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Senior School Certificate Examination

March 2019

Marking Scheme — Mathematics (041) 65/2/1, 65/2/2, 65/2/3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.
12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/2/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A'|\,|A| = |I| \Rightarrow |A|^2 = 1$ $\frac{1}{2}$

$\therefore$ $|A| = 1$ or $|A| = -1$ $\frac{1}{2}$

2. For $x < 0$, $y = x\,|x| = -x^2$ $\frac{1}{2}$

$\therefore$ $\frac{dy}{dx} = -2x$ $\frac{1}{2}$

3. Order = 2, Degree not defined $\frac{1}{2}+\frac{1}{2}$

4. D. Rs are 1, 1, 1
$\therefore$ Direction cosines of the line are:

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ 1

OR

Equation of the line is:

$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$ 1

SECTION B

5. $\left.\begin{array}{l}\forall\, a, b \in \mathbb{R}, \sqrt{a^2+b^2} \in \mathbb{R} \\ \therefore * \text{ is a binary operation on } \mathbb{R}\end{array}\right\}$ 1

Also,

$\left.\begin{array}{l} a*(b*c) = a*\sqrt{b^2+c^2} = \sqrt{a^2+b^2+c^2} \\ (a*b)*c = \sqrt{a^2+b^2}*c = \sqrt{a^2+b^2+c^2}\end{array}\right\} \left.\begin{array}{l}\Rightarrow a*(b*c) = (a*b)*c \\ \therefore\ * \text{ is Associative}\end{array}\right\}$ 1

6. $(A-2I)(A-3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ 1

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$ 1

7. $\int \sqrt{3-2x-x^2}\,dx = \int \sqrt{2^2-(x+1)^2}\,dx$ 1

$$= \frac{x+1}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right) + c$$ 1

8. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}dx = \int (\sec x \cdot \tan x + \operatorname{cosec} x \cdot \cot x)\,dx$ 1

$$= \sec x - \operatorname{cosec} x + c$$ 1

OR

$\int \frac{x-3}{(x-1)^3}e^x dx = \int e^x\{(x-1)^{-2} - 2(x-1)^{-3}\}dx$ 1

$$\left.\begin{array}{l} = e^x(x-1)^{-2} + c \\ \quad\text{or} \\ \dfrac{e^x}{(x-1)^2} + c \end{array}\right\}$$ 1

9. Differentiating $y = Ae^{2x} + Be^{-2x}$, we get

$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$, differentiate again to get, 1

$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x} = 4y$ or $\frac{d^2y}{dx^2} - 4y = 0$ 1

10. Let θ be the angle between $\vec{a}$ & $\vec{b}$, then

$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{7}{2.7} = \frac{1}{2}$ $1\frac{1}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\frac{1}{2}$

OR

$(-3\hat{i}+7\hat{j}+5\hat{k})\cdot\{(-5\hat{i}+7\hat{j}-3\hat{k})\times(7\hat{i}-5\hat{j}-3\hat{k})\} = \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$ 1

$$= -264$$ $\frac{1}{2}$

∴ Volume of cuboid = 264 cubic units $\frac{1}{2}$

11. $P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$ $\frac{1}{2}$

$P(A \cap B) = P(A) \cdot P(B|A) = 0.3 \times 0.5 = 0.15$ $\frac{1}{2}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70}$ or $\frac{3}{14}$ 1

12. (i) $P(3 \text{ heads}) = {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{5}{16}$ 1

(ii) $P(\text{At most 3 heads}) = P(r \le 3)$

$$\left.\begin{aligned} &= 1 - P(4 \text{ heads or } 5 \text{ heads}) \\ &= 1 - \left\{{}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) + {}^5C_5\left(\frac{1}{2}\right)^5\right\} \\ &= \frac{26}{32} \text{ or } \frac{13}{16} \end{aligned}\right\}$$ 1

OR

X = No. of heads in simultaneous toss of two coins.

X:	0	1	2	$\frac{1}{2}$
P(x):	1/4	1/2	1/4	$1\frac{1}{2}$

SECTION C

13. R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

For $1 \in A$, $(1, 1) \notin R \Rightarrow$ R is not reflexive 1

For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow$ R is not symmetric $1\frac{1}{2}$

For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow$ R is not transitive $1\frac{1}{2}$

OR

One-One: Let for $x_1, x_2 \in N$, $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$ 2

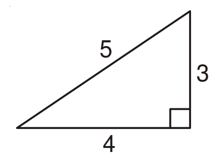
$\therefore$ 'f' is one-one

Onto: co-domain of f = Range of f = Y 1

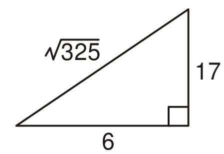
$\therefore$ 'f' is onto

$\therefore$ f is invertible with, $f^{-1}: Y \to N$ and $f^{-1}(y) = \dfrac{y-3}{4}$ or $f^{-1}(x) = \dfrac{x-3}{4}, x \in Y$ 1

14. $\sin\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$



$= \sin\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ 1



$= \sin\left[\tan^{-1}\left(\frac{3/4 + 2/3}{1 - 3/4 \cdot 2/3}\right)\right] = \sin\left[\tan^{-1}\left(\frac{17}{6}\right)\right]$ $1\frac{1}{2}$

$= \sin\left[\sin^{-1}\left(\frac{17}{\sqrt{325}}\right)\right] = \frac{17}{\sqrt{325}}$ $1\frac{1}{2}$

15. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \begin{pmatrix} \text{By applying,} \\ C_1 \to C_1 + C_2 + C_3 \end{pmatrix}$ 1

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \left\{ \begin{array}{l} \text{By applying,} \\ R_2 \to R_2 - R_1; \\ R_3 \to R_3 - R_1 \end{array} \right.$ 2

$= (a + b + c)\{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\}$

$= 3(a + b + c)(ab + bc + ca)$ 1

16. $x\sqrt{1+y}+y\sqrt{1+x}=0 \Rightarrow x\sqrt{1+y}=-y\sqrt{1+x}$ $\frac{1}{2}$

Squaring to get: $x^2(1+y)=y^2(1+x)$ $\frac{1}{2}$

Simplifying to get: $(x-y)(x+y+xy)=0$ 1

As, $x \neq y \quad \therefore \quad y=-\frac{x}{1+x}$ 1

Differentiating w.r.t. 'x', we get:

$$\frac{dy}{dx}=\frac{-1(1+x)-(-x)\cdot 1}{(1+x)^2}=\frac{-1}{(1+x)^2}$$ 1

OR

$(\cos x)^y=(\sin y)^x \Rightarrow y\cdot\log(\cos x)=x\cdot\log(\sin y)$ 1

Differentiating w.r.t 'x'

$$\Rightarrow \frac{dy}{dx}\cdot\log(\cos x)+y(-\tan x)=\log(\sin y)+x\cdot\cot y\cdot\frac{dy}{dx}$$ 2

$$\Rightarrow \frac{dy}{dx}=\frac{y\cdot\tan x+\log(\sin y)}{\log(\cos x)-x\cot y}$$ 1

17. $(x-a)^2+(y-b)^2=c^2,\ c>0$

Differentiating both sides with respect to 'x', we get

$$2(x-a)+2(y-b)\cdot\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{x-a}{y-b}$$ $1\frac{1}{2}$

Differentiating again with respect to 'x', we get;

$$\frac{d^2y}{dx^2}=-\frac{(y-b)-(x-a)\cdot\frac{dy}{dx}}{(y-b)^2}=\frac{-c^2}{(y-b)^3}$$ (By substituting '$\frac{dy}{dx}$') $1+\frac{1}{2}$

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}=\frac{\left[1+\frac{(x-a)^2}{(y-b)^2}\right]^{3/2}}{-\frac{c^2}{(y-b)^3}}=\frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}}=-c$$ 1

Which is a constant independent of 'a' & 'b'.

