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Senior School Certificate Examination

March 2017

Marking Scheme — Mathematics 65/1/1, 65/1/2, 65/1/3

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/ Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A^{-1}| = \frac{1}{|A|} \Rightarrow k = -1$ 1

2. $\lim_{x \to 0_-} f(x) = \lim_{x \to 0_-} \frac{kx}{|x|} = -k$ $\frac{1}{2}$

$k = -3$ $\frac{1}{2}$

3. $\int_2^3 3^x dx = \left[\frac{3^x}{\log 3}\right]_2^3 = \frac{18}{\log 3}$ $\frac{1}{2}+\frac{1}{2}$

4. $\cos^2 90° + \cos^2 60° + \cos^2 \gamma = 1$ $\frac{1}{2}$

$\cos \gamma = \pm\frac{\sqrt{3}}{2}, \gamma = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\frac{1}{2}$

SECTION B

5. Let $A = [a_{ij}]_{n \times n}$ be skew symmetric matrix

A is skew symmetric

$\therefore \quad A = -A'$ 1

$\Rightarrow \quad a_{ij} = -a_{ji} \; \forall \; i, j$

For diagonal elements i = j,

$\Rightarrow \quad 2a_{ii} = 0$

$\Rightarrow \quad a_{ii} = 0 \Rightarrow$ diagonal elements are zero. 1

6. From the given equation

$2\sin y \cos y \cdot \frac{dy}{dx} - \sin xy \cdot \left[x \cdot \frac{dy}{dx} + y \cdot 1\right] = 0$ 1

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin(xy)}$$

$$\therefore \quad \left.\frac{dy}{dx}\right|_{x=1,\, y=\frac{\pi}{4}} = \frac{\pi}{4(\sqrt{2}-1)}$$ 1

7. $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \quad \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{3}{4\pi r^2}$$ 1

$S = 4\pi r^2$

$$\Rightarrow \quad \frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$$ $\frac{1}{2}$

$$\Rightarrow \quad \left.\frac{dS}{dt}\right|_{r=2} = 3\text{cm}^2/\text{s}$$ $\frac{1}{2}$

8. $f(x) = 4x^3 - 18x^2 + 27x - 7$

$f'(x) = 12x^2 - 36x + 27$ $\frac{1}{2}$

$= 3(2x-3)^2 \geq 0 \;\forall\; x \in R$ 1

$\Rightarrow$ f(x) is increasing on R $\frac{1}{2}$

9. Equation of given line is $\frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35}$ $\frac{1}{2}$

Its DR's $\left\langle \frac{1}{5}, -\frac{1}{7}, \frac{1}{35} \right\rangle$ or $\langle 7, -5, 1 \rangle$ $\frac{1}{2}$

Equation of required line is

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$$ 1

10. $P(E \cap F') = P(E) - P(E \cap F)$ 1

$= P(E) - P(E) \cdot P(F)$ $\frac{1}{2}$

$= P(E)[1 - P(F)]$

$= P(E)P(F')$ $\frac{1}{2}$

$\Rightarrow$ E and F' are independent events.

11. Let x necklaces and y bracelets are manufactured

$\therefore$ L.P.P. is

Maximize profit, P = 100x + 300y $\frac{1}{2}$

subject to constraints

$x + y \leq 24$

$\frac{1}{2}x + y \leq 16$ or $x + 2y \leq 32$ $\frac{1}{2} \times 3 = 1\frac{1}{2}$

$x, y, \geq 1$

12. $\int \frac{dx}{x^2 + 4x + 8} = \int \frac{dx}{(x+2)^2 + (2)^2}$ 1

$= \frac{1}{2} \tan^{-1} \frac{x+2}{2} + C$ 1

SECTION C

13. Let $\frac{1}{2} \cos^{-1} \frac{a}{b} = x$ $\frac{1}{2}$

LHS = $\tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right) = \frac{1 + \tan x}{1 - \tan x} + \frac{1 - \tan x}{1 + \tan x}$ $1\frac{1}{2}$

