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Senior School Certificate Examination

March 2019

Marking Scheme — Mathematics (041) 65/3/1, 65/3/2, 65/3/3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.
12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/3/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|-2A| = (-2)^3 \cdot |A|$ $\frac{1}{2}$

$= -8 \cdot 4 = -32$ $\frac{1}{2}$

2. $y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$ $\frac{1}{2}+\frac{1}{2}$

3. order 4, degree 2 $\frac{1}{2}+\frac{1}{2}$

4. $\sqrt{(-18)^2+(12)^2+(-4)^2} = 22$ $\frac{1}{2}$

$\therefore$ DC's are $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$ or $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$ $\frac{1}{2}$

OR

D·R's of required line are 3, –5, 6 $\frac{1}{2}$

Equation of line is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ $\frac{1}{2}$

SECTION B

5. Let $e \in \mathbb{R}$ be the identity element.

then $a*e = e*a = a$ 1

$\Rightarrow a^2 + e^2 = e^2 + a^2 = a^2 \Rightarrow e^2 = 0 \Rightarrow e = 0.$ 1

$\therefore$ Identity element is $0 \in \mathbb{R}$

6. $kA = k\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$

$\therefore \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} \Rightarrow 2k = 3a, 3k = 2b \text{ and } -4k = 24$ $\frac{1}{2}$

$\Rightarrow k = -6,\ a = \frac{-12}{3} = -4,\ b = \frac{-18}{2} = -9$ $1\frac{1}{2}$

7. $\int \frac{\sin x - \cos x}{\sqrt{1+\sin 2x}} dx = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ 1

$= -\log|\sin x + \cos x| + c$ 1

8. $\int \frac{\sin(x-a)}{\sin(x+a)}dx = \int \frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$= \int \left[\frac{\sin(x+a)\cos 2a}{\sin(x+a)} - \frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$ $\frac{1}{2}$

$= x\cdot\cos 2a - \sin 2a \cdot \log|\sin(x+a)| + c$ $\frac{1}{2}+\frac{1}{2}$

OR

$\int (\log x)^2 \cdot 1dx = (\log x)^2 \cdot x - \int 2\cdot \log x \cdot \frac{1}{x}\cdot x\ dx$ 1

$= x\cdot(\log x)^2 - \left\{\log x \cdot 2x - \int \frac{1}{x}\cdot 2x dx\right\}$ $\frac{1}{2}$

$= x(\log x)^2 - 2x\log x + 2x + c$ $\frac{1}{2}$

9. $y^2 = m(a^2 - x^2) \Rightarrow 2y\frac{dy}{dx} = -2mx$ $\frac{1}{2}$

or $y\frac{dy}{dx} = -mx$ ...(i)

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m$ ...(ii) $\frac{1}{2}$

form (i) and (ii) we get $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$ 1

or $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

10. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\vec{a}\times\vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j}+\hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a},\vec{b},\vec{c}$ are coplanar if $\vec{a}\cdot\vec{b}\times\vec{c} = 0$ $\frac{1}{2}$

$$\vec{a}\cdot\vec{b}\times\vec{c} = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3)+2(-6)+3(3) = 3-12+9 = 0$$ $1+\frac{1}{2}$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

11. $A = \{(S, F, M), (S, M, F), (M, F, S), (F, M, S)\}$, $B = \{(S, F, M), (M, F, S)\}$ 1

Total number of possible arrangements = 6

$$P(B|A) = \frac{P(B\cap A)}{P(A)}$$

$$= \frac{2/6}{4/6} = \frac{1}{2}$$ 1

12. Given $2\,P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$

Let $P(X = x_3) = k$, then $P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3}$ and $P(X = x_4) = \frac{k}{5}$ $\frac{1}{2}$

$$\therefore\; k + \frac{k}{2} + \frac{k}{3} + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61}$$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r \geq 4) = P(4) + P(5)$

$$= {}^5C_4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^5C_5\left(\frac{1}{2}\right)^5 = 6\left(\frac{1}{2}\right)^5 = \frac{6}{32} \text{ or } \frac{3}{16}$$ 1

(ii) P(at most 4 heads) = $P(r \leq 4) = 1 - P(5)$

$$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$$ 1

SECTION C

13. (i) For $a \in Z, (a,a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let (a, b) $\in$ R for a, b $\in$ Z, then a – b is divisible by 2

$\Rightarrow$ (b – a) is also divisible by 2

$\therefore$ (b, a) $\in$ R $\Rightarrow$ R is symmetric ...(ii) 1

For a, b, c, $\in$ Z, Let (a, b) $\in$ R and (b, c) $\in$ R

$\therefore$ a – b = 2p, p $\in$ Z, and b – c = 2q, q $\in$ Z,

adding, a – c = 2 (p + q) $\Rightarrow$ (a – c) is divisible by 2

$\Rightarrow$ (a, c) $\in$ R, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$\text{fof}(x) = f\left(\frac{4x+3}{6x-4}\right)$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$\Rightarrow \text{fof}(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$ 1

Since fof(x) = x $\Rightarrow$ fof = I $\Rightarrow$ $f^{-1} = f$ 1

14. Given $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right), x > 0$

$\Rightarrow \tan^{-1}x - \left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{\pi}{6}$ 1

$\Rightarrow 2\tan^{-1}x = \frac{2\pi}{3} \Rightarrow \tan^{-1}x = \frac{\pi}{3}$ 1

$\Rightarrow x = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ $\therefore \sec^{-1}\frac{2}{x} = \sec^{-1}\frac{2}{\sqrt{3}} = \frac{\pi}{6}$ 1+1

15. Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$R_1 \to R_1 - (R_2 + R_3) \Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ 1

$\therefore \Delta = -2\begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = -\frac{2}{b}\begin{vmatrix} 0 & bc & b \\ b & bc+ab & b \\ c & bc & a+b \end{vmatrix}$ $1+\frac{1}{2}$

$C_2 \to C_2 - cC_3$

$\Rightarrow \Delta = -\frac{2}{b}\begin{vmatrix} 0 & 0 & b \\ b & ab & b \\ c & -ac & a+b \end{vmatrix}$ 1

$= -\frac{2}{b} \cdot b \cdot (-abc - abc) = 4abc.$ $\frac{1}{2}$

16. $\sin y = x \cdot \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$ $\frac{1}{2}$

differentiating w.r.t. y, we get

$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$ $1\frac{1}{2}$

$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$ $1\frac{1}{2}$

$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ $\frac{1}{2}$

OR

$(\sin x)^y = (x+y) \Rightarrow y \cdot \log \sin x = \log(x+y)$ 1

differentiating w.r.t. x, we get

$y \cdot \cot x + \log \sin x \cdot \frac{dy}{dx} = \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right)$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y\cot x}{\log\sin x - \frac{1}{x+y}}$$ 1

$$= \frac{1 - y(x+y)\cot x}{(x+y)\log\sin x - 1}$$ $\frac{1}{2}$

17. $y = (\sec^{-1} x)^2, x > 0$

$$\frac{dy}{dx} = 2\sec^{-1} x \cdot \frac{1}{x\sqrt{x^2 - 1}}$$ 1

$$\Rightarrow x\sqrt{x^2 - 1}\frac{dy}{dx} = 2\sqrt{y}$$ $\frac{1}{2}$

squaring both sides, we get

$$x^2(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y \quad \text{or} \quad (x^4 - x^2)\left(\frac{dy}{dx}\right)^2 = 4y$$ $\frac{1}{2}$

differentiating w.r.t. x.