18. Let (α, β) be the point on the curve where normal passes through (–1, 4) $\therefore\ \alpha^2 = 4\beta$, also $\frac{dy}{dx} = \frac{x}{2}$

Slope of normal at (α, β) = $\frac{-1}{\left.\frac{dy}{dx}\right]_{(\alpha,\beta)}} = \frac{-1}{\frac{\alpha}{2}} = \frac{-2}{\alpha}$ 1

Equation of normal: $y - 4 = \frac{-2}{\alpha}(x+1)$ $\frac{1}{2}$

(α, β) lies on the normal $\Rightarrow \beta - 4 = \frac{-2}{\alpha}(\alpha + 1)$

Putting $\beta = \frac{\alpha^2}{4}$, we get; $\alpha^3 - 8\alpha + 8 = 0 \Rightarrow (\alpha - 2)(\alpha^2 + 2\alpha - 4) = 0$ 1

For α = 2 Equation of normal is: x + y – 3 = 0 1

For $\alpha = \pm\sqrt{5} - 1$; Equation of normal is: $y - 4 = \frac{-2}{\pm\sqrt{5}-1}(x+1)$ $\frac{1}{2}$

19. $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx = \frac{3}{5}\int \frac{1}{x+2}dx + \frac{1}{5}\int \frac{2x}{x^2+1}dx + \frac{1}{5}\int \frac{1}{x^2+1}dx$ 2

$= \frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + c$ 2

20. $\int_0^a f(x)\,dx = -\int_a^0 f(a-t)dx$ Put x = a – t, dx = – dt

Upper limit = t = a – x = a – a = 0

Lower limit = t = a – x = a – 0 = a

$= \int_0^a f(a-t)\,dt = \int_0^a f(a-x)\,dx$ 1

Let I = $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}dx$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)}dx$ $\therefore\ I = \int_0^{\pi/2} \frac{\pi/2 - x}{\cos x + \sin x}dx$ ...(ii) 1

Adding, (i) and (ii) we get

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\cos x + \sin x} dx = \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx$$

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx$$ $\frac{1}{2}$

$$\Rightarrow \quad 2I = \frac{\pi}{2\sqrt{2}}\left\{\log\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|\right\}_0^{\frac{\pi}{2}}$$ $\frac{1}{2}$

$$= \frac{\pi}{2\sqrt{2}}\{\log(\sqrt{2}+1) - \log(\sqrt{2}-1)\}$$ $\frac{1}{2}$

$$\Rightarrow \quad I = \frac{\pi}{4\sqrt{2}}\{\log(\sqrt{2}+1) - \log(\sqrt{2}-1)\} \text{ or } \frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$ $\frac{1}{2}$

21. The given differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$$ $\frac{1}{2}$

Put $\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, to get 1

$$v + x\frac{dv}{dx} = v - \tan v \Rightarrow x\frac{dv}{dx} = -\tan v$$

$$\Rightarrow \quad \cot v \; dv = -\frac{1}{x}dx,$$ 1

Integrating both sides we get,

$$\left.\begin{aligned} &\log|\sin v| = -\log|x| + \log c \\ \Rightarrow \quad &\log|\sin v| = \log\left|\frac{c}{x}\right| \end{aligned}\right\}$$ 1

$\therefore$ Solution of differential equation is

$$\sin\left(\frac{y}{x}\right) = \frac{c}{x} \text{ or } x \cdot \sin\left(\frac{y}{x}\right) = c$$ $\frac{1}{2}$

OR

The given differential equation can be written as:

$$\frac{dy}{dx} + \frac{\cos x}{1+\sin x} \cdot y = \frac{-x}{1+\sin x};$$ $1\frac{1}{2}$

$$\text{I.F.} = e^{\int \frac{\cos x}{1+\sin x} dx} = e^{\log(1+\sin x)} = 1+\sin x$$ 1

$\therefore$ Solution of the given differential equation is:

$$y(1 + \sin x) = \int \frac{-x}{1+\sin x} \times (1+\sin x)\, dx + c$$ 1

$$\Rightarrow y(1 + \sin x) = \frac{-x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1+\sin x)} + \frac{c}{1+\sin x}$$ $\frac{1}{2}$

22. $\vec{a} \cdot \frac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|} = 1$ 1

$$\Rightarrow (\hat{i}+\hat{j}+\hat{k}) \cdot \{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}\} = \sqrt{(2+\lambda)^2 + 36 + 4}$$ $1\frac{1}{2}$

$$\Rightarrow \lambda + 6 = \sqrt{(2+\lambda)^2 + 40}$$

Squaring to get

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda = 1$$ $\frac{1}{2}$

$\therefore$ Unit vector along $(\vec{b}+\vec{c})$ is $\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$ 1

23. Lines are perpendicular

$\therefore$ $-3(3\lambda) + 2\lambda(2) + 2(-5) = 0 \Rightarrow \lambda = -2$ 2

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_3 \end{vmatrix} = \begin{vmatrix} 1-1 & 1-2 & 6-3 \\ -3 & 2(-2) & 2 \\ 3(-2) & 2 & -5 \end{vmatrix} = -63 \neq 0$$ $1\frac{1}{2}$

$\therefore$ Lines are not intersecting $\frac{1}{2}$

SECTION D

24. $|A| = 11;\ \text{Adj}\ (A) = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ 1+2

$\therefore\ A^{-1} = \frac{1}{|A|} \cdot \text{Adj}\, A = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ $\frac{1}{2}$

Taking; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix};\ B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

The system of equations in matrix form is

$A \cdot X = B \qquad \therefore\ X = A^{-1} \cdot B$ 1

$\therefore$ Solution is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ 1

$\therefore\ x = 1,\ y = 1,\ z = 1$ $\frac{1}{2}$

OR

We know: A = IA

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1

$R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + 2R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \leftrightarrow R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_1 \to R_1 - R_2 - 2R_3$

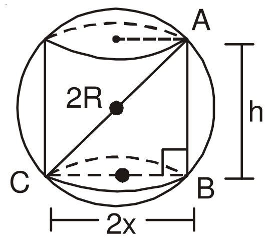
$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

4

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

1

25. Correct Figure 1



In rt. ΔABC; $4x^2 + h^2 = 4R^2$, $x^2 = \dfrac{4R^2 - h^2}{4}$ 1

V(Volume of cylinder) = $\pi x^2 h = \dfrac{\pi}{4}(4R^2 h - h^3)$ 1

$V'(h) = \dfrac{\pi}{4}(4R^2 - 3h^2)$; $V''(h) = \dfrac{\pi}{4}(-6h)$ $\frac{1}{2}+\frac{1}{2}$

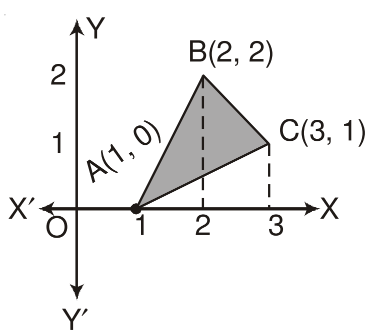
$V'(h) = 0 \Rightarrow h = \dfrac{2R}{\sqrt{3}}$ 1

$V''\left(\dfrac{2R}{\sqrt{3}}\right) = \dfrac{-6\pi}{4}\left(\dfrac{2R}{\sqrt{3}}\right) < 0 \Rightarrow$ Volume 'V' is max. $\frac{1}{2}$

for $h = \dfrac{2R}{\sqrt{3}}$

Max. Volume: $V = \dfrac{4}{3\sqrt{3}}\pi R^3$ $\frac{1}{2}$

26. Correct Figure 1



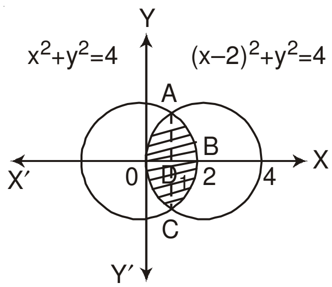
Equation of line AB : $y = 2(x-1)$
Equation of line BC : $y = 4 - x$
Equation of line AC : $y = \dfrac{1}{2}(x-1)$
$1\frac{1}{2}$