$= \frac{2(1 + \tan^2 x)}{1 - \tan^2 x} = \frac{2}{\cos 2x}$ 1

$= \frac{2b}{a}$ = RHS 1

14. $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$= 3(x+y)\begin{vmatrix} 1 & x+y & x+2y \\ 1 & x & x+y \\ 1 & x+2y & x \end{vmatrix}$$ 1

$R_1 \to R_1 - R_2,\ R_3 \to R_3 - R_2$

$$= 3(x+y)\begin{vmatrix} 0 & y & y \\ 1 & x & x+y \\ 0 & 2y & -y \end{vmatrix}$$ 1+1

$= -3(x+y)(-y^2 - 2y^2) = 9y^2(x+y)$ 1

OR

Let $D = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ $\frac{1}{2}$

$CD = AB \Rightarrow \begin{bmatrix} 2x+5z & 2y+5w \\ 3x+8z & 3y+8w \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ 1+1

$2x + 5z = 3,\ 3x + 8z = 43;\ 2y + 5w = 0,\ 3y + 8w = 22.$

Solving, we get $x = -191,\ y = -110,\ z = 77,\ w = 44$ 1

$\therefore\ D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$ $\frac{1}{2}$

15. $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

$y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ 1

$u = (\sin x)^x$

$\Rightarrow\ \log u = x \log \sin x$ $\frac{1}{2}$

$\Rightarrow\ \frac{du}{dx} = (\sin x)^x\ [x \cot x + \log \sin x]$ 1

$v = \sin^{-1}\sqrt{x}$

$\Rightarrow \quad \frac{dv}{dx} = \frac{1}{2\sqrt{x - x^2}}$ 1

$\therefore \quad \frac{dy}{dx} = (\sin x)^x[x \cot x + \log \sin x] + \frac{1}{2\sqrt{x - x^2}}$ $\frac{1}{2}$

OR

$x^m \cdot y^n = (x + y)^{m + n}$

$\Rightarrow \quad m \log x + n \log y = (m + n) \log (x + y)$ 1

$\Rightarrow \quad \frac{m}{x} + \frac{n}{y} \cdot \frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$ 1

$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$ ...(i) 1

$\frac{d^2y}{dx^2} = \frac{x\frac{dy}{dx} - y}{x^2} = 0$ ...(ii) (using (i)) 1

16. $\int \frac{2x}{(x^2+1)(x^2+2)^2} = \int \frac{dy}{(y+1)(y+2)^2}$ [by substituting $x^2 = y$] 1

$= \int \frac{dy}{y+1} - \int \frac{dy}{y+2} - \int \frac{dy}{(y+2)^2}$ (using partial fraction) $1\frac{1}{2}$

$= \log(y+1) - \log(y+2) + \frac{1}{y+2} + C$ 1

$= \log(x^2+1) - \log(x^2+2) + \frac{1}{x^2+2} + C$ $\frac{1}{2}$

17. $I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

$= \int_0^{\pi} \frac{(\pi - x)\sin x}{1 + \cos^2 x} dx$ 1

$$\Rightarrow 2I = \pi\int_0^{\pi} \frac{\sin x\, dx}{1+\cos^2 x}$$

Put $\cos x = t$ and $-\sin x\, dx = dt$ 1

$$= -\pi\int_1^{-1} \frac{dt}{1+t^2}$$

$$= \pi[\tan^{-1} t]_{-1}^{1} = \frac{\pi^2}{2}$$ $1\frac{1}{2}$

$$\Rightarrow \quad I = \frac{\pi^2}{4}$$ $\frac{1}{2}$

OR

$$I = \int_0^{3/2} |x \sin \pi x|\, dx$$

$$= \int_0^{1} x \sin \pi x \cdot dx - \int_1^{3/2} x \sin \pi x\, dx$$ $1\frac{1}{2}$