$$(x^4 - x^2)2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + (4x^3 - 2x)\left(\frac{dy}{dx}\right)^2 = 4 \cdot \frac{dy}{dx}$$ $1\frac{1}{2}$

$$\Rightarrow x^2(x^2 - 1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0$$ $\frac{1}{2}$

18. Curve $y = \frac{x-7}{(x-2)(x-3)}$ cuts at x – axis at the point x = 7, y = 0 i.e. (7, 0) $\frac{1}{2}$

$$\frac{dy}{dx} = \frac{(x^2 - 5x + 6) \cdot 1 - (x-7)(2x-5)}{(x^2 - 5x + 6)^2}$$ $\frac{1}{2}$

at (7, 0) $\frac{dy}{dx} = \frac{20}{(20)^2} = \frac{1}{20}$ $\frac{1}{2}$

$\therefore$ Slope of tangent at (7, 0) is $\frac{1}{20}$ $\frac{1}{2}$

and slope of Normal at (7, 0) is –20 $\frac{1}{2}$

Equation of tangent at (7, 0) is $y - 0 = \frac{1}{20}(x - 7)$

or $x - 20y - 7 = 0$ 1

Equation of Normal at (7, 0) is $y - 0 = -20(x - 7)$

or $20x + y = 140$. $\frac{1}{2}$

19. $I = \int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$

Put $\sin^2 x = t \Rightarrow \sin 2x\, dx = dt$ $\frac{1}{2}$

$\therefore I = \int \frac{dt}{(t+1)(t+3)} = \int \left(\frac{1/2}{t+1} + \frac{-1/2}{t+3} \right) dt$ $1\frac{1}{2}$

$= \frac{1}{2}\log|t+1| - \frac{1}{2}\log|t+3| + c$ $1\frac{1}{2}$

$= \frac{1}{2}\log(\sin^2 x + 1) - \frac{1}{2}\log(\sin^2 + 3) + c.$ $\frac{1}{2}$

20. RHS $= \int_a^b f(a+b-x)\, dx = -\int_b^a f(t)\, dt$, where $a + b - x = t$, $dx = -dt$ $\frac{1}{2}$

$= \int_a^b f(t)\, dt = \int_a^b f(x)\, dx = \text{LHS}$ $\frac{1}{2}$

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ ...(i) $\frac{1}{2}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos(\pi/2 - x)}}{\sqrt{\cos(\pi/2 - x)} + \sqrt{\sin(\pi/2 - x)}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 \cdot dx = x]_{\pi/6}^{\pi/3} = \pi/6.$ $\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{12}$ $\frac{1}{2}$

21. $\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore\ v + x\frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$

$\Rightarrow \int \frac{1-v}{1+v^2}dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$ $1+\frac{1}{2}$

$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$ 1

$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$ 1

or $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$

OR

$(1+x^2)dy + 2xy\,dx = \cot x.dx.$

$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\cot x}{1+x^2}$ 1

I.F. = $e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$ 1

$\therefore$ Solution is, $y\cdot(1+x^2) = \int \cot x\,dx = \log|\sin x| + c$ 1+1

or $y = \frac{1}{1+x^2}\cdot\log|\sin x| + \frac{c}{1+x^2}$

22. Given $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}$ $\therefore\ \vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b}\perp\vec{c} \Rightarrow \vec{b}\cdot\vec{c} = 0$ ...(ii) $\frac{1}{2}$

$(|3\vec{a}-2\vec{b}+2\vec{c}\,|)^2 = 9\,|\vec{a}\,|^2 + 4\,|\vec{b}\,|^2 + 4\,|\vec{c}\,|^2 - 12\,\vec{a}\cdot\vec{b} - 8\,\vec{b}\cdot\vec{c} + 12\,\vec{a}\cdot\vec{c}$ 1

$= 9(1)^2 + 4(2)^2 + 4(3)^2$ [using (i) and (ii)] 1

$= 9 + 16 + 36 = 61$

$\Rightarrow\ |3\vec{a}-2\vec{b}+2\vec{c}\,| = \sqrt{61}$ 1

23. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1};\ \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$ $\frac{1}{2}$

lines are perpendicular to each other,

$\Rightarrow\ (5\lambda+2)\cdot 1 + (-5)(2\lambda) + 1(3) = 0$ $\frac{1}{2}$

$-5\lambda + 5 = 0 \ \Rightarrow\ \lambda = 1$ $\frac{1}{2}$

$\therefore$ lines are $\frac{x-5}{7} = \frac{y-2}{-5} - \frac{z-1}{1};\ \frac{x}{1} = \frac{y+\frac{1}{2}}{2} = \frac{z-1}{3}$ $\frac{1}{2}$

Shortest distance between these lines $= \frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|} = \frac{\left|\left(5\hat{i}+\frac{5}{2}\hat{j}\right)\cdot(-17\hat{i}-20\hat{j}+19\hat{k})\right|}{|\vec{b}_1\times\vec{b}_2|}$ 1

$= \frac{135}{|\vec{b}_1\times\vec{b}_2|} \neq 0$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24. $|A| = 1(7) - 1(-3) + 1(-1) = 9$ 1

$$(\text{adj } A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$ 2

$$\Rightarrow A^{-1} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$ $\frac{1}{2}$

Given equations can be written as $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

or $AX = B \Rightarrow X = A^{-1}B$ $\frac{1}{2}$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $1\frac{1}{2}$

$\therefore x = 1, y = 2, z = 3$ $\frac{1}{2}$

OR

Let: $\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_1 \leftrightarrow R_3 \begin{bmatrix} 3 & 7 & 2 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$

$\begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{matrix} \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$

$R_3 \to R_3 - 2R_1 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & -5 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix} A$

$R_3 \to R_3 + 5R_2 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ -2 & 5 & -2 \end{bmatrix} A$ 4

$\begin{matrix} R_1 \to R_1 - 4R_2 \\ R_3 \to -R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$

$R_1 \to R_1 - R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$ 1

25. Let Given surface area of open cylinder be S.

Then $S = 2\pi rh + \pi r^2$

$$\Rightarrow h = \frac{S - \pi r^2}{2\pi r}$$ 1

Volume $V = \pi r^2 h$ $\frac{1}{2}$

$$V = \pi r^2\left[\frac{S-\pi r^2}{2\pi r}\right] = \frac{1}{2}[Sr - \pi r^3]$$ 1

$$\frac{dV}{dr} = \frac{1}{2}[S - 3\pi r^2]$$ $\frac{1}{2}$

$$\frac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2 \text{ or } 2\pi rh + \pi r^2 = 3\pi r^2$$ 1

