax+2a^2$ is a solution of the differential equation $2\left(\frac{dy}{dx}\right)^2+x\left(\frac{dy}{dx}\right)-y=0$. **Answer:** $y=ax+2a^2\Rightarrow\frac{dy}{dx}=a$ $\text{LHS}=2\left(\frac{dy}{dx}\right)^2+x\left(\frac{dy}{dx}\right)-y$ $=2(a)^2+x(a)-(ax+2a^2)=0=\text{RHS}$	$\frac{1}{2}$ $\frac{1}{2}$
17	Find $\text{adj}A$, if $A=\begin{bmatrix}2 & -1\\ 4 & 3\end{bmatrix}$. **Answer:** $\text{adj}A=\begin{bmatrix}3 & 1\\ -4 & 2\end{bmatrix}$ ($\frac{1}{2}$ mark for any two correct co-factors)	1
18	If $\int_0^a\frac{dx}{1+4x^2}=\frac{\pi}{8}$, then find the value of a. **Answer:** $\int_0^a\frac{dx}{(2x)^2+1}=\frac{\pi}{8}$ $\Rightarrow\frac{1}{2}\left[\tan^{-1}(2x)\right]_0^a=\frac{\pi}{8}$ $\Rightarrow a=\frac{1}{2}$ **OR** Find $\int\frac{dx}{\sqrt{x}+x}$ **Answer:** Put $\sqrt{x}=t$ $\therefore\int\frac{dx}{\sqrt{x}+x}=2\log\left(1+\sqrt{x}\right)+C$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
19	Find $\int\frac{1}{x(1+x^2)}dx$ **Answer:** $I=\int\frac{1}{x(1+x^2)}dx=\int\left(\frac{1}{x}-\frac{x}{1+x^2}\right)dx$ $=\log\|x\|-\frac{1}{2}\log\left(1+x^2\right)+C$	$\frac{1}{2}$ $\frac{1}{2}$
20	If $[x]$ denotes the greatest integer function, then find $\int_0^{3/2}\left[x^2\right]dx$ **Answer:** $\int_0^{\frac{3}{2}}\left[x^2\right]dx=\int_0^1 0\,dx+\int_1^{\sqrt{2}}1dx+\int_{\sqrt{2}}^{\frac{3}{2}}2\,dx$ $=0+\left(\sqrt{2}-1\right)+2\left(\frac{3}{2}-\sqrt{2}\right)=2-\sqrt{2}$	$\frac{1}{2}$ $\frac{1}{2}$

SECTION – B

Q. Nos. 21 to 26 carry 2 marks each.

21	Find $\lvert\vec{a}\rvert$ and $\lvert\vec{b}\rvert$, if $\lvert\vec{a}\rvert = 2\lvert\vec{b}\rvert$ and $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 12$. **Answer:** $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = 12 \Rightarrow \lvert\vec{a}\rvert^2 - \lvert\vec{b}\rvert^2 = 12$ $\Rightarrow 3\lvert\vec{b}\rvert^2 = 12 \Rightarrow \lvert\vec{b}\rvert = 2$ Now, $\lvert\vec{a}\rvert^2 = 12 + \lvert\vec{b}\rvert^2 = 16 \Rightarrow \lvert\vec{a}\rvert = 4$ **OR** Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$. **Answer:** $\vec{a}\times\vec{b} = 7\hat{i} - 6\hat{j} - 10\hat{k}$ and $\lvert\vec{a}\times\vec{b}\rvert = \sqrt{185}$ Required unit vector $= \frac{1}{\sqrt{185}}\left(7\hat{i} - 6\hat{j} - 10\hat{k}\right)$	 1 1 $1+\frac{1}{2}$ $\frac{1}{2}$
22	Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$, if $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$. **Answer:** $\frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$ $\frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$ $\therefore \frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$ $\therefore \left.\frac{dy}{dx}\right\rvert_{\theta=\frac{\pi}{3}} = \sqrt{3}$	 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
23	Find the equation of the plane with intercept 3 on the y-axis and parallel to xz – plane. **Answer:** Let required plane parallel to xz -plane is $y = k$ Given y -intercept is 3 $\Rightarrow k = 3$ $\Rightarrow$ Equation of required plane is $y = 3$	 1 1
24	Check if the relation R on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y) : y \text{ is divisible by } x\}$ is (i) symmetric (ii) transitive. **Answer:** (i) As $(2,4) \in R$ but $(4,2) \notin R \Rightarrow R$ is not symmetric. (ii) Let $(a, b) \in R$ and $(b, c) \in R$ $\Rightarrow b = \lambda a$ and $c = \mu b$ Now, $c = \mu b = \mu(\lambda a) \Rightarrow (a, c) \in R$ $\Rightarrow R$ is transitive. **OR** Prove that: $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ **Answer:** LHS $= \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$ $= \frac{9}{4}\left[\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right] = \frac{9}{4}\cos^{-1}\frac{1}{3}$ $= \frac{9}{4}\sin^{-1}\left(\sqrt{1-\left(\frac{1}{3}\right)^2}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) = \text{RHS}$	 1 1 1 1