$ar(\Delta ABC) = 2\int_1^2 (x-1)\,dx + \int_2^3 (4-x)\,dx - \dfrac{1}{2}\int_1^3 (x-1)\,dx$ $1\frac{1}{2}$

$= (x-1)^2\Big]_1^2 - \dfrac{1}{2}(4-x)^2\Big]_2^3 - \dfrac{1}{4}(x-1)^2\Big]_1^3$ $1\frac{1}{2}$

$= 1 + \dfrac{3}{2} - 1 = \dfrac{3}{2}$ $\frac{1}{2}$

OR



Correct Figure 1

Getting the point of intersection as x = 1 1

Area (OABCO) = 4 × ar(ABD)

$$= 4\int_1^2 \sqrt{2^2 - x^2}\, dx$$ 2

$$= 4\left\{\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right\}_1^2$$ 1

$$= \left(\frac{8\pi}{3} - 2\sqrt{3}\right)$$ 1

27. Let $\vec{a} = \hat{i} + \hat{j} - 2\hat{k},\ \vec{b} = 2\hat{i} - \hat{j} + \hat{k},\ \vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

$$(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix} = -9\hat{i} - 3\hat{j} + \hat{k}$$ 2

Vector equation of plane is:

$$\{\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})\} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = 0$$

$\Rightarrow\ \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0$ 1

Cartesian Equation of plane is: $-9x - 3y + z + 14 = 0$ 1

Equation of plane through (2, 3, 7) and parallel to above plane is:

$$-9(x - 2) - 3(y - 3) + (z - 7) = 0$$

$\Rightarrow\ -9x - 3y + z + 20 = 0$ 1

Distance between parallel planes = $\left|\frac{-14+20}{\sqrt{91}}\right| = \frac{6}{\sqrt{91}}$ 1

OR

Equation of line: $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ 1

Equation of plane: $\begin{vmatrix} x-2 & y & z-3 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}=0$ 1

$\Rightarrow$ $-3(x-2)+3y-3(z-3)=0$

$\Rightarrow$ $x-y+z-5=0$ 1

General point on line: $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$

is: $P(3k+2, 4k-1, 2k+2)$; Putting in the equation of plane 1

we get, $3k+2-4k+1+2k+2=5 \Rightarrow k=0$ 1

$\therefore$ Point of intersection is: $(2, -1, 2)$ 1

28. Let E_1 = Event that two-headed coin is chosen
E_2 = Event that biased coin is chosen
E_3 = Event that unbiased coin is chosen
A = Event that coin tossed shows head } 1

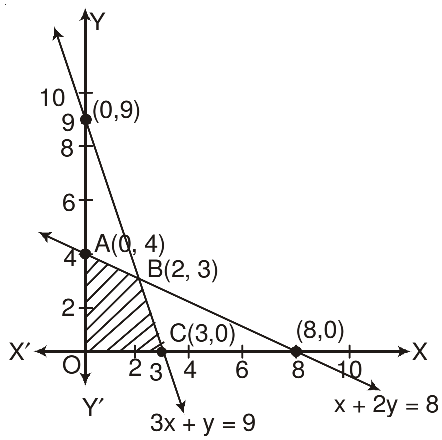
Then, $P(E_1)=P(E_2)=P(E_3)=1/3$ 1

$P(A|E_1)=1,\ P(A|E_2)=\frac{75}{100}=\frac{3}{4},\ P(A|E_3)=\frac{1}{2}$ $1\frac{1}{2}$

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)+P(E_3)\cdot P(A|E_3)}$$

$$=\frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1+\frac{1}{3}\cdot\frac{3}{4}+\frac{1}{3}\cdot\frac{1}{2}}=\frac{4}{9}$$ $2+\frac{1}{2}$

29.



let the company produce: Goods A = x units

Goods B = y units

then, the linear programming problem is:

Maximize profit: z = 40x + 50y (In ₹) $\frac{1}{2}$

Subject to constraints:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x, y \ge 0 \end{array}\right\}$$ $2\frac{1}{2}$

Correct graph: 2

Corner point	Value of z (₹)	
A(0, 4)	200	
B(2, 3)	230 (Max)	$\frac{1}{2}$
C(3, 0)	120	

∴ Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units $\frac{1}{2}$

QUESTION PAPER CODE 65/2/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A| = |B| = 0$ $\frac{1}{2}$

$\Rightarrow |AB| = 0$ $\frac{1}{2}$

2. $\frac{d}{dx}(e^{\sqrt{3x}}) = \frac{\sqrt{3}}{2\sqrt{x}} e^{\sqrt{3x}}$ 1

3. Order = 2, Degree not defined $\frac{1}{2}+\frac{1}{2}$

4. D. Rs are 1, 1, 1
$\therefore$ Direction cosines of the line are:

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ 1

OR

Equation of the line is:

$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$ 1

SECTION B

5. $\int \sqrt{3-2x-x^2}\, dx = \int \sqrt{2^2-(x+1)^2}\, dx$ 1

$= \frac{x+1}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right) + c$ 1

6. $|A| = p^2 - 4$ $\frac{1}{2}$

$|A^3| = 125 \Rightarrow |A|^3 = 125 \Rightarrow |A| = 5$ 1

$\therefore p^2 - 4 = 5 \Rightarrow p = \pm 3$ $\frac{1}{2}$

7. $\left.\begin{array}{l}\forall\, a, b \in \mathbb{R}, \sqrt{a^2+b^2} \in \mathbb{R} \\ \therefore * \text{ is a binary operation on } \mathbb{R}\end{array}\right\}$ 1

Also,

$$\left.\begin{array}{l} a*(b*c)=a*\sqrt{b^2+c^2}=\sqrt{a^2+b^2+c^2} \\ (a*b)*c=\sqrt{a^2+b^2}*c=\sqrt{a^2+b^2+c^2} \end{array}\right\} \begin{array}{l} \Rightarrow\ a*(b*c)=(a*b)*c \\ \therefore\ * \text{ is Associative} \end{array}\Bigg\}$$ 1

8. Let θ be the angle between $\vec{a}$ & $\vec{b}$, then

$$\sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}=\frac{7}{2.7}=\frac{1}{2}$$ $1\frac{1}{2}$

$$\Rightarrow\quad \theta=\frac{\pi}{6}\ \text{or}\ \frac{5\pi}{6}$$ $\frac{1}{2}$

OR

$$(-3\hat{i}+7\hat{j}+5\hat{k})\cdot\{(-5\hat{i}+7\hat{j}-3\hat{k})\times(7\hat{i}-5\hat{j}-3\hat{k})\} = \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$ 1

$$= -264$$ $\frac{1}{2}$

∴ Volume of cuboid = 264 cubic units $\frac{1}{2}$

9. (i) $P(3\text{ heads}) = {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2=\frac{5}{16}$ 1

(ii) P(At most 3 heads) = P (r ≤ 3)

$$\left.\begin{array}{l} =1-P(4\text{ heads or }5\text{ heads}) \\ =1-\left\{{}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)+{}^5C_5\left(\frac{1}{2}\right)^5\right\} \\ =\frac{26}{32}\ \text{or}\ \frac{13}{16} \end{array}\right\}$$ 1

OR

X = No. of heads in simultaneous toss of two coins.