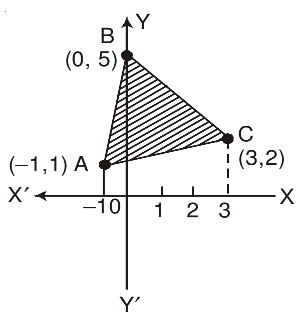
$$\Rightarrow 2\pi rh = 2\pi r^2 \qquad \Rightarrow h = r$$ 1

$$\frac{d^2V}{dr^2} = -6\pi r < 0$$ 1

$\therefore$ For volume to be maximum, height = radius

26. Let the points be A (–1, 1), B (0, 5) and C (3, 2)

Correct Figure 1



Equation of AB : $y = 4x + 5$

BC : $y = 5 - x$

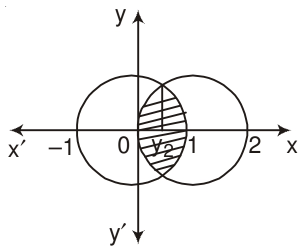
AC : $y = \frac{1}{4}(x+5)$ $1\frac{1}{2}$

$$\text{Req. Area} = \int_{-1}^{0}(4x+5)\,dx + \int_{0}^{3}(5-x)\,dx - \int_{-1}^{3}\frac{1}{4}(x+5)\,dx$$ $1\frac{1}{2}$

$$\therefore A = \left[\frac{(4x+5)^2}{8}\right]_{-1}^{0} + \left[\frac{(5-x)^2}{-2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{(x+5)^2}{2}\right]_{-1}^{3}$$ 1

$$= 3+\frac{21}{2}-6=\frac{15}{2}$$ 1

OR



Correct Figure 1

$$(x-1)^2+y^2 = 1$$

and $x^2 + y^2 = 1 \Rightarrow (x-1)^2 = x^2$

$$\Rightarrow x = \frac{1}{2}$$ 1

$$\therefore \text{ Required area } = 2\left[\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}\,dx+\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}\,dx\right]$$ 2

$$= 2\left[\frac{x-1}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}+2\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$ 1

$$= 2\left[\frac{-\sqrt{3}}{8}+\frac{\pi}{6}\right]+2\left[\frac{-\sqrt{3}}{8}+\frac{\pi}{6}\right]=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$$ 1

27. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix}=0$$ 1

$$\Rightarrow 16(x-2)+24(y-5)+32(z+3)=0$$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$$\frac{x-3}{4}=\frac{y-1}{4}=\frac{z-5}{6} \text{ or } \frac{x-3}{2}=\frac{y-1}{2}=\frac{z-5}{3}$$ ...(ii) 1

Any point on (ii) is $(2\lambda+3, 2\lambda+1, 3\lambda+5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+5)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0,$ is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4]=0$ 1

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda=0$ ...(i) 1

Plane (i) is ∥ to x–axis $\Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3=0$ $1\frac{1}{2}$

or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6=0$

Distance of this plane from x–axis

$=\frac{|-6|}{\sqrt{(-1)^2+(3)^2}}=\frac{6}{\sqrt{10}}$ units 1

28. Let the events be

E_1 : bag I is selected
E_2 : bag II is selected
A : getting a red ball
} 1

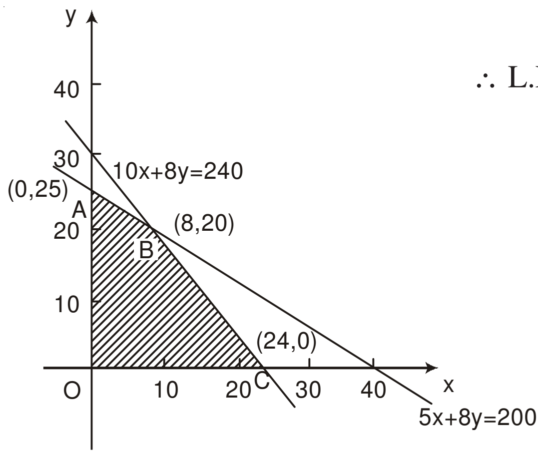
$P(E_1)=P(E_2)=\frac{1}{2}$ $\frac{1}{2}$

$P(A/E_1)=\frac{3}{9}=\frac{1}{3};\ P(A/E_2)=\frac{5}{5+n}$ $\frac{1}{2}+1$

$$P(E_2/A) = \frac{3}{5} = \frac{\frac{1}{2}\cdot\frac{5}{5+n}}{\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{5}{5+n}}$$ 2

$$\Rightarrow \frac{3}{5} = \frac{15}{5+n+15} \Rightarrow n = 5.$$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ $\frac{1}{2}$

such that $5x + 8y \leq 200$
$10x + 8y \leq 240$
$x, y \geq 0$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20. 1

QUESTION PAPER CODE 65/3/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{dy}{dx}-\frac{2}{x}\cdot y = 2x \Rightarrow$ I.F. $= e^{-2\log x} = \frac{1}{x^2}$ $\frac{1}{2}+\frac{1}{2}$

2. $y^2+2xy\frac{dy}{dx}-2x = 0 \Rightarrow \frac{dy}{dx}=\frac{2x-y^2}{2xy}$ $\frac{1}{2}+\frac{1}{2}$

3. $|-2A| = (-2)^3\cdot|A|$ $\frac{1}{2}$

$= -8 \times 4 = -32$ $\frac{1}{2}$

4. $\sqrt{(-18)^2+(12)^2+(-4)^2} = 22$ $\frac{1}{2}$

$\therefore$ DC's are $\frac{-18}{22},\frac{12}{22},\frac{-4}{22}$ or $\frac{-9}{11},\frac{6}{11},\frac{-2}{11}$ $\frac{1}{2}$

OR

D·R's of required line are 3, –5, 6 $\frac{1}{2}$

Equation of line is $\frac{x+2}{3} = \frac{y-4}{-5}=\frac{z+5}{6}$ $\frac{1}{2}$

SECTION B

5. Let a = 2, b = 3 $\Rightarrow 2*3 = \frac{2}{4}=\frac{1}{2},\ 3*2 = \frac{3}{3}=1 \Rightarrow 2*3 \neq 3*2.$ 1

$$(2*3)*4=\frac{1}{2}*4=\frac{\frac{1}{2}}{4+1}=\frac{1}{10}, 2*(3*4)=2*\frac{3}{5}=\frac{2}{8/5}=\frac{5}{4}$$

$\Rightarrow (2*3)*4 \neq 2*(3*4)$ 1

6. $A^2 = \begin{bmatrix}-3 & 6\\ -2 & 4\end{bmatrix}\begin{bmatrix}-3 & 6\\ -2 & 4\end{bmatrix}=\begin{bmatrix}-3 & 6\\ -2 & 4\end{bmatrix}=A$ 1

$\Rightarrow A^3 = A^2\cdot A = A\cdot A = A^2 = A$ 1

7. $y^2 = m(a^2-x^2) \Rightarrow 2y\frac{dy}{dx}=-2mx$ $\frac{1}{2}$

or $y\frac{dy}{dx} = -mx$ ...(i)

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = -m \quad ...(ii)$$ $\frac{1}{2}$

form (i) and (ii) we get $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$ 1

or $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

8. $\int\frac{\sin x-\cos x}{\sqrt{1+\sin 2x}}dx = \int\frac{\sin x-\cos x}{\sin x+\cos x}dx$ 1

$= -\log|\sin x+\cos x|+c$ 1

9. $\int\frac{\sin(x-a)}{\sin(x+a)}dx = \int\frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$= \int\left[\frac{\sin(x+a)\cdot\cos 2a}{\sin(x+a)}-\frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$ $\frac{1}{2}$