25 Show that the function $f(x)=\frac{x}{3}+\frac{3}{x}$ decreases in the intervals $(-3,0)\cup(0,3)$.

Answer:

$f'(x)=\frac{1}{3}-\frac{3}{x^2}$ — $\frac{1}{2}$

for decreasing, $f'(x)<0 \Rightarrow \frac{1}{3}-\frac{3}{x^2}<0$ — $\frac{1}{2}$

$\Rightarrow x^2<9 \Rightarrow -3<x<3$

since $f(x)$ is not defined at $x=0$, — 1

so $f(x)$ decreases in $(-3,0)\cup(0,3)$.

26 Three distinct numbers are chosen randomly from the first 50 natural numbers. Find the probability that all the three numbers are divisible by both 2 and 3.

Answer:

Since there are only 8 numbers (in first 50 natural numbers) which are divisible by 6,

$\therefore$ favourable number of outcomes are ${}^{8}C_3$. — $\frac{1}{2}$

Total number of possible outcomes are ${}^{50}C_3$. — $\frac{1}{2}$

Required probability = $\frac{{}^{8}C_3}{{}^{50}C_3}=\frac{1}{350}$ — 1

SECTION – C

Q. Nos. 27 to 32 carry 4 marks each.

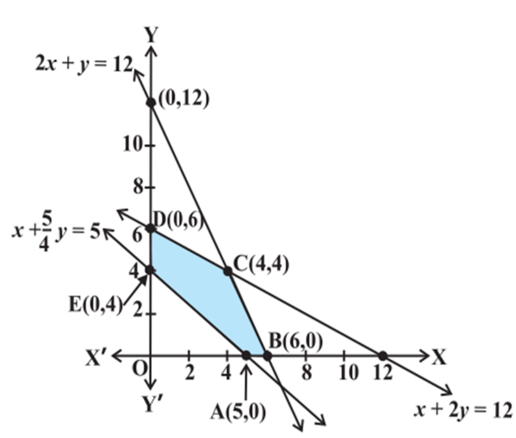
27 A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day . He produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on Machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹ 600 and ₹ 400 on items M and N respectively. How many of each item should he produce so as to maximize his profit assuming that she can sell all the items that he produced? What will be the maximum profit?

Answer:

Let x units of item M and y units of item N are produced.



For correct graph : — $1\frac{1}{2}$ marks

Maximize $Z=600x+400y$

subject to

$x+2y\le 12$

$2x+y\le 12$ — $1\frac{1}{2}$

$x+1.25y\ge 5$

$x\ge 0, y\ge 0$

Corner points values:

$Z_{A(5,0)}=3000, Z_{B(6,0)}=3600$

$Z_{C(4,4)}=4000, Z_{D(0,6)}=2400$ — $\frac{1}{2}$

$Z_{E(0,4)}=1600$

$\therefore$ 4 units each of M and N must be produced to get maximum profit of Rs. 4,000 — $\frac{1}{2}$

28 Prove that the relation R on Z, defined by $R=\{(x,y):(x-y)\text{ is divisible by } 5\}$ is an equivalence relation.

Answer:

For reflexive

$x-x=0$, for every $x\in Z$ is divisible by 5 $\Rightarrow (x,x)\in R$ — **1**

For symmetric

$(x,y)\in R \Rightarrow x-y \text{ is divisible by } 5 \Rightarrow y-x \text{ is divisible by } 5$

$\Rightarrow (y,x)\in R \Rightarrow R \text{ is symmetric.}$ — **1**

For transitive

Let $(x,y)\in R$ and $(y,z)\in R$

$(x,y)\in R \Rightarrow x-y=5\lambda \quad ...(i)$

$(y,z)\in R \Rightarrow y-z=5\mu \quad ...(ii)$

adding (i) and (ii), $x-z=5(\lambda+\mu)=5k$

$\Rightarrow (x,z)\in R \Rightarrow R$ is transitive. — **2**

Hence R is an equivalence relation.

---

29 A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.
Hence find the mean of the number of tails.

Answer:

$P(\text{Head})=\frac{3}{4}, P(\text{Tail})=\frac{1}{4}$ — **1**

Let X = number of tails. Clearly X can be $0,1,2$ — **$\frac{1}{2}$**

Probability distribution is given by

X	0	1	2
$P(X)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

— **$1\frac{1}{2}$**

$\text{Mean}=\sum X.P(X)=\frac{1}{2}$ — **1**

OR

Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator.