X:	0	1	2	$\frac{1}{2}$
P(x):	1/4	1/2	1/4	$1\frac{1}{2}$

10. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\sec x \cdot \tan x + \text{cosec } x \cdot \cot x) dx$ 1

$= \sec x - \text{cosec } x + c$ 1

OR

$\int \frac{x-3}{(x-1)^3} e^x dx = \int e^x \{(x-1)^{-2} - 2(x-1)^{-3}\} dx$ 1

$= e^x (x-1)^{-2} + c$ or $\frac{e^x}{(x-1)^2} + c$ 1

11. $P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$ $\frac{1}{2}$

$P(A \cap B) = P(A) \cdot P(B/A) = 0.3 \times 0.5 = 0.15$ $\frac{1}{2}$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70}$ or $\frac{3}{14}$ 1

12. Given differential equation can be written as:

$\frac{dy}{dx} = e^x \cdot e^y \Rightarrow e^{-y} dy = e^x dx$ 1

Integrating both sides, we get

$-e^{-y} = e^x + c$ 1

SECTION C

13. $\sin\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$

$= \sin\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ 1

$= \sin\left[\tan^{-1}\left(\frac{3/4 + 2/3}{1 - 3/4 \cdot 2/3}\right)\right] = \sin\left[\tan^{-1}\left(\frac{17}{6}\right)\right]$ $1\frac{1}{2}$

$= \sin\left[\sin^{-1}\left(\frac{17}{\sqrt{325}}\right)\right] = \frac{17}{\sqrt{325}}$ $1\frac{1}{2}$

14. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \begin{pmatrix} \text{By applying,} \\ C_1 \to C_1 + C_2 + C_3 \end{pmatrix}$ 1

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \begin{cases} \text{By applying,} \\ R_2 \to R_2 - R_1; \\ R_3 \to R_3 - R_1 \end{cases}$ 2

$= (a + b + c)\ \{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\}$

$= 3(a + b + c)\ (ab + bc + ca)$ 1

15. $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

For $1 \in A$, $(1, 1) \notin R \Rightarrow$ R is not reflexive 1

For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow$ R is not symmetric $1\frac{1}{2}$

For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow$ R is not transitive $1\frac{1}{2}$

OR

One-One: Let for $x_1, x_2 \in N$, $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$ 2

$\therefore$ 'f ' is one-one

Onto: co-domain of f = Range of f = Y 1

$\therefore$ 'f ' is onto

$\therefore$ f is invertible with, $f^{-1}: Y \to N$ and $f^{-1}(y) = \frac{y-3}{4}$ or $f^{-1}(x) = \frac{x-3}{4}, x \in Y$ 1

16. $x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ $\frac{1}{2}$

Squaring to get: $x^2(1 + y) = y^2(1 + x)$ $\frac{1}{2}$

Simplifying to get: $(x - y)(x + y + xy) = 0$ 1

As, $x \neq y \quad \therefore \quad y = -\frac{x}{1+x}$ 1

Differentiating w.r.t. 'x', we get:

$$\frac{dy}{dx} = \frac{-1(1+x)-(-x)\cdot 1}{(1+x)^2} = \frac{-1}{(1+x)^2}$$ 1

OR

$(\cos x)^y = (\sin y)^x \Rightarrow y \cdot \log(\cos x) = x \cdot \log(\sin y)$ 1

Differentiating w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} \cdot \log(\cos x) + y(-\tan x) = \log(\sin y) + x \cdot \cot y \cdot \frac{dy}{dx}$$ 2

$$\Rightarrow \frac{dy}{dx} = \frac{y \cdot \tan x + \log(\sin y)}{\log(\cos x) - x \cot y}$$ 1

17. $\int_0^a f(x)\,dx = -\int_a^0 f(a-t)\,dx$ Put $x = a - t$, $dx = -dt$

Upper limit = $t = a - x = a - a = 0$

Lower limit = $t = a - x = a - 0 = a$

$$= \int_0^a f(a-t)\,dt = \int_0^a f(a-x)\,dx$$ 1

Let $I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$ ...(i)

$$\Rightarrow I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx \qquad \therefore\ I = \int_0^{\pi/2} \frac{\pi/2 - x}{\cos x + \sin x} dx$$ ...(ii) 1

Adding (i) and (ii) we get

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\cos x + \sin x} dx = \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx$$

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx$$ $\frac{1}{2}$

$$\Rightarrow \ 2I = \frac{\pi}{2\sqrt{2}}\left\{\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right\}_0^{\pi/2}$$ $\frac{1}{2}$

$$= \frac{\pi}{2\sqrt{2}}\{\log|\sqrt{2}+1| - \log|\sqrt{2}-1|\}$$ $\frac{1}{2}$

$$\Rightarrow \ I = \frac{\pi}{4\sqrt{2}}\{\log(\sqrt{2}+1) - \log(\sqrt{2}-1)\} \text{ or } \frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$ $\frac{1}{2}$

18. $$\int\frac{x^2+x+1}{(x+2)(x^2+1)}dx = \frac{3}{5}\int\frac{1}{x+2}dx + \frac{1}{5}\int\frac{2x}{x^2+1}dx + \frac{1}{5}\int\frac{1}{x^2+1}dx$$ 2

$$= \frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + c$$ 2

19. The given differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$$ $\frac{1}{2}$

Put $\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, to get 1

$$v + x\frac{dv}{dx} = v - \tan v \Rightarrow x\frac{dv}{dx} = -\tan v$$

$$\Rightarrow \ \cot v \, dv = -\frac{1}{x}dx,$$ 1

Integrating both sides we get,

$$\left.\begin{aligned} &\log|\sin v| = -\log|x| + \log c \\ \Rightarrow \ &\log|\sin v| = \log\left|\frac{c}{x}\right| \end{aligned}\right\}$$ 1

$\therefore$ Solution of differential equation is

$$\sin\left(\frac{y}{x}\right) = \frac{c}{x} \text{ or } x\cdot\sin\left(\frac{y}{x}\right) = c$$ $\frac{1}{2}$

OR

The given differential equation can be written as:

$$\frac{dy}{dx} + \frac{\cos x}{1+\sin x}\cdot y = \frac{-x}{1+\sin x};$$ $1\frac{1}{2}$

$$\text{I.F.} = e^{\int \frac{\cos x}{1+\sin x}dx} = e^{\log(1+\sin x)} = 1+\sin x$$ 1

$\therefore$ Solution of the given differential equation is:

$$y(1 + \sin x) = \int \frac{-x}{1+\sin x} \times (1+\sin x)\,dx + c$$ 1

$$\Rightarrow y(1 + \sin x) = \frac{-x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1+\sin x)} + \frac{c}{(1+\sin x)}$$ $\frac{1}{2}$

20. $\vec{a} \cdot \frac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|} = 1$ 1

$$\Rightarrow (\hat{i}+\hat{j}+\hat{k}) \cdot \{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}\} = \sqrt{(2+\lambda)^2 + 36 + 4}$$ $1\frac{1}{2}$

$$\Rightarrow \lambda + 6 = \sqrt{(2+\lambda)^2 + 40}$$

Squaring to get

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda = 1$$ $\frac{1}{2}$

$\therefore$ Unit vector along $(\vec{b}+\vec{c})$ is $\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$ 1

21. $e^{y/x} = \frac{x}{a+bx}$, taking log, on both sides, we get $\frac{y}{x} = \log x - \log(a+bx)$ 1

Differentiating with respect to 'x'

$$\left.\begin{aligned} &\frac{x \cdot y' - y}{x^2} = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{(a+bx)x} \\ &\Rightarrow x \cdot y' - y = \frac{ax}{a+bx} \quad \text{...(i)} \end{aligned}\right\}$$ $1\frac{1}{2}$