$= x\cdot\cos 2a-\sin 2a\cdot\log|\sin(x+a)|+c$ $\frac{1}{2}+\frac{1}{2}$

OR

$\int(\log x)^2\cdot 1dx = (\log x)^2\cdot x-\int 2\cdot\log x\cdot\frac{1}{x}\cdot x\,dx$ 1

$= x\cdot(\log x)^2-\left\{\log x\cdot 2x-\int\frac{1}{x}\cdot 2xdx\right\}$ $\frac{1}{2}$

$= x(\log x)^2-2x\log x+2x+c$ $\frac{1}{2}$

10. $A = \{(S, F, M), (S, M, F), (M, F, S), (F, M, S)\}$
$B = \{(S, F, M), (M, F, S)\}$ 1

Total number of possible arrangements = 6

$P(B|A) = \frac{P(B\cap A)}{P(A)}$

$= \frac{2/6}{4/6} = \frac{1}{2}$ 1

11. Given $2\,P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$

Let $P(X = x_3) = k$, then $P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3}$ and $P(X = x_4) = \frac{k}{5}$ $\frac{1}{2}$

$\therefore\; k + \frac{k}{2} + \frac{k}{3} + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61}$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r \geq 4) = P(4) + P(5)$

$= {}^5C_4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^5C_5\left(\frac{1}{2}\right)^5 = 6\left(\frac{1}{2}\right)^5 = \frac{6}{32}$ or $\frac{3}{16}$ 1

(ii) P(at most 4 heads) = $P(r \leq 4) = 1 - P(5)$

$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$ 1

12. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\vec{a} \times \vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j} + \hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a} \cdot \vec{b} \times \vec{c} = 0$ $\frac{1}{2}$

$$\vec{a} \cdot \vec{b} \times \vec{c} = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3) + 2(-6) + 3(3)$$

$$= 3 - 12 + 9 = 0$$

$1 + \frac{1}{2}$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

SECTION C

13. (i) For $a \in Z, (a, a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let (a, b) $\in$ R for a, b $\in$ Z, then a – b is divisible by 2

$\Rightarrow$ (b – a) is also divisible by 2

$\therefore$ (b, a) $\in$ R $\Rightarrow$ R is symmetric ...(ii) 1

For a, b, c, $\in$ Z, Let (a, b) $\in$ R and (b, c) $\in$ R

$\therefore$ a – b = 2p, p $\in$ Z, and b – c = 2q, q $\in$ Z,

adding, a – c = 2 (p + q) $\Rightarrow$ (a – c) is divisible by 2

$\Rightarrow$ (a, c) $\in$ R, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$$\text{fof}(x) = f\left(\frac{4x+3}{6x-4}\right)$$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$$\Rightarrow \text{fof}(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$$ 1

Since fof(x) = x $\Rightarrow$ fof = I $\Rightarrow$ $f^{-1} = f$ 1

14. $\sin y = x \cdot \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$ $\frac{1}{2}$

differentiating w.r.t. y, we get

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin^2(a+y)}$$ $1\frac{1}{2}$

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$ $1\frac{1}{2}$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$ $\frac{1}{2}$

OR

$(\sin x)^y = (x+y) \Rightarrow y \cdot \log \sin x = \log(x+y)$ 1

differentiating w.r.t. x, we get

$$y \cdot \cot x + \log \sin x \cdot \frac{dy}{dx} = \frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y\cot x}{\log \sin x - \frac{1}{x+y}}$$ 1

$$= \frac{1-y(x+y)\cot x}{(x+y)\log \sin x - 1}$$ $\frac{1}{2}$

15. $\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2} \Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \frac{\pi}{2} - \sin^{-1}\frac{4}{x} = \cos^{-1}\frac{4}{x}$ 1

$$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \sin^{-1}\left(\sqrt{1-\frac{16}{x^2}}\right) \Rightarrow \left(\frac{3}{x}\right)^2 = \frac{x^2-16}{x^2}$$ $1\frac{1}{2}$

$\Rightarrow x^2 = 25 \Rightarrow x = \pm 5,\ x = -5$ (rejected) $\therefore\ x = 5$ $\frac{1}{2}+1$

16. $$\text{LHS} = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ ac^2 & bc^2 & c(c^2+1) \end{vmatrix} \left\{\begin{array}{l} \text{Applying} \\ R_1 \to aR_1 \\ R_2 \to bR_2 \\ R_3 \to cR_3 \end{array}\right.$$ 1

$$= \begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} \{R_1 \to R_1 + R_2 + R_3$$ $\frac{1}{2}+1$

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix} = 1+a^2+b^2+c^2. \quad \begin{cases} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{cases}$$ $1\frac{1}{2}$

$= \text{RHS}$

17. $y = (\cot^{-1} x)^2 \Rightarrow \dfrac{dy}{dx} = 2\cot^{-1} x \cdot \left(\dfrac{-1}{1+x^2}\right)$ 1

$\Rightarrow (1+x^2)\dfrac{dy}{dx} = -2\cot^{-1} x = -2\sqrt{y}$ $\frac{1}{2}$

squaring both sides, we get

$(1+x^2)^2 \cdot \left(\dfrac{dy}{dx}\right)^2 = 4y$ $\frac{1}{2}$

differentiating, w.r.t. x,

$2(1+x^2)2x \cdot \left(\dfrac{dy}{dx}\right)^2 + 2(1+x^2)^2 \cdot \dfrac{dy}{dx} \cdot \dfrac{d^2y}{dx^2} = 4 \cdot \dfrac{dy}{dx}$ $1\frac{1}{2}$

$\Rightarrow 2x(1+x^2)\dfrac{dy}{dx} + (1+x^2)^2 \dfrac{d^2y}{dx^2} = 2.$ $\frac{1}{2}$

18. $I = \displaystyle\int \frac{\sin 2x}{(\sin^2 x+1)(\sin^2 x+3)}\, dx$

Put $\sin^2 x = t \Rightarrow \sin 2x\, dx = dt$ $\frac{1}{2}$

$\therefore\ I = \displaystyle\int \frac{dt}{(t+1)(t+3)} = \int \left(\frac{1/2}{t+1} + \frac{-1/2}{t+3}\right) dt$ $1\frac{1}{2}$

$= \dfrac{1}{2}\log|t+1| - \dfrac{1}{2}\log|t+3| + c$ $1\frac{1}{2}$

$= \dfrac{1}{2}\log(\sin^2 x+1) - \dfrac{1}{2}\log(\sin^2 + 3) + c.$ $\frac{1}{2}$

19. RHS $= \displaystyle\int_a^b f(a+b-x)\, dx = -\int_b^a f(t)\, dt$, where $a + b - x = t$, $dx = -dt$ $\frac{1}{2}$

$= \displaystyle\int_a^b f(t)\, dt = \int_a^b f(x)\, dx = \text{LHS}$ $\frac{1}{2}$

Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx$ ...(i) $\frac{1}{2}$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\, dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get $2I = \int_{\pi/6}^{\pi/3} 1 \cdot dx = x]_{\pi/6}^{\pi/3} = \pi/6.$ $\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{12}$ $\frac{1}{2}$

20. $\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore\ v + x\frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$

$\Rightarrow \int \frac{1-v}{1+v^2} dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2} dv - \frac{1}{2}\int \frac{2v}{1+v^2} dv = \int \frac{dx}{x}$ $1+\frac{1}{2}$