Answer:

Let M be an event of choosing a man and N be an event of choosing a women.

A be an event of choosing a good orator. — **$\frac{1}{2}$**

$P(M)=P(W)=\frac{1}{2};$

$P(A|M)=\frac{5}{100}=\frac{1}{20}, P(A|W)=\frac{25}{1000}=\frac{1}{40}$ — **2**

$P(A)=P(A|M).P(M)+P(A|W).P(W)$

$=\frac{1}{20}\times\frac{1}{2}+\frac{1}{40}\times\frac{1}{2}=\frac{3}{80}$ — **$1+\frac{1}{2}$**

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30 If $y=\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$, then show that $\frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$

Answer:

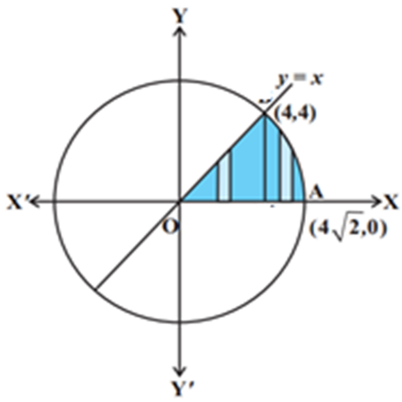
	Put $x=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}x$	1
	$\therefore y=\sin^{-1}\left(\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{2}\right)=\sin^{-1}\left(\sin\left(\frac{\pi}{4}+\theta\right)\right)$	2
	$\Rightarrow y=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$	$\frac{1}{2}$
	$\Rightarrow \frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$	$\frac{1}{2}$
	OR	
	Verify the Rolle's Theorem for the function $f(x)=e^x\cos x$ in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.	
	Answer:	
	f is continuous in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.	
	f is differentiable in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ with $f'(x)=e^x(\cos x-\sin x)$	2
	Also, $f\left(\frac{-\pi}{2}\right)=f\left(\frac{\pi}{2}\right)=0$	
	All conditions of Rolle's Theorem are satisfied. So, there exist $c\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	
	such that $f'(c)=0\Rightarrow e^c(\cos c-\sin c)=0$	1
	$\Rightarrow c=\frac{\pi}{4}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	1
31	Evaluate $\int_0^1\sqrt{3-2x-x^2}\,dx$	
	Answer:	
	$I=\int_0^1\sqrt{3-2x-x^2}\,dx=\int_0^1\sqrt{4-(x+1)^2}\,dx$	1
	Put $x+1=t\Rightarrow dx=dt$. when $x=0, t=1$ and when $x=1, t=2$	$\frac{1}{2}$
	$\therefore I=\int_1^2\sqrt{4-t^2}\,dt=\left[\frac{t}{2}\sqrt{4-t^2}+\frac{4}{2}\sin^{-1}\left(\frac{t}{2}\right)\right]_1^2$	$1\frac{1}{2}$
	$=\left[\left(0+2\sin^{-1}1\right)-\left(\frac{\sqrt{3}}{2}+2\sin^{-1}\frac{1}{2}\right)\right]$	
	$=\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$	1
32	Find the general solution of the differential equation $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x}$.	
	Answer:	
	Given differential equation can be written as	
	$\frac{dy}{dx}=\frac{1}{x}\left(e^y-1\right)\Rightarrow\frac{dy}{e^y-1}=\frac{dx}{x}$	1
	$\Rightarrow\int\frac{dy}{e^y-1}=\int\frac{dx}{x}$	
	$\Rightarrow\int\frac{e^{-y}}{1-e^{-y}}dy=\int\frac{dx}{x}$	1
	$\Rightarrow\log\left\|1-e^{-y}\right\|=\log\left\|x\right\|+\log C$	2
	$\Rightarrow 1-e^{-y}=Cx$	

SECTION – D

Q. Nos. 33 to 36 carry 6 marks each.

33 Find the area of the region lying in the first quadrant and enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

Answer: Point of intersection of $y = x$ and $x^2 + y^2 = 32$ in first quadrant is $(4,4)$. — 1



Correct figure — 1

$$\text{Area} = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$$ — 1

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}}$$ — 2

$$= 8 + (4\pi - 8) = 4\pi$$ — 1

34 Using properties of determinants prove that: $\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$.

Answer:

$$\text{LHS} = \Delta = \begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$$

$C_3 \to C_3 + C_2$

$$\Delta = \begin{vmatrix} a-b & b+c & a+b+c \\ b-c & c+a & a+b+c \\ c-a & a+b & a+b+c \end{vmatrix}$$ — 1

taking $(a+b+c)$ common from C_3 and applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\Delta = (a+b+c)\begin{vmatrix} a-2b+c & b-a & 0 \\ b-2c+a & c-b & 0 \\ c-a & a+b & 1 \end{vmatrix}$$ — 1+1+1

expanding along C_3,

$$\Delta = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$ — 1

$$= a^3 + b^3 + c^3 - 3abc = \text{RHS}$$ — 1

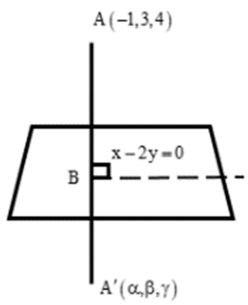
OR

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = O$. Hence find A^{-1}.