Differentiating with respect to 'x'

$$\Rightarrow x \cdot y'' + y' - y' = \frac{(a+bx)\cdot a - ax \cdot b}{(a+bx)^2} = \left(\frac{a}{a+bx}\right)^2$$ 1

$$\left.\begin{aligned} &\Rightarrow x \cdot y'' = \left(\frac{a}{a+bx}\right)^2 \Rightarrow x^3 \cdot y'' = \left(\frac{ax}{a+bx}\right)^2 \\ &\Rightarrow x^3 \frac{d^2y}{dx^2} = \left\{x \cdot \frac{dy}{dx} - y\right\}^2 \quad \text{(Using (i))} \end{aligned}\right\}$$ $\frac{1}{2}$

22. Let Edge = x cm, then

V(Volume of cube) = x^3, S(Surface area) = $6x^2$ 1

$$\frac{dv}{dt} = 8\,cm^3/s \;\Rightarrow\; 3x^2\frac{dx}{dt} = 8 \;\Rightarrow\; \frac{dx}{dt} = \frac{8}{3x^2}$$ 1

$$\frac{ds}{dt} = 12x\frac{dx}{dt} = 12x\cdot\frac{8}{3x^2} = \frac{32}{x}$$ 1

$$\therefore\; \left.\frac{ds}{dt}\right]_{x=12} = \frac{32}{12} = \frac{8}{3}\,cm^2/s$$ 1

23. Equation of plane:

$$\left.\begin{aligned} &\begin{vmatrix} x-2 & y-5 & z+3 \\ -2-2 & -3-5 & 5+3 \\ 5-2 & 3-5 & -3+3 \end{vmatrix} = 0 \\ \Rightarrow &\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0 \end{aligned}\right\}$$ 2

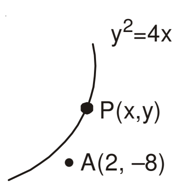
$\Rightarrow\; (x-2)(16) - (y-5)(-24) + (z+3)(32) = 0$

$\Rightarrow\; 2x + 3y + 4z - 7 = 0$ 1

Vector form: $\vec{r}\cdot(2\hat{i} + 3\hat{j} + 4\hat{k}) - 7 = 0$ 1

SECTION D

24.



Let P(x, y) be any point on the curve $y^2 = 4x$

$z = AP = \sqrt{(x-2)^2 + (y+8)^2}$ 1

let $s = z^2 = \left(\frac{y^2}{4} - 2\right)^2 + (y+8)^2$ 1

$$\frac{ds}{dy} = 2\left(\frac{y^2}{4} - 2\right)\left(\frac{y}{2}\right) + 2(y+8) = \frac{y^3}{4} + 16$$ 1

$$\frac{d^2s}{dy^2} = \frac{3y^2}{4}$$ $\frac{1}{2}$

Let $\frac{ds}{dy} = 0 \Rightarrow y^3 = -64 \Rightarrow y = -4$ 1

$$\left.\frac{d^2s}{dy^2}\right]_{y=-4} = \frac{3(16)}{4} > 0$$ $\frac{1}{2}$

$\therefore$ s or z is minimum at y = –4; $x = \frac{y^2}{4} = 4$

$\therefore$ The nearest point is P(4, –4) 1

25. Let $f(x) = x^2 + 2 + e^{2x}$, a = 1, b = 3, nh = 2 1

then,

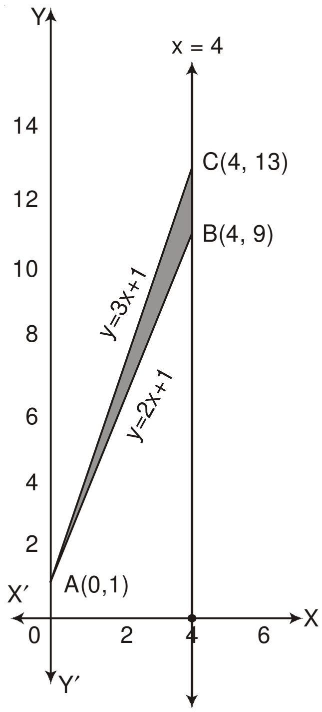
$$f(1) + f(1+h) + f(1+2h) + \ldots + f(1+\overline{n-1}\,h)$$

$$= 3n + h^2[1^2 + 2^2 + \ldots + (n-1)^2] + 2h[1 + 2 + \ldots + (n-1)] + e^2[1 + e^{2h} + e^{4h} + \ldots + e^{2(n-1)h}]$$ 1

$$= 3n + h^2\frac{(n-1)(n)(2n-1)}{6} + \frac{2h(n-1)(n)}{2} + \frac{e^2(e^{2nh}-1)}{e^{2h}-1}$$ 2

$$\int_1^3 (x^2 + 2 + e^{2x})dx = \lim_{h\to 0}\left[3(nh) + \frac{(nh-h)(nh)(2nh-h)}{6} + (nh)(nh-h) + \frac{e^2}{2}\cdot\frac{2h}{e^{2h}-1}(e^{2nh}-1)\right]$$ 1

$$\left.\begin{aligned} &= 6 + \frac{2\times 2\times 4}{6} + 2\times 2 + \frac{e^2}{2}\times 1\times(e^4-1) \\ &= \frac{38}{3} + \frac{e^2(e^4-1)}{2} \end{aligned}\right\}$$ 1



OR

Correct figure 1

Points of intersection of given lines

are A(0, 1), B(4, 9), C(4, 13) $1\frac{1}{2}$

$\therefore$ Req. Area $= \int_0^4 (3x+1)\,dx - \int_0^4 (2x+1)\,dx$ 2

$= \int_0^4 x\,dx = \frac{1}{2}x^2\Big]_0^4 = 8$ $1\frac{1}{2}$

26. $|A| = 11$; Adj (A) $= \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ 1+2

$\therefore \quad A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ $\frac{1}{2}$

Taking; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

The system of equations in matrix form is

$A \cdot X = B \qquad \therefore X = A^{-1} \cdot B$ 1

$\therefore$ Solution is:

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ 1

$\therefore \quad x = 1,\ y = 1,\ z = 1$ $\frac{1}{2}$

OR

We know: A = IA

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1

$R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \rightarrow R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \rightarrow R_2 + 2R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} A$$ 4

$R_2 \leftrightarrow R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_2 \rightarrow R_2 - 3R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_1 \rightarrow R_1 - R_2 - 2R_3$

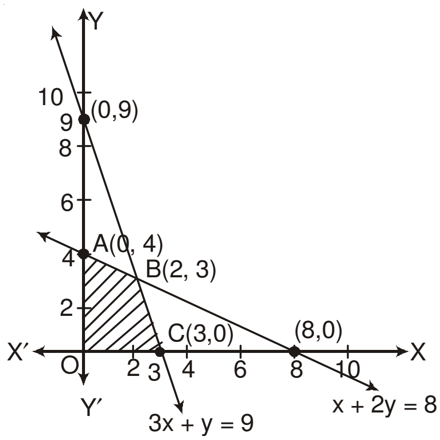
$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$ 1

27. let the company produce: Goods A = x units

Goods B = y units

then, the linear programming problem is:



Maximize profit: z = 40x + 50y (In ₹) $\frac{1}{2}$

Subject to constraints:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x, y \ge 0 \end{array}\right\}$$ $2\frac{1}{2}$

Correct graph. 2

Corner point	Value of z (₹)
A(0, 4)	200
B(2, 3)	230 (Max)
C(3, 0)	120

$\frac{1}{2}$

∴ Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units $\frac{1}{2}$

28. Let E_1 = Event that two-headed coin is chosen

E_2 = Event that biased coin is chosen

E_3 = Event that unbiased coin is chosen

A = Event that coin tossed shows head 1

Then, $P(E_1) = P(E_2) = P(E_3) = 1/3$ 1

$P(A|E_1) = 1,\ P(A|E_2) = \frac{75}{100} = \frac{3}{4},\ P(A|E_3) = \frac{1}{2}$ $1\frac{1}{2}$