$$= \left[-x\frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\right]_0^1 - \left[-\frac{x\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\right]_1^{3/2}$$ $1\frac{1}{2}$

$$= \frac{2}{\pi} + \frac{1}{\pi^2}$$ 1

18. $x^2 - y^2 = C(x^2 + y^2)^2 \Rightarrow 2x - 2yy' = 2C(x^2 + y^2)(2x + 2yy')$ 1

$$\Rightarrow \quad (x - yy') = \frac{x^2 - y^2}{y^2 + x^2}(2x + 2yy') \Rightarrow (y^2 + x^2)(x - yy') = (x^2 - y^2)(2x + 2yy')$$ 1

$$\Rightarrow \quad [-2y(x^2 - y^2) - y(y^2 + x^2)]\frac{dy}{dx} = 2x(x^2 - y^2) - x(y^2 + x^2)$$ 1

$$\Rightarrow \quad (y^3 - 3x^2y)\frac{dy}{dx} = (x^3 - 3xy^2)$$

$$\Rightarrow \quad (y^3 - 3x^2y)dy = (x^3 - 3xy^2)dx$$ 1

Hence $x^2 - y^2 = C(x^2 + y^2)^2$ is the solution of given differential equation.

19. $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = c_2 - c_3$ 1

(a) $c_1 = 1,\ \ c_2 = 2$

$[\vec{a}\ \vec{b}\ \vec{c}] = 2 - c_3$ 1

$\because\ \vec{a}, \vec{b}, \vec{c}$ are coplanar $[\vec{a}\ \vec{b}\ \vec{c}] = 0 \Rightarrow c_3 = 2$ 1

(b) $c_2 = -1, c_3 = 1$

$[\vec{a}\ \vec{b}\ \vec{c}] = c_2 - c_3 = -2 \neq 0$

$\Rightarrow$ No value of c_1 can make $\vec{a}, \vec{b}, \vec{c}$ coplanar 1

20. $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}\cdot\vec{b} = 0 = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a}$ ...(i) 1

Let α, β and γ be the angles made by $(\vec{a}+\vec{b}+\vec{c})$ with $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively

$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a} = |\vec{a}+\vec{b}+\vec{c}|\,|\vec{a}|\cos\alpha$

$\Rightarrow \alpha = \cos^{-1}\left(\dfrac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$

Similarly, $\beta = \cos^{-1}\left(\dfrac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$ and $\gamma = \cos^{-1}\left(\dfrac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$ 1

using (i), we get $\alpha = \beta = \gamma$ $\frac{1}{2}$

Now $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$ 1

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 3|\vec{a}|^2$ (using (i))

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}|\vec{a}|$

$\therefore\ \alpha = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = \beta = \gamma$ $\frac{1}{2}$

21.

x	P(x)
0	p
1	p
2	k
3	k

$\Sigma p(x) = 1 \Rightarrow 2p + 2k = 1 \Rightarrow k = \frac{1}{2} - p$ 1

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	p	0	0
1	p	p	p
2	$\frac{1}{2} - p$	$1 - 2p$	$2 - 4p$
3	$\frac{1}{2} - p$	$\frac{3}{2} - 3p$	$\frac{9}{2} - 9p$
		$\frac{5}{2} - 4p$	$\frac{13}{2} - 12p$