$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$ 1

$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$ 1

or $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$

OR

$(1+x^2)dy + 2xy\, dx = \cot x.dx.$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{\cot x}{1+x^2}$$ 1

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = (1+x^2)$$ 1

$$\therefore \text{ Solution is, } y \cdot (1+x^2) = \int \cot x \, dx = \log|\sin x| + c$$ 1+1

$$\text{or } y = \frac{1}{1+x^2} \cdot \log|\sin x| + \frac{c}{1+x^2}$$

21. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$ $\frac{1}{2}$

lines are perpendicular to each other,

$$\Rightarrow (5\lambda+2) \cdot 1 + (-5)(2\lambda) + 1(3) = 0$$ $\frac{1}{2}$

$$-5\lambda + 5 = 0 \Rightarrow \lambda = 1$$ $\frac{1}{2}$

$$\therefore \text{ lines are } \frac{x-5}{7} = \frac{y-2}{-5} - \frac{z-1}{1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2} = \frac{z-1}{3}$$ $\frac{1}{2}$

$$\text{Shortest distance between these lines } = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{\left|\left(5\hat{i} + \frac{5}{2}\hat{j}\right) \cdot (-17\hat{i} - 20\hat{j} + 19\hat{k})\right|}{|\vec{b}_1 \times \vec{b}_2|}$$ 1

$$= \frac{135}{|\vec{b}_1 \times \vec{b}_2|} \neq 0$$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

22. Given $\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} = \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} \therefore \vec{b} \cdot \vec{a} = \vec{c} \cdot \vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b} \perp \vec{c} \Rightarrow \vec{b} \cdot \vec{c} = 0$ ...(ii) $\frac{1}{2}$

$$(|3\vec{a} - 2\vec{b} + 2\vec{c}|)^2 = 9|\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 - 12\,\vec{a} \cdot \vec{b} - 8\,\vec{b} \cdot \vec{c} + 12\,\vec{a} \cdot \vec{c}$$ 1

$= 9(1)^2 + 4(2)^2 + 4(3)^2$ [using (i) and (ii)] 1

$= 9 + 16 + 36 = 61$

$\Rightarrow |3\vec{a} - 2\vec{b} + 2\vec{c}| = \sqrt{61}$ 1

23. Curve $y = \dfrac{x-7}{(x-2)(x-3)}$ cuts at x – axis at the point x = 7, y = 0 i.e. (7, 0) $\frac{1}{2}$

$\dfrac{dy}{dx} = \dfrac{(x^2 - 5x + 6)\cdot 1 - (x-7)(2x-5)}{(x^2 - 5x + 6)^2}$ $\frac{1}{2}$

at (7, 0) $\dfrac{dy}{dx} = \dfrac{20}{(20)^2} = \dfrac{1}{20}$ $\frac{1}{2}$

$\therefore$ Slope of tangent at (7, 0) is $\dfrac{1}{20}$ $\frac{1}{2}$

and slope of Normal at (7, 0) is –20 $\frac{1}{2}$

Equation of tangent at (7, 0) is $y - 0 = \dfrac{1}{20}(x - 7)$

or $x - 20y - 7 = 0$ 1

Equation of Normal at (7, 0) is $y - 0 = -20(x - 7)$

or $20x + y = 140.$ $\frac{1}{2}$

SECTION D

24. $f(x) = \sin x + \dfrac{1}{2}\cos 2x \Rightarrow f'(x) = \cos x - \sin 2x$ 1

$f'(x) = 0 \Rightarrow \cos x - 2\sin x \cos x = 0$

$\Rightarrow \cos x(1 - 2\sin x) = 0$

$\Rightarrow x = \dfrac{\pi}{2}$ or $x = \dfrac{\pi}{6}$ 1

$x = \dfrac{\pi}{6} \in \left(0, \dfrac{\pi}{2}\right)$ 1

$f''(x) = -\sin x - 2\cos 2x$ 1

$f''(\pi/6) < 0 \Rightarrow \quad x = \frac{\pi}{6}$ is a local maxima. 1

Local Max. Value = $f(\pi/6) = \frac{3}{4}$ 1

Local extreme values do exist at end points x = 0, x = $\frac{\pi}{2}$ but no marks are alotted here for that

25. $A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ 2

$A^2 \cdot A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ 3

or $A \cdot A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow A^2 = A^{-1}$ 1

OR

Given System of equation can be written as

$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \\ -2 \end{bmatrix}$ or $AX = B$ 1

$|A| = 2(0) + 3\ (-2) + 5\ (1) = -1 \neq 0$ 1

$\therefore\ X = A^{-1} \cdot B$

$(\text{adj. } A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ 2

$A^{-1} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ $\frac{1}{2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 13 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 2, y = 2, z = 3.$ $1\frac{1}{2}$

26. Let the events be

$$\left.\begin{array}{l} E_1 : \text{bag I is selected} \\ E_2 : \text{bag II is selected} \\ A : \text{getting a red ball} \end{array}\right\}$$ 1

$P(E_1) = P(E_2) = \frac{1}{2}$ $\frac{1}{2}$

$P(A / E_1) = \frac{3}{9} = \frac{1}{3};\ P(A / E_2) = \frac{5}{5+n}$ $\frac{1}{2}+1$

$$P(E_2/A) = \frac{3}{5} = \frac{\frac{1}{2}\cdot\frac{5}{5+n}}{\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{5}{5+n}}$$ 2

$\Rightarrow \frac{3}{5} = \frac{15}{5+n+15} \Rightarrow n = 5.$ 1

27. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$ 1

$\Rightarrow 16(x-2) + 24(y-5) + 32(z+3) = 0$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k}) = 7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$\frac{x-3}{4} = \frac{y-1}{4} = \frac{z-5}{6}$ or $\frac{x-3}{2} = \frac{y-1}{2} = \frac{z-5}{3}$ ...(ii) 1

Any point on (ii) is $(2\lambda + 3, 2\lambda + 1, 3\lambda + 5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+5)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$, is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4]=0$ 1

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda=0$ ...(i) 1

Plane (i) is || to x–axis $\Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3=0$ $1\frac{1}{2}$

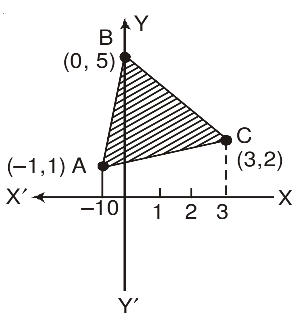
or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6=0$

Distance of this plane from x–axis

$=\frac{|-6|}{\sqrt{(-1)^2+(3)^2}}=\frac{6}{\sqrt{10}}$ units 1

28. Let the points be A (–1, 1), B (0, 5) and C (3, 2)

Correct Figure 1



Equation of AB : $y=4x+5$

BC : $y=5-x$

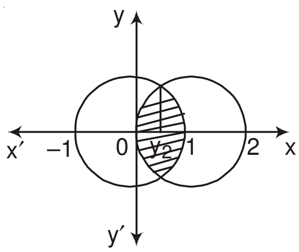
AC : $y=\frac{1}{4}(x+5)$ $1\frac{1}{2}$