	Answer: $A^2=\begin{bmatrix}9&7&5\\1&4&1\\8&9&9\end{bmatrix}$	1
	$A^3=\begin{bmatrix}28&37&26\\10&5&1\\35&42&34\end{bmatrix}$	1
	$LHS=A^3-4A^2-3A+11I$ $=\begin{bmatrix}28&37&26\\10&5&1\\35&42&34\end{bmatrix}-4\begin{bmatrix}9&7&5\\1&4&1\\8&9&9\end{bmatrix}-3\begin{bmatrix}1&3&2\\2&0&-1\\1&2&3\end{bmatrix}+11\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$	2
	Now, $A^{-1}=-\frac{1}{11}\left(A^2-4A-3I\right)$	1
	$=-\frac{1}{11}\begin{bmatrix}2&-5&-3\\-7&1&5\\4&1&-6\end{bmatrix}$	1
35	Find the intervals on which the function $f(x)=(x-1)^3(x-2)^2$ is (a) strictly increasing (b) strictly decreasing. **Answer:**	
	$f(x)=(x-1)^3(x-2)^2$ $\Rightarrow f'(x)=(x-1)^2(x-2)(5x-8)$	2
	(a) for strictly increasing, $f'(x)>0$ $\Rightarrow (x-1)^2(x-2)(5x-8)>0$	1
	$\Rightarrow (x-2)(5x-8)>0 \qquad (\text{as } x\neq 1)$ $\Rightarrow x\in\left(-\infty,\frac{8}{5}\right)\cup(2,\infty) \qquad (\text{as } x\neq 1)$ $\therefore x\in(-\infty,1)\cup\left(1,\frac{8}{5}\right)\cup(2,\infty)$	$1\frac{1}{2}$
	(b) for strictly decreasing, $f'(x)<0$ $\Rightarrow x\in\left(\frac{8}{5},2\right)$	$1\frac{1}{2}$
	OR	
	Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its side. Also, find the maximum volume. **Answer:** Let the dimensions of the rectangle be x cm and y cm.	
	Given $2x+2y=36\Rightarrow y=18-x$	1
	Volume, $V=\pi x^2 y=\pi x^2(18-x)=\pi\left(18x^2-x^3\right)$	1
	$\Rightarrow \frac{dV}{dx}=\pi\left(36x-3x^2\right)$	1
	For maxima/minima, put $\frac{dV}{dx}=0$ $\Rightarrow x=12\,\text{cm} \quad (\because x\neq 0)$	1

<table>
<tr><td></td><td>Again, $\frac{d^2V}{dx^2} = \pi(36-6x)$
$\Rightarrow \left.\frac{d^2V}{dx^2}\right|_{x=12\,\text{cm}} = -36\pi < 0$
$\therefore$ Volume is maximum when $x = 12$ cm.
Also, $y = (18-x)$ cm $= 6$ cm
Dimension of rectangle are $12\text{ cm} \times 6\text{cm}$
Maximum volume $= \pi x^2 y = 864\pi \text{ cm}^3$</td><td>1

$\frac{1}{2}+\frac{1}{2}$</td></tr>
<tr><td>36</td><td>Find the image of the point $(-1,3,4)$ in the plane $x-2y=0$.

Answer:


Let the required image be $A'(\alpha,\beta,\gamma)$.
Equation of AB is $\frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0} = \lambda$
Any point on AB is $(\lambda-1, -2\lambda+3, 4)$.
Let $B(\lambda-1, -2\lambda+3, 4)$.
As B lies on the plane $x-2y=0$
$\Rightarrow (\lambda-1) - 2(-2\lambda+3) = 0 \Rightarrow \lambda = \frac{7}{5}$
$\therefore B\left(\frac{2}{5}, \frac{1}{5}, 4\right)$
Using mid-point formula,
$\left(\frac{\alpha-1}{2}, \frac{\beta+3}{2}, \frac{\gamma+4}{2}\right) = \left(\frac{2}{5}, \frac{1}{5}, 4\right)$
$\Rightarrow \alpha = \frac{9}{5}, \beta = -\frac{13}{5}, \gamma = 4$
$\therefore$ Required image is $A'\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$</td><td>$1\frac{1}{2}$
1
1
1
1
$\frac{1}{2}$</td></tr>
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