2

As per problem, $\Sigma\ p_i\ x_i^2 = 2\Sigma\ p_i\ x_i$

$\Rightarrow \quad p = \frac{3}{8}$ 1

22. Let H_1 be the event that 6 appears on throwing a die

H_2 be the event that 6 does not appear on throwing a die

E be the event that he reports it is six 1

$P(H_1) = \frac{1}{6}, P(H_2) = 1 - \frac{1}{6} = \frac{5}{6}$

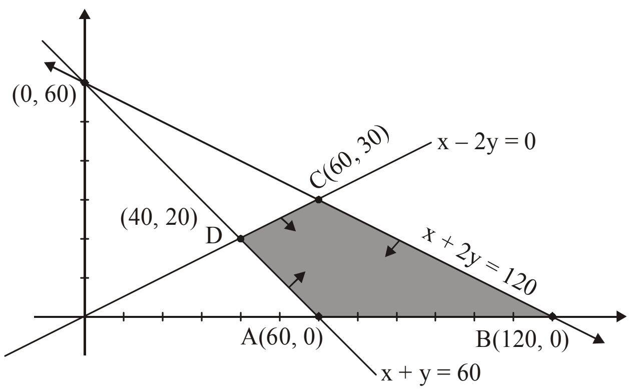
$P(E/H_1) = \frac{4}{5}, P(E/H_2) = \frac{1}{5}$ 1

$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) P(E/H_2)}$ $\frac{1}{2}$

$= \frac{4}{9}$ $\frac{1}{2}$

Relevant value: Yes, Truthness leads to more respect in society. 1

23.



Correct graph of 3 lines $1\frac{1}{2}$

Correct shade of 3 lines $1\frac{1}{2}$

$Z = 5x + 10y$

$Z|_{A(60,\ 0)} = 300$

$Z|_{B(120,\ 0)} = 600$

$Z|_{C(60,\ 30)} = 600$

$Z|_{D(40,\ 20)} = 400$

Minimum value of Z = 300 at x = 60, y = 0 1

SECTION D

24. $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 1

$AB = I \Rightarrow A^{-1} = B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ 1

Given equations in matrix form are:

$\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$ 1

$A'X = C$ $\frac{1}{2}$

$\Rightarrow \quad X = (A')^{-1} C = (A^{-1})'C$ 1

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$ 1

$\Rightarrow \quad x = 0,\ y = 5,\ z = 3$ $\frac{1}{2}$

25. Clearly $f^{-1}(y) = g(y): [-5, \infty) \to R_+$ and,

$$fog(y) = f\left(\frac{\sqrt{y+6}-1}{3}\right) = 9\left(\frac{\sqrt{y+6}-1}{3}\right)^2 + 6\left(\frac{\sqrt{y+6}-1}{3}\right) - 5 = y$$ 2

and $(gof)(x) = g(9x^2+6x-5) = \dfrac{\sqrt{9x^2+6x+1}-1}{3} = x$ 2

$\therefore\ g = f^{-1}$ 1

(i) $f^{-1}(10) = \dfrac{\sqrt{16}-1}{3} = 1$ $\frac{1}{2}$

(ii) $f^{-1}(y) = \dfrac{4}{3} \Rightarrow y = 19$ $\frac{1}{2}$

OR

Note: Some short comings have been observed in this question which makes the question unsolvable.

So, 6 marks may be given for a genuine attempt.

$a * b = a - b + ab \ \forall a, b \in A = Q - [1]$

$b * a = b - a + ba$

$(a * b) \neq b * a \Rightarrow *$ is not commutative. $1\frac{1}{2}$

$(a * b) * c = (a - b + ab) * c$

$= a - b - c + ab + ac - bc + abc$

$a * (b * c) = a * (b - c + bc)$

$= a - b + c + ab - ac - bc + abc$

$(a * b) * c \neq a * (b * c)$ $1\frac{1}{2}$

$\Rightarrow *$ is not associative.

Existence of identity

$a * e = a - e + ae = a$ $\quad\quad$ $e * a = e - a + ea = a$

$\Rightarrow e\ (a - 1) = 0$ $\quad\quad$ $\Rightarrow e(1 + a) = 2a$

$\Rightarrow e = 0$ $\quad\quad$ $\Rightarrow e = \dfrac{2a}{1+a}$ $\quad 1\frac{1}{2}$

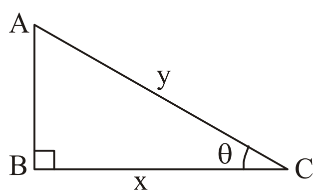
$\because$ e is not unique

$\therefore$ No idendity element exists.