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)}$$

$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{2}} = \frac{4}{9}$$ $2+\frac{1}{2}$

29. Let $\vec{a} = \hat{i} + \hat{j} - 2\hat{k},\ \vec{b} = 2\hat{i} - \hat{j} + \hat{k},\ \vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

$$(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix} = -9\hat{i} - 3\hat{j} + \hat{k}$$ 2

Vector equation of plane is:

$$\{\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})\} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = 0$$

$\Rightarrow\ \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0$ 1

Cartesian Equation of plane is: $-9x - 3y + z + 14 = 0$ 1

Equation of plane through (2, 3, 7) and parallel to above plane is:

$$-9(x - 2) - 3(y - 3) + (z - 7) = 0$$

$\Rightarrow\ -9x - 3y + z + 20 = 0$ 1

Distance between parallel planes = $\left|\dfrac{-14 + 20}{\sqrt{91}}\right| = \dfrac{6}{\sqrt{91}}$ 1

OR

Equation of line: $\dfrac{x - 2}{3} = \dfrac{y + 1}{4} = \dfrac{z - 2}{2}$ 1

Equation of plane: $\begin{vmatrix} x - 2 & y & z - 3 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$ 1

$\Rightarrow\ -3(x - 2) + 3y - 3(z - 3) = 0$

$\Rightarrow\ x - y + z - 5 = 0$ 1

General point on line: $\dfrac{x - 2}{3} = \dfrac{y + 1}{4} = \dfrac{z - 2}{2}$

is: P(3k + 2, 4k – 1, 2k + 2); Putting in the equation of plane 1

we get, $3k + 2 - 4k + 1 + 2k + 2 = 5 \Rightarrow k = 0$ 1

$\therefore$ Point of intersection is: (2, –1, 2) 1

QUESTION PAPER CODE 65/2/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{dy}{dx} = 2ae^{2x} \Rightarrow \frac{dy}{dx} = 2(y-5)$ $\frac{1}{2}+\frac{1}{2}$

2. $\frac{dy}{dx} = -\frac{\sqrt{3}\sin(\sqrt{3x})}{2\sqrt{x}}$ 1

3. $|A'|\,|A| = |I| \Rightarrow |A|^2 = 1$ $\frac{1}{2}$

$\Rightarrow |A| = 1$ or $|A| = -1$ $\frac{1}{2}$

4. D. Rs are 1, 1, 1

$\therefore$ Direction cosines of the line are:

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ 1

OR

Equation of the line is:

$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$ 1

SECTION B

5. $\overrightarrow{AB} = 3\hat{i} - \hat{j} - 2\hat{k}$; $\overrightarrow{AC} = 9\hat{i} - 3\hat{j} - 6\hat{k}$ 1

Clearly, $\overrightarrow{AC} = 3 \cdot \overrightarrow{AB} \Rightarrow \overrightarrow{AC} \parallel \overrightarrow{AB}$, $\therefore$ A, B & C are Collinear 1

OR

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix} = -17\hat{i} + 13\hat{j} + 7\hat{k}$ $1\frac{1}{2}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{289 + 169 + 49} = \sqrt{507}$ $\frac{1}{2}$

6. $\int \frac{x-5}{(x-3)^3} \cdot e^x \, dx = \int e^x \left[\frac{(x-3)-2}{(x-3)^3} \right] dx$ $\frac{1}{2}$

$= \int e^x [(x-3)^{-2} - 2(x-3)^{-3}] dx$ $\frac{1}{2}$

$= e^x (x-3)^{-2} + c \text{ or } \frac{e^x}{(x-3)^2} + c$ 1

7. $\left.\begin{array}{l} \forall\, a, b \in \mathbb{R}, \sqrt{a^2+b^2} \in \mathbb{R} \\ \therefore * \text{ is a binary operation on } \mathbb{R} \end{array}\right\}$ 1

Also,

$\left.\begin{array}{l} a*(b*c) = a*\sqrt{b^2+c^2} = \sqrt{a^2+b^2+c^2} \\ (a*b)*c = \sqrt{a^2+b^2}*c = \sqrt{a^2+b^2+c^2} \end{array}\right\} \left.\begin{array}{l} \Rightarrow a*(b*c) = (a*b)*c \\ \therefore \; * \text{ is Associative} \end{array}\right\}$ 1

8. $(A - 2I)\,(A - 3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ 1

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$ 1

9. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\sec x \cdot \tan x + \operatorname{cosec} x \cdot \cot x)\, dx$ 1

$= \sec x - \operatorname{cosec} x + c$ 1

OR

$\int \frac{x-3}{(x-1)^3} e^x dx = \int e^x \{(x-1)^{-2} - 2(x-1)^{-3}\} dx$ 1

$\left.\begin{array}{l} = e^x (x-1)^{-2} + c \\ \text{or} \\ \frac{e^x}{(x-1)^2} + c \end{array}\right\}$ 1

10. (i) $P(3 \text{ heads}) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \frac{5}{16}$ 1

(ii) P(At most 3 heads) = P (r ≤ 3)

$$= 1 - P(4 \text{ heads or } 5 \text{ heads})$$
$$= 1 - \left\{ {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + {}^5C_5 \left(\frac{1}{2}\right)^5 \right\}$$
$$= \frac{26}{32} \text{ or } \frac{13}{16}$$ 1

OR

X = No. of heads in simultaneous toss of two coins.

X:	0	1	2	$\frac{1}{2}$
P(x):	1/4	1/2	1/4	$1\frac{1}{2}$

11. $P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$ $\frac{1}{2}$

$P(A \cap B) = P(A) \cdot P(B/A) = 0.5 \times 0.3 = 0.15$ $\frac{1}{2}$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70} \text{ or } \frac{3}{14}$ 1

12. Differenatiae $y = Ae^{2x} + Be^{-2x}$, we get

$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$, differenatiate again to get, 1

$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x} = 4y \text{ or } \frac{d^2y}{dx^2} - 4y = 0$ 1

SECTION C

13. $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$

$\Rightarrow \tan^{-1}\left\{\frac{2x}{1-(x^2-1)}\right\} = \tan^{-1}\frac{8}{31}$ 2

$\Rightarrow 4x^2 + 31x - 8 = 0 \Rightarrow x = \frac{1}{4} \text{ or } x = -8$ 1

x = –8 does not satisty the given equation so $x = \frac{1}{4}$ 1

14. $x = ae^t (\sin t + \cos t);\ y = ae^t (\sin t - \cos t)$

then

$$\left.\begin{aligned}\frac{dy}{dt} &= ae^t(\sin t - \cos t) + ae^t(\cos t + \sin t)\\ &= y + x\end{aligned}\right\}$$ $1\frac{1}{2}$

$$\left.\begin{aligned}\frac{dx}{dt} &= ae^t(\sin t + \cos t) + ae^t(\cos t - \sin t)\\ &= x - y\end{aligned}\right\}$$ $1\frac{1}{2}$

$\therefore \quad \frac{dy}{dx} = \frac{y+x}{x-y}$ or $\frac{x+y}{x-y}$ 1

OR

Let, $y = x^{\sin x} + (\sin x)^{\cos x} = u + v;\ \therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ 1

$u = x^{\sin x} \Rightarrow \log u = \sin x \cdot \log x \Rightarrow \frac{du}{dx} = x^{\sin x}\left\{\cos x \cdot \log x + \frac{\sin x}{x}\right\}$ $1\frac{1}{2}$

$v = (\sin x)^{\cos x} \Rightarrow \log v = \cos x \cdot \log(\sin x) \Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\{\cos x \cdot \cot x - \sin x \cdot \log \sin x\}$ 1

$\therefore \quad \frac{dy}{dx} = x^{\sin x}\left\{\cos x \cdot \log x + \frac{\sin x}{x}\right\} + (\sin x)^{\cos x}\{\cos x \cdot \cot x - \sin x . \log \sin x\}$ $\frac{1}{2}$