Req. Area $=\int_{-1}^{0}(4x+5)dx+\int_{0}^{3}(5-x)dx-\int_{-1}^{3}\frac{1}{4}(x+5)dx$ $1\frac{1}{2}$

$$\therefore\ A = \left[\frac{(4x+5)^2}{8}\right]_{-1}^{0} + \left[\frac{(5-x)^2}{-2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{(x+5)^2}{2}\right]_{-1}^{3}$$ 1

$$= 3 + \frac{21}{2} - 6 = \frac{15}{2}$$ 1

OR



Correct Figure 1

$$(x-1)^2 + y^2 = 1$$

and $x^2 + y^2 = 1 \Rightarrow (x-1)^2 = x^2$

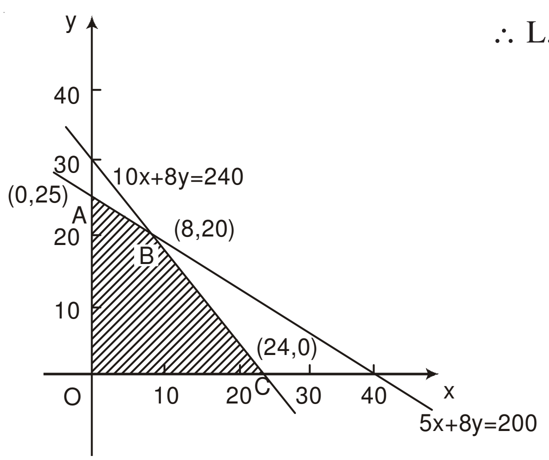
$$\Rightarrow x = \frac{1}{2}$$ 1

$$\therefore \text{ Required area} = 2\left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}\,dx + \int_{\frac{1}{2}}^{1} \sqrt{1-x^2}\,dx\right]$$ 2

$$= 2\left[\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}} + 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$ 1

$$= 2\left[\frac{-\sqrt{3}}{8} + \frac{\pi}{6}\right] + 2\left[\frac{-\sqrt{3}}{8} + \frac{\pi}{6}\right] = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ $\frac{1}{2}$

such that
$$\left.\begin{aligned} 5x + 8y &\le 200 \\ 10x + 8y &\le 240 \\ x, y &\ge 0 \end{aligned}\right\}$$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20.

QUESTION PAPER CODE 65/3/3
EXPECTED ANSWER/VALUE POINTS
SECTION A

1. DRs are (6, 2, 3) $\therefore$ DC's are $\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$ $\frac{1}{2}+\frac{1}{2}$

OR

$\frac{x-1}{-3}=\frac{y-7}{p}=\frac{z-3}{2}; \frac{x-1}{-3p}=\frac{y-5}{1}=\frac{z-6}{-5}$ $\frac{1}{2}$

$\Rightarrow 9p + p - 10 = 0 \Rightarrow p = 1$ $\frac{1}{2}$

2. $\frac{dy}{dx}-\frac{2}{x}\cdot y = 2x$ $\frac{1}{2}$

$\Rightarrow$ I.F. $= e^{-2\log x} = x^{-2} = \frac{1}{x^2}$ $\frac{1}{2}$

3. $|-2A| = (-2)^3 \cdot |A|$ $\frac{1}{2}$

$= -8 \times 4 = -32$ $\frac{1}{2}$

4. $y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$ $\frac{1}{2}+\frac{1}{2}$

SECTION B

5. $B'A' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix}\begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$ 1

$= \begin{bmatrix} 75 & 56 & 71 \\ 9 & 4 & 13 \\ 42 & 22 & 58 \end{bmatrix}$ 1

6. $\int_a^b \frac{\log x}{x} dx = \left[\frac{1}{2}(\log x)^2\right]_a^b$ 1

$= \frac{1}{2}[(\log b)^2 - (\log a)^2]$ 1

7. $y^2 = m(a^2 - x^2) \Rightarrow 2y\frac{dy}{dx} = -2mx$ $\frac{1}{2}$

or $y\frac{dy}{dx} = -mx$ ...(i)

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m$ ...(ii) $\frac{1}{2}$

form (i) and (ii) we get $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$ 1

or $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

8. $\int \frac{\sin(x-a)}{\sin(x+a)}dx = \int \frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$= \int \left[\frac{\sin(x+a)\cdot\cos 2a}{\sin(x+a)} - \frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$ $\frac{1}{2}$

$= x\cdot\cos 2a - \sin 2a \cdot \log|\sin(x+a)| + c$ $\frac{1}{2}+\frac{1}{2}$

OR

$\int (\log x)^2 \cdot 1 dx = (\log x)^2 \cdot x - \int 2\cdot \log x \cdot \frac{1}{x}\cdot x\, dx$ 1

$= x\cdot(\log x)^2 - \left\{\log x \cdot 2x - \int \frac{1}{x}\cdot 2x dx\right\}$ $\frac{1}{2}$

$= x(\log x)^2 - 2x\log x + 2x + c$ $\frac{1}{2}$

9. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\vec{a}\times\vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j}+\hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a}\cdot\vec{b}\times\vec{c} = 0$ $\frac{1}{2}$

$$\vec{a}\cdot\vec{b}\times\vec{c} = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3) + 2(-6) + 3(3)$$

$$= 3 - 12 + 9 = 0$$ $1+\frac{1}{2}$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

10. A = {(S, F, M), (S, M, F), (M, F, S), (F, M, S)}
B = {(S, F, M), (M, F, S)} 1

Total number of possible arrangements = 6

$$P(B|A) = \frac{P(B\cap A)}{P(A)}$$

$$= \frac{2/6}{4/6} = \frac{1}{2}$$ 1

11. Given 2 $P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$

Let $P(X = x_3) = k$, then $P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3}$ and $P(X = x_4) = \frac{k}{5}$ $\frac{1}{2}$

$\therefore\ k + \frac{k}{2} + \frac{k}{3} + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61}$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r \geq 4) = P(4) + P(5)$

$$= {}^5C_4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^5C_5\left(\frac{1}{2}\right)^5 = 6\left(\frac{1}{2}\right)^5 = \frac{6}{32} \text{ or } \frac{3}{16}$$ 1

(ii) P(at most 4 heads) = $P(r \leq 4) = 1 - P(5)$

$$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$$ 1

12. Let $e \in \mathbb{R}$ be the identity element.

then $a*e = e*a = a$ 1

$\Rightarrow a^2 + e^2 = e^2 + a^2 = a^2 \Rightarrow e^2 = 0 \Rightarrow e = 0.$ 1

$\therefore$ Identity element is $0 \in \mathbb{R}$

SECTION C

13. $\tan(\sec^{-1}\frac{1}{x}) = \sin(\tan^{-1} 2) \Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$ 2

$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \frac{1-x^2}{x^2} = \frac{4}{5}$ 1

$\Rightarrow 9x^2 = 5 \Rightarrow x^2 = \frac{5}{9} \Rightarrow x = \frac{\sqrt{5}}{3}, \{x > 0\}$ 1

14. $e^y \cdot (x+1) = 1 \Rightarrow e^y \cdot 1 + (x+1)e^y \cdot \frac{dy}{dx} = 0$ 1

$\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1}$ 1

$\Rightarrow \frac{d^2y}{dx^2} = +\frac{1}{(x+1)^2}$ 1

$\therefore \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ 1

OR

$y = \sin^{-1}\left(\frac{2 \cdot 2^x}{1+(2^x)^2}\right) = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$, where $t = 2^x$ 1