$a * b = e = b * a$

$\therefore$ No identity element exists. $\quad 1\frac{1}{2}$

$\Rightarrow$ Inverse element does not exist.

26. 

Given $x + y = k$ $\quad 1$

Area of $\Delta = \dfrac{1}{2}x\sqrt{y^2 - x^2}$

Let $Z = \dfrac{1}{4}x^2(y^2 - x^2)$

$= \dfrac{1}{4}x^2[(k - x)^2 - x^2]$

$= \dfrac{1}{4}[k^2x^2 - 2kx^3]$ $\quad 1$

$\dfrac{dz}{dx} = \dfrac{1}{4}[2k^2x - 6kx^2] = 0 \Rightarrow k - 3x = 0 \Rightarrow x = \dfrac{k}{3}$ $\quad 1\frac{1}{2}$

$\Rightarrow x + y - 3x = 0$ or $y = 2x$

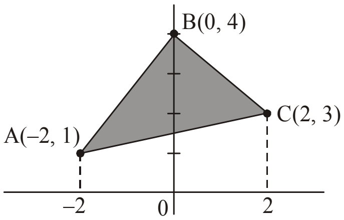
$\dfrac{d^2z}{dx^2} = \dfrac{1}{4}[2k^2 - 12kx]$ $\quad 1$

$\left.\dfrac{d^2z}{dx^2}\right|_{x = \frac{k}{3}} = \dfrac{1}{4}[2k^2 - 4k^2] = -\dfrac{k^2}{2} < 0$

$\therefore$ Area will be maximum for $2x = y$ $\quad 1$

but $\dfrac{x}{y} = \cos\theta \Rightarrow \cos\theta = \dfrac{x}{2x} = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$ $\quad \frac{1}{2}$

27.



Equation of AB: $y = \frac{3}{2}x + 4$ — Correct Figure: 1

Equation of BC; $y = 4 - \frac{x}{2}$

Equation of AC; $y = \frac{1}{2}x + 2$ — $1\frac{1}{2}$

Required area $= \int_{-2}^{0}\left(\frac{3}{2}x+4\right)dx + \int_{0}^{2}\left(4-\frac{x}{2}\right)dx - \int_{-2}^{2}\left(\frac{1}{2}x+2\right)dx$ — 1

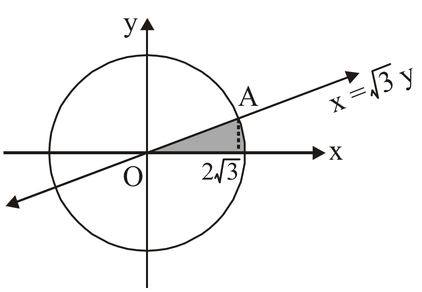
$= \left[\frac{3x^2}{4}+4x\right]_{-2}^{0} + \left[4x-\frac{x^2}{4}\right]_{0}^{2} - \left[\frac{x^2}{4}+2x\right]_{-2}^{2}$ — $1\frac{1}{2}$

$= 5 + 7 - 8$ — 1

$= 4$ sq.units

OR

Note: In this problem, two regions are possible instead of a unique one, so full 6 marks may be given for finding the area of either region correctly.