15. $\int \frac{2\cos x}{(1-\sin x)(2-\cos^2 x)}dx = \int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}dx = \int \frac{2}{(1-t)(1+t^2)}dt$, where $\sin x = t$, $\cos x\, dx = dt$

1

$\int \frac{2}{(1-t)(1+t^2)}dt = \int \frac{1}{1-t}dt + \int \frac{t+1}{t^2+1}dt$ $1\frac{1}{2}$

$= -\log(1-t) + \frac{1}{2}\log(t^2+1) + \tan^{-1}(t) + c$ 1

$= -\log|1-\sin x| + \frac{1}{2}\log(\sin^2 x + 1) + \tan^{-1}(\sin x) + c$ $\frac{1}{2}$

16. $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

For $1 \in A$, $(1, 1) \notin R \Rightarrow R$ is not reflexive 1

For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric $1\frac{1}{2}$

For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow R$ is not transitive $1\frac{1}{2}$

OR

One-One: Let for $x_1, x_2 \in N$, $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$ 2

$\therefore$ 'f ' is one-one

Onto: co-domain of f = Range of f = Y 1

$\therefore$ 'f ' is onto

$\therefore$ f is invertible with, $f^{-1}: Y \to N$ and $f^{-1}(y) = \frac{y-3}{4}$ or $f^{-1}(x) = \frac{x-3}{4}, x \in Y$ 1

17. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \begin{pmatrix} \text{By applying,} \\ C_1 \to C_1 + C_2 + C_3 \end{pmatrix}$ 1

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \begin{cases} \text{By applying,} \\ R_2 \to R_2 - R_1; \\ R_3 \to R_3 - R_1 \end{cases}$ 2

$= (a + b + c)\{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\}$

$= 3(a + b + c)(ab + bc + ca)$ 1

18. $(x - a)^2 + (y - b)^2 = c^2, c > 0$

Differentiating both sides with respect to 'x', we get

$2(x - a) + 2(y - b) \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x-a}{y-b}$ $1\frac{1}{2}$

Differentiating again with respect to 'x', we get;

$$\frac{d^2y}{dx^2} = -\frac{(y-b)-(x-a)\cdot\frac{dy}{dx}}{(y-b)^2} = \frac{-c^2}{(y-b)^3} \quad \text{(By substituting '}\tfrac{dy}{dx}\text{')}$$ $1+\frac{1}{2}$

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+\frac{(x-a)^2}{(y-b)^2}\right]^{3/2}}{-\frac{c^2}{(y-b)^3}} = \frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}} = -c$$ 1

Which is a constant independent of 'a' & 'b'.

19. Let (α, β) be the point on the curve where normal

passes through $(-1, 4)$ $\therefore$ $\alpha^2 = 4\beta$, also $\frac{dy}{dx} = \frac{x}{2}$

Slope of normal at (α, β) = $\frac{-1}{\left.\frac{dy}{dx}\right]_{(\alpha,\beta)}} = \frac{-1}{\frac{\alpha}{2}} = \frac{-2}{\alpha}$ 1

Equation of normal: $y - 4 = \frac{-2}{\alpha}(x+1)$ $\frac{1}{2}$

(α, β) lies on normal $\Rightarrow \beta - 4 = \frac{-2}{\alpha}(\alpha+1)$

Putting $\beta = \frac{\alpha^2}{4}$, we get; $\alpha^3 - 8\alpha + 8 = 0 \Rightarrow (\alpha - 2)(\alpha^2 + 2\alpha - 4) = 0$ 1

For $\alpha = 2$ Equation of normal is: $x + y - 3 = 0$ 1

For $\alpha = \pm\sqrt{5} - 1$; Equation of normal is: $y - 4 = \frac{-2}{\pm\sqrt{5}-1}(x+1)$ $\frac{1}{2}$

20. $\int_0^a f(x)\,dx = -\int_a^0 f(a-t)dx$ Put $x = a - t$, $dx = -dt$

Upper limit = $t = a - x = a - a = 0$

Lower limit = $t = a - x = a - 0 = a$

$= \int_0^a f(a-t)\,dt = \int_0^a f(a-x)\,dx$ 1

Let $I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$ ...(i)

$$\Rightarrow I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$\therefore \ I = \int_0^{\pi/2} \frac{\pi/2 - x}{\cos x + \sin x} dx$ ...(ii) 1

Adding (i) and (ii) we get

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\cos x + \sin x} dx = \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx$$

$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx$ $\frac{1}{2}$

$\Rightarrow \ 2I = \frac{\pi}{2\sqrt{2}}\left\{\log\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|\right\}_0^{\pi/2}$ $\frac{1}{2}$

$= \frac{\pi}{2\sqrt{2}}\{\log|\sqrt{2}+1| - \log|\sqrt{2}-1|\}$ $\frac{1}{2}$

$\Rightarrow \ I = \frac{\pi}{4\sqrt{2}}\{\log(\sqrt{2}+1) - \log(\sqrt{2}-1)$ or $\frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ $\frac{1}{2}$

21. The given differential equation can be written as:

$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$ $\frac{1}{2}$

Put $\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, to get 1

$$v + x\frac{dv}{dx} = v - \tan v \Rightarrow x\frac{dv}{dx} = -\tan v$$

$\Rightarrow \ \cot v \ dv = -\frac{1}{x}dx,$ 1

Integrating both sides we get,

$$\left.\begin{aligned} &\log|\sin v| = -\log|x| + \log c \\ \Rightarrow\ &\log|\sin v| = \log\left|\frac{c}{x}\right| \end{aligned}\right\} \qquad 1$$

$\therefore$ Solution of differential equation is

$$\sin\left(\frac{y}{x}\right) = \frac{c}{x} \text{ or } x\cdot\sin\left(\frac{y}{x}\right) = c \qquad \frac{1}{2}$$

OR

The given differential equation can be written as:

$$\frac{dy}{dx} + \frac{\cos x}{1+\sin x}\cdot y = \frac{-x}{1+\sin x}; \qquad 1\frac{1}{2}$$

$$\text{I.F.} = e^{\int\frac{\cos x}{1+\sin x}dx} = e^{\log(1+\sin x)} = 1+\sin x \qquad 1$$

$\therefore$ Solution of the given differential equation is:

$$y(1+\sin x) = \int\frac{-x}{1+\sin x}\times(1+\sin x)\,dx + c \qquad 1$$

$$\Rightarrow y(1+\sin x) = \frac{-x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1+\sin x)} + \frac{c}{1+\sin x} \qquad \frac{1}{2}$$

22. Lines are perpendicular

$\therefore$ $-3(3\lambda) + 2\lambda(2) + 2(-5) = 0 \Rightarrow \lambda = -2$ 2

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_3 \end{vmatrix} = \begin{vmatrix} 1-1 & 1-2 & 6-3 \\ -3 & 2(-2) & 2 \\ 3(-2) & 2 & -5 \end{vmatrix} = -63 \neq 0 \qquad 1\frac{1}{2}$$

$\therefore$ Lines are not intersecting $\frac{1}{2}$

23. $\vec{a}\cdot\frac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|} = 1$ 1

$$\Rightarrow (\hat{i}+\hat{j}+\hat{k})\cdot\{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}\} = \sqrt{(2+\lambda)^2+36+4} \qquad 1\frac{1}{2}$$

$$\Rightarrow \lambda + 6 = \sqrt{(2+\lambda)^2+40}$$

Squaring to get

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda = 1$$ $\frac{1}{2}$

$\therefore$ Unit vector along $(\vec{b} + \vec{c})$ is $\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$ 1

SECTION D

24. $A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$ $1\frac{1}{2}$

$A^3 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$ $1\frac{1}{2}$

LHS = $A^3 - 6A^2 + 5A + 11I$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ 1

$= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} = O = \text{R.H.S.}$

$A^3 - 6A^2 + 5A + 11I = O$, Pre-multiplying by A^{-1}

$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = O \Rightarrow A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I)$ 1

$\therefore A^{-1} = \begin{bmatrix} -3/11 & 4/11 & 5/11 \\ 9/11 & -1/11 & -4/11 \\ 5/11 & -3/11 & -1/11 \end{bmatrix}$ 1

OR

Let,

$A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

System of equation in Matrix form: A · X = B

$|A| = 3(2 - 3) + 2(4 + 4) + 3(-6 - 4) = -17 \neq 0$ 1

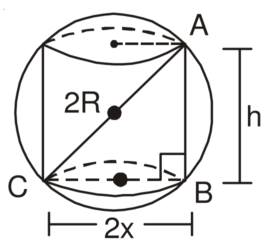
Solution matrix, $X = A^{-1} \cdot B$. 1

$$(\text{adj } A) = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$ 2

$$\therefore \quad A^{-1} = \frac{-1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$ $\frac{1}{2}$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow x = 1, y = 2, x = 3$$ $1\frac{1}{2}$

25. Correct Figure 1



ln rt. ΔABC; $4x^2 + h^2 = 4R^2, x^2 = \frac{4R^2 - h^2}{4}$ 1

V(Volume of cylinder) = $\pi x^2 h = \frac{\pi}{4}(4R^2 h - h^3)$ 1

$V'(h) = \frac{\pi}{4}(4R^2 - 3h^2); \; V''(h) = \frac{\pi}{4}(-6h)$ $\frac{1}{2} + \frac{1}{2}$

$V'(h) = 0 \Rightarrow h = \frac{2R}{\sqrt{3}}$ 1

$V''\left(\frac{2R}{\sqrt{3}}\right) = \frac{-6\pi}{4}\left(\frac{2R}{\sqrt{3}}\right) < 0 \Rightarrow$ Volume 'V' is max. $\frac{1}{2}$

for $h = \frac{2R}{\sqrt{3}}$

Max. Volume: $V = \frac{4}{3\sqrt{3}}\pi R^3$ $\frac{1}{2}$

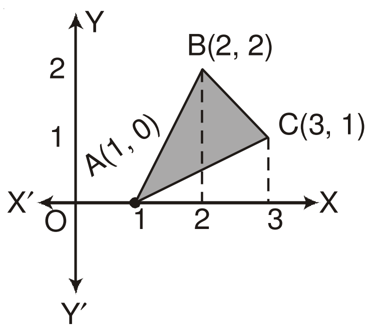
26. E_1 = Event of selecting first bag
E_2 = Event of selecting second bag
A = Event both balls drawn are red. } 1

$$P(E_1) = P(E_2) = \frac{1}{2};\ P(A|E_1) = \frac{^5C_2}{^9C_2} = \frac{20}{72};\ P(A|E_2) = \frac{^3C_2}{^9C_2} = \frac{6}{72}$$ 1+2

$$P(E_2|A) = \frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{6}{72}}{\frac{1}{2}\cdot\frac{20}{72} + \frac{1}{2}\cdot\frac{6}{72}} = \frac{6}{26} = \frac{3}{13}$$ $1\frac{1}{2}+\frac{1}{2}$

27. Correct Figure 1



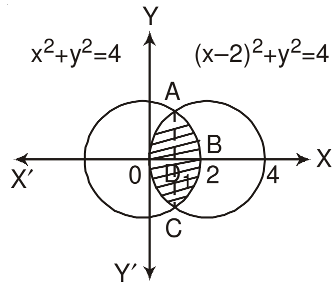
Equation of line AB : $y = 2(x-1)$
Equation of line BC : $y = 4 - x$
Equation of line AC : $y = \frac{1}{2}(x-1)$ } $1\frac{1}{2}$

$$\text{ar}(\Delta ABC) = 2\int_1^2 (x-1)\,dx + \int_2^3 (4-x)\,dx - \frac{1}{2}\int_1^3 (x-1)\,dx$$ $1\frac{1}{2}$

$$= (x-1)^2\Big]_1^2 - \frac{1}{2}(4-x)^2\Big]_2^3 - \frac{1}{4}(x-1)^2\Big]_1^3$$ $1\frac{1}{2}$

$$= 1 + \frac{3}{2} - 1 = \frac{3}{2}\text{ sq. units}$$ $\frac{1}{2}$

OR



Correct Figure 1

Getting the point of intersection as x = 1 1

Area (OABCO) = 4 × ar(ABD)

$$= 4\int_1^2 \sqrt{2^2 - x^2}\, dx \qquad 2$$

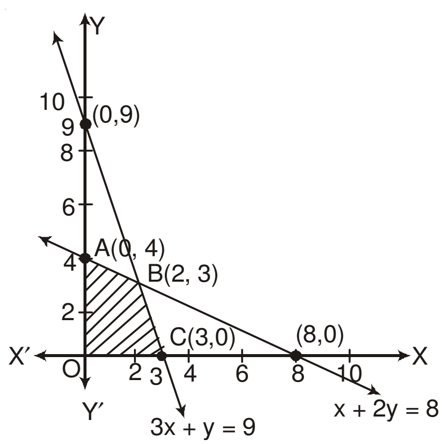
$$= 4\left\{\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right\}_1^2 \qquad 1$$

$$= \left(\frac{8\pi}{3} - 2\sqrt{3}\right) \qquad 1$$

28. let the company produce: Goods A = x units

Goods B = y units

then, the linear programming problem is:



Maximize profit: z = 40x + 50y (In ₹) $\frac{1}{2}$

Subject to constraints:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x, y \ge 0 \end{array}\right\} \qquad 2\frac{1}{2}$$

Correct graph. 2

Corner point	Value of z (₹)	
A(0, 4)	200	
B(2, 3)	230 (Max)	$\frac{1}{2}$
C(3, 0)	120	

∴ Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units $\frac{1}{2}$

29. Let $\vec{a}=\hat{i}+\hat{j}-2\hat{k},\ \vec{b}=2\hat{i}-\hat{j}+\hat{k},\ \vec{c}=\hat{i}+2\hat{j}+\hat{k}$

$$(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 0 & 1 & 3\end{vmatrix}=-9\hat{i}-3\hat{j}+\hat{k}$$ 2

Vector equation of plane is:

$$\{\vec{r}-(\hat{i}+\hat{j}-2\hat{k})\}\cdot(-9\hat{i}-3\hat{j}+\hat{k})=0$$

$\Rightarrow\ \vec{r}\cdot(-9\hat{i}-3\hat{j}+\hat{k})+14=0$ 1

Cartesian Equation of plane is: $-9x-3y+z+14=0$ 1

Equation of plane through (2, 3, 7) and parallel to above plane is:

$-9(x-2)-3(y-3)+(z-7)=0$

$\Rightarrow\ -9x-3y+z+20=0$ 1

Distance between parallel planes = $\left|\dfrac{-14+20}{\sqrt{91}}\right|=\dfrac{6}{\sqrt{91}}$ 1

OR

Equation of line: $\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{2}$ 1

Equation of plane: $\begin{vmatrix}x-2 & y & z-3\\ -1 & 1 & 2\\ 1 & 2 & 1\end{vmatrix}=0$ 1

$\Rightarrow\ -3(x-2)+3y-3(z-3)=0$

$\Rightarrow\ x-y+z-5=0$ 1

General point on line: $\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{2}=k$ (say)

is: P(3k + 2, 4k – 1, 2k + 2); Putting in the equation of plane 1

we get, $3k+2-4k+1+2k+2=5\Rightarrow k=0$ 1

$\therefore$ Point of intersection is: (2, –1, 2) 1