$\Rightarrow y = 2\tan^{-1} t$ 1

$\frac{dy}{dt} = \frac{2}{1+t^2}$ and $\frac{dt}{dx} = 2^x \cdot \log 2.$ 1

$\Rightarrow \frac{dy}{dx} = \frac{2}{1+t^2} \cdot 2^x \cdot \log 2 = \frac{2^{x+1} \cdot \log 2}{1+4^x}$ 1

15. $f(x) = 4x^3 - 6x^2 - 72x + 30$

$\Rightarrow f'(x) = 12x^2 - 12x - 72 = 12(x-3)(x+2)$ 1

$f'(x) = 0 \Rightarrow x = -2, x = 3$ 1

$\therefore$ possible intervals are $(-\infty, -2), (-2, 3), (3, \infty)$ $\frac{1}{2}$

$f'(x) < 0$ in $(-2, 3)$

and $f'(x) > 0$ in $(-\infty, -2)$ and $(3, \infty)$

$\Rightarrow$ f(x) is strictly increasing in $(-\infty, -2), (3, \infty)$ or $(-\infty, 2], [3, \infty)$ 1

and strictly decreasing in $(-2, 3)$ or $[-2, 3]$ $\frac{1}{2}$

16. (i) For $a \in Z, (a, a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let $(a, b) \in R$ for $a, b \in Z$, then $a - b$ is divisible by 2

$\Rightarrow (b - a)$ is also divisible by 2

$\therefore (b, a) \in R \Rightarrow$ R is symmetric ...(ii) 1

For $a, b, c, \in Z$, Let $(a, b) \in R$ and $(b, c) \in R$

$\therefore a - b = 2p, p \in Z$, and $b - c = 2q, q \in Z$,

adding, $a - c = 2(p + q) \Rightarrow (a - c)$ is divisible by 2

$\Rightarrow (a, c) \in R$, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$fof(x) = f\left(\frac{4x+3}{6x-4}\right)$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$$\Rightarrow fof(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$$ 1

Since fof(x) = x ⇒ fof = I ⇒ $f^{-1} = f$ 1

17. Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$$R_1 \to R_1 - (R_2 + R_3) \Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$ 1

$$\therefore \Delta = -2\begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = -\frac{2}{b}\begin{vmatrix} 0 & bc & b \\ b & bc+ab & b \\ c & bc & a+b \end{vmatrix}$$ $1+\frac{1}{2}$

$C_2 \to C_2 - cC_3$

$$\Rightarrow \Delta = -\frac{2}{b}\begin{vmatrix} 0 & 0 & b \\ b & ab & b \\ c & -ac & a+b \end{vmatrix}$$ 1

$$= -\frac{2}{b} \cdot b \cdot (-abc - abc) = 4abc.$$ $\frac{1}{2}$

18. $y = (\sec^{-1} x)^2, x > 0$

$$\frac{dy}{dx} = 2\sec^{-1} x \cdot \frac{1}{x\sqrt{x^2-1}}$$ 1

$$\Rightarrow x\sqrt{x^2-1}\frac{dy}{dx} = 2\sqrt{y}$$ $\frac{1}{2}$

squaring both sides, we get

$$x^2(x^2-1)\left(\frac{dy}{dx}\right)^2 = 4y \quad \text{or} \quad (x^4-x^2)\left(\frac{dy}{dx}\right)^2 = 4y$$ $\frac{1}{2}$

differentiating w.r.t. x.

$$(x^4-x^2)2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + (4x^3-2x)\left(\frac{dy}{dx}\right)^2 = 4 \cdot \frac{dy}{dx}$$ 1

$$\Rightarrow x^2(x^2-1)\frac{d^2y}{dx^2}+(2x^3-x)\frac{dy}{dx}-2=0$$ 1

19. RHS $= \int_a^b f(a+b-x)\,dx = -\int_b^a f(t)\,dt$, where a + b – x = t, dx = –dt $\frac{1}{2}$

$= \int_a^b f(t)\,dt = \int_a^b f(x)\,dx = \text{LHS}$ $\frac{1}{2}$

Let I $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ ...(i) $\frac{1}{2}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get 2I $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\cdot dx = x]_{\pi/6}^{\pi/3} = \pi/6.$ $\frac{1}{2}$

$\Rightarrow$ I $= \frac{\pi}{12}$ $\frac{1}{2}$

20. I $= \int \frac{\sin 2x}{(\sin^2 x+1)(\sin^2 x+3)}\,dx$

Put $\sin^2 x = t \Rightarrow \sin 2x\,dx = dt$ $\frac{1}{2}$

$\therefore$ I $= \int \frac{dt}{(t+1)(t+3)} = \int\left(\frac{1/2}{t+1}+\frac{-1/2}{t+3}\right)dt$ $1\frac{1}{2}$

$= \frac{1}{2}\log|t+1| - \frac{1}{2}\log|t+3| + c$ $1\frac{1}{2}$

$= \frac{1}{2}\log(\sin^2 x+1) - \frac{1}{2}\log(\sin^2 +3) + c.$ $\frac{1}{2}$

21. Writing the equations of given lines in standard form, as

$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1};\ \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$ $\frac{1}{2}$

lines are perpendicular to each other,

$\Rightarrow\ (5\lambda+2)\cdot 1+(-5)(2\lambda)+1(3)=0$ $\frac{1}{2}$

$-5\lambda+5=0\ \Rightarrow\ \lambda=1$ $\frac{1}{2}$

$\therefore$ lines are $\frac{x-5}{7}=\frac{y-2}{-5}-\frac{z-1}{1};\ \frac{x}{1}=\frac{y+\frac{1}{2}}{2}=\frac{z-1}{3}$ $\frac{1}{2}$

Shortest distance between these lines $=\frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}=\frac{\left|\left(5\hat{i}+\frac{5}{2}\hat{j}\right)\cdot(-17\hat{i}-20\hat{j}+19\hat{k})\right|}{|\vec{b}_1\times\vec{b}_2|}$ 1

$$=\frac{135}{|\vec{b}_1\times\vec{b}_2|}\neq 0$$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

22. Given $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}\ \therefore\ \vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b}\perp\vec{c}\ \Rightarrow\ \vec{b}\cdot\vec{c}=0$ ...(ii) $\frac{1}{2}$

$(|3\vec{a}-2\vec{b}+2\vec{c}|)^2=9|\vec{a}|^2+4|\vec{b}|^2+4|\vec{c}|^2-12\,\vec{a}\cdot\vec{b}-8\,\vec{b}\cdot\vec{c}+12\,\vec{a}\cdot\vec{c}$ 1

$=9(1)^2+4(2)^2+4(3)^2$ [using (i) and (ii)] 1

$=9+16+36=61$

$\Rightarrow\ |3\vec{a}-2\vec{b}+2\vec{c}|=\sqrt{61}$ 1

23. $\frac{dy}{dx}=\frac{x+y}{x-y}=\frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore\ v+x\frac{dv}{dx}=\frac{1+v}{1-v}\ \Rightarrow\ x\frac{dv}{dx}=\frac{1+v}{1-v}-v=\frac{1+v-v+v^2}{1-v}$

$$\Rightarrow \int \frac{1-v}{1+v^2}dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$$ $1+\frac{1}{2}$

$$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$$ 1

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$$ 1

or $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$

OR

$(1+x^2)dy + 2xy\,dx = \cot x.dx.$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\cot x}{1+x^2}$$ 1