Correct Figure — 1

x-coordinate of points of intersection is $x = \pm 2\sqrt{3}$ — 1

Required area

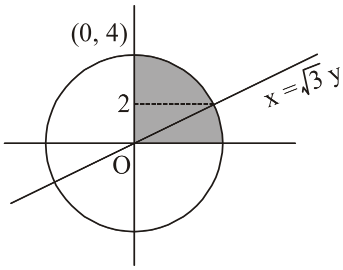
$= \int_{0}^{2\sqrt{3}} \frac{x}{\sqrt{3}}\cdot dx + \int_{2\sqrt{3}}^{4}\sqrt{4^2 - x^2}\,dx$ — $1\frac{1}{2}$

$= \left[\frac{x^2}{2\sqrt{3}}\right]_{0}^{2\sqrt{3}} + \left[\frac{x\sqrt{16-x^2}}{2} + 8\sin^{-1}\frac{x}{4}\right]_{2\sqrt{3}}^{4}$ — $1\frac{1}{2}$

$= 2\sqrt{3} + 8\left(\frac{\pi}{2} - \frac{\pi}{3}\right) - 2\sqrt{3}$

$= \frac{4\pi}{3}$ sq.units — 1

Alternate Solution



Correct figure 1

y-co-ordinate of point of intersection is y = 2 1

Required Area

$$= \sqrt{3}\int_0^2 y\,dx + \int_2^4 \sqrt{(4)^2 - y^2}\,dy$$ $1\frac{1}{2}$

$$= \sqrt{3}\left[\frac{y^2}{2}\right]_0^2 + \left[\frac{y\sqrt{16-y^2}}{2} + 8\sin^{-1}\frac{y}{4}\right]_2^4$$ $1\frac{1}{2}$

$$= 2\sqrt{3} + 4\pi - 2\sqrt{3} - \frac{4\pi}{3}$$

$$= \frac{8\pi}{3}\text{ sq.units}$$ 1

28. The given equation can be written as

$$\frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$$ 1

$$\text{I.F.} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$ 1

$\therefore$ Solution is

$$y \cdot x = \int (x\cos x + \sin x)dx + c$$ 1

$$\Rightarrow y \cdot x = x \sin x + c$$ 1

$$\text{or } y = \sin x + \frac{c}{x}$$

when $x = \frac{\pi}{2}, y = 1$, we get $c = 0$ 1

Required solution is y = sin x 1

29. Equation of family of planes

$$\vec{r}\cdot[(2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(\hat{i} - \hat{j})] = 1 - 4\lambda$$ 1

$\Rightarrow \vec{r}\cdot[(2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}] = 1-4\lambda$ ...(i) 1

plane (i) is perpendicular to

$\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+8 = 0$

$2(2+\lambda)-1(-3-\lambda)+1(4) = 0 \Rightarrow \lambda = -\frac{11}{3}$ 1+1

Substituting $\lambda = -\frac{11}{3}$ in equation (i), we get

$\vec{r}\cdot\left(-\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+4\hat{k}\right) = \frac{47}{3}$

$\Rightarrow \boxed{\vec{r}\cdot(-5\hat{i}+2\hat{j}+12\hat{k}) = 47}$ (vector equation)

or $\boxed{-5x+2y+12z-47 = 0}$ (cartesian equation)

} (ii) 1

Line $\frac{x-1}{1} = \frac{y-2}{1/2} = \frac{z-4}{1/3}$ lies on the plane

$\because$ (i) Point P(1, 2, 4) satisfies equation (ii) $\frac{1}{2}$

and $a_1a_2+b_1b_2+c_1c_2 = -5+1+4 = 0$ $\frac{1}{2}$

$\Rightarrow$ Line is perpendicular to the normal of plane $\therefore$ Plane contains the given line

OR

Equation of line L_1 passing through (1, 2, –4) is

$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$ 1

L_2: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$

L_3: $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

$\because L_1 \perp L_2 \Rightarrow 3a-16b+7c = 0$ 1

$L_1 \perp L_3 \Rightarrow 3a+8b-5c = 0$ 1

Solving, we get

$$\frac{a}{24} = \frac{b}{36} = \frac{c}{72} \Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6}$$ 1

$\therefore$ Required cartesian equation of line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$ 1

Vector equation

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$ 1

QUESTION PAPER CODE 65/1/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\cos^2 90° + \cos^2 60° + \cos^2 \gamma = 1$ $\frac{1}{2}$