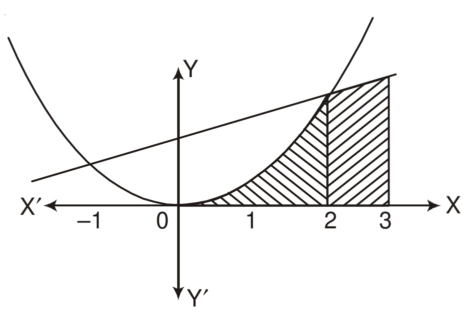
I.F. $= e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$ 1

$\therefore$ Solution is, $y\cdot(1+x^2) = \int \cot x\,dx = \log|\sin x| + c$ 1+1

or $y = \frac{1}{1+x^2}\cdot\log|\sin x| + \frac{c}{1+x^2}$

SECTION D

24. $\{(x,y): 0 \le y \le x^2, 0 \le y \le x+2, -1 \le x \le 3\}$



Correct Figure 1

$$\text{Area} = \int_{-1}^{2} x^2dx + \int_{2}^{3}(x+2)dx$$ 2

$$= \left.\frac{x^3}{3}\right]_{-1}^{2} + \left.\frac{(x+2)^2}{2}\right]_{2}^{3}$$ 2

$$= 3 + \frac{9}{2} = \frac{15}{2}$$ 1

OR

$$I = \lim_{h\to 0} h[f(1) + f(1+h) + f(1+2h) + \cdots + f(1+\overline{n-1})n)]$$ 1

where $h = \frac{3}{n}$ or $nh = 3$ $\frac{1}{2}$

$$= \lim_{h\to 0} h[(2+e^2) + (2+h+e^{2+2h}) + (2+2h+e^{2+4h}) + \cdots + (2+(n-1)h) + e^{2+2(n-1)h}]$$ 1

$$= \lim_{h\to 0} h\left[2n + h\cdot\frac{n(n-1)}{2}\right] + \lim_{h\to 0} h\cdot e^2[1+e^{2h}+e^{4h}+\cdots e^{2(n-1)h}]$$ 1

$$= \lim_{h\to 0}\left[2nh + \frac{nh(nh-h)}{2}\right] + \lim_{h\to 0} \not{h}\cdot\frac{e^2}{2\not{h}}\,\frac{e^{2nh}-1}{\frac{e^{2h}-1}{2h}}$$ $1\frac{1}{2}$

$$= 6 + \frac{9}{2} + \frac{e^2(e^6-1)}{2} = \frac{21}{2} + \frac{e^2(e^6-1)}{2}$$ 1

25. p = (prob. of doublet) = 1/6 $\therefore$ q = 5/6 1

X	0	1	2	3	4
P(X)	$\left(\frac{5}{6}\right)^4 = \frac{625}{1296}$	$4\cdot\left(\frac{5}{6}\right)^3\frac{1}{6} = \frac{500}{1296}$	$6\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2$ $\frac{150}{1296}$	$4\frac{5}{6}\left(\frac{1}{6}\right)^3$ $\frac{20}{1296}$	$\left(\frac{1}{6}\right)^4$ $\frac{1}{1296}$

1

XP(X)	0	$\frac{500}{1296}$	$\frac{300}{1296}$	$\frac{60}{1296}$	$\frac{4}{1296}$
$X^2P(X)$	0	$\frac{500}{1296}$	$\frac{600}{1296}$	$\frac{180}{1296}$	$\frac{16}{1296}$

1

1

Mean $= \Sigma XP(X) = \frac{864}{1296} = \frac{2}{3}$ 1

Variance $= \Sigma X^2\cdot P(X) - [\Sigma X\cdot P(X)]^2 = 1 - \frac{4}{9} = \frac{5}{9}$ 1

26. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$ 1

$\Rightarrow\ 16(x-2)+24(y-5)+32(z+3)=0$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$\frac{x-3}{4}=\frac{y-1}{4}=\frac{z-5}{6}$ or $\frac{x-3}{2}=\frac{y-1}{2}=\frac{z-5}{3}$ ...(ii) 1

Any point on (ii) is $(2\lambda+3, 2\lambda+1, 3\lambda+5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+4)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0,$ is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4] = 0$ 1

$\Rightarrow\ \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda = 0$ ...(i) 1

Plane (i) is || to x–axis $\Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3 = 0$ $1\frac{1}{2}$

or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6 = 0$

Distance of this plane from x–axis

$= \frac{|-6|}{\sqrt{(-1)^2+(3)^2}}=\frac{6}{\sqrt{10}}$ units 1

27. Let Given surface area of open cylinder be S.

Then S = $2\pi rh + \pi r^2$

$\Rightarrow$ h = $\dfrac{S-\pi r^2}{2\pi r}$ 1

Volume V = $\pi r^2 h$ $\frac{1}{2}$

V = $\pi r^2\left[\dfrac{S-\pi r^2}{2\pi r}\right] = \dfrac{1}{2}[Sr - \pi r^3]$ 1

$\dfrac{dV}{dr} = \dfrac{1}{2}[S - 3\pi r^2]$ $\frac{1}{2}$

$\dfrac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2$ or $2\pi rh + \pi r^2 = 3\pi r^2$ 1

$\Rightarrow 2\pi rh = 2\pi r^2$ $\Rightarrow$ h = r 1

$\dfrac{d^2V}{dr^2} = -6\pi r < 0$ 1

$\therefore$ For volume to be maximum, height = radius

28. |A| = $1(7) - 1(-3) + 1(-1) = 9$ 1

(adj A) = $\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ 2

$\Rightarrow A^{-1} = \dfrac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ $\frac{1}{2}$

Given equations can be written as $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

or AX = B $\Rightarrow$ X = $A^{-1}B$ $\frac{1}{2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ $1\frac{1}{2}$

$\therefore$ x = 1, y = 2, z = 3 $\frac{1}{2}$

OR

Let: $\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_1 \leftrightarrow R_3 \begin{bmatrix} 3 & 7 & 2 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$

$\begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{matrix} \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$

$R_3 \to R_3 - 2R_1 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & -5 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix} A$

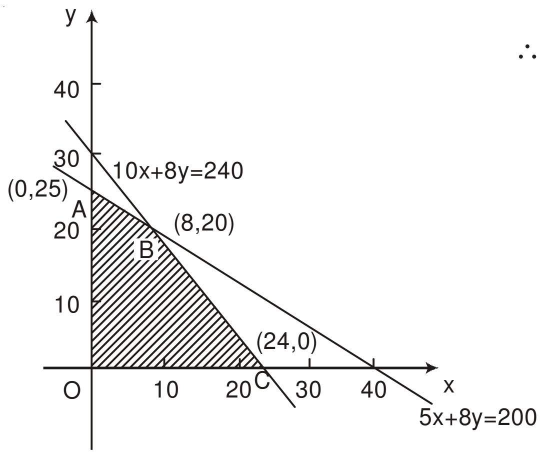
$R_3 \to R_3 + 5R_2 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ -2 & 5 & -2 \end{bmatrix} A$ 4

$\begin{matrix} R_1 \to R_1 - 4R_2 \\ R_3 \to -R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$

$R_1 \to R_1 - R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ $\frac{1}{2}$

such that $5x + 8y \le 200$

$10x + 8y \le 240$

$x, y \ge 0$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20. 1