$\cos \gamma = \pm\frac{\sqrt{3}}{2}, \ \gamma = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$ $\frac{1}{2}$

2. $\int_2^3 3^x dx = \left[\frac{3^x}{\log 3}\right]_2^3 = \frac{18}{\log 3}$ $\frac{1}{2}+\frac{1}{2}$

3. $|A^{-1}| = \frac{1}{|A|} \Rightarrow k = -1$ 1

4. $\lim\limits_{x \to 0_-} f(x) = \lim\limits_{x \to 0_-} \frac{kx}{|x|} = -k$ $\frac{1}{2}$

$k = -3$ $\frac{1}{2}$

SECTION B

5. $P(E \cap F') = P(E) - P(E \cap F)$ 1

$= P(E) - P(E) \cdot P(F)$ $\frac{1}{2}$

$= P(E)[1 - P(F)]$

$= P(E)P(F')$ $\frac{1}{2}$

$\Rightarrow$ E and F' are independent events.

6. Let x necklaces and y bracelets are manufactured

$\therefore$ L.P.P. is

Maximize profit, P = 100x + 300y $\frac{1}{2}$

subject to constraints

$x + y \leq 24$

$\frac{1}{2}x + y \leq 16$ or $x + 2y \leq 32$ $\frac{1}{2} \times 3 = 1\frac{1}{2}$

$x, y, \geq 1$

7. $\int \frac{dx}{x^2 + 4x + 8} = \int \frac{dx}{(x+2)^2 + (2)^2}$ 1

$= \frac{1}{2}\tan^{-1}\frac{x+2}{2} + C$ 1

8. Let $A = [a_{ij}]_{n \times n}$ be skew symmetric matrix

A is skew symmetric

$\therefore \quad A = -A'$ 1

$\Rightarrow \quad a_{ij} = -a_{ji} \ \forall \ i, j$

For diagonal elements i = j,

$\Rightarrow \quad 2a_{ii} = 0$

$\Rightarrow \quad a_{ii} = 0 \Rightarrow$ diagonal elements are zero. 1

9. From the given equation

$2\sin y \cos y \cdot \frac{dy}{dx} - \sin xy \cdot \left[x \cdot \frac{dy}{dx} + y \cdot 1\right] = 0$ 1

$\Rightarrow \quad \frac{dy}{dx} = \frac{y \sin xy}{\sin 2y - x\sin(xy)}$

$\therefore \quad \left.\frac{dy}{dx}\right|_{x = 1, y = \frac{\pi}{4}} = \frac{\pi}{4(\sqrt{2} - 1)}$ 1

10. $f(x) = 4x^3 - 18x^2 + 27x - 7$

$f'(x) = 12x^2 - 36x + 27$ $\frac{1}{2}$

$= 3(2x - 3)^2 \geq 0 \ \forall \ x \in R$ 1

$\Rightarrow$ f(x) is increasing on R $\frac{1}{2}$

11. Equation of given line $\frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35}$ $\frac{1}{2}$

Its DR's $\left\langle \frac{1}{5}, -\frac{1}{7}, \frac{1}{35} \right\rangle$ or $\langle 7, -5, 1 \rangle$ $\frac{1}{2}$

Equation of required line

$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$ 1

12. Given curve is $y = 5x - 2x^3$

$\Rightarrow \quad \frac{dy}{dx} = 5 - 6x^2$

$\Rightarrow \quad m = 5 - 6x^2$ 1

$\frac{dm}{dt} = -12x\frac{dx}{dt} = -24x$

$\left.\frac{dm}{dt}\right|_{x=3} = -72$ 1

SECTION C

13. $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

$= \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx$ 1

$\Rightarrow 2I = \pi\int_0^{\pi} \frac{\sin x \, dx}{1+\cos^2 x}$

Put $\cos x = t$ and $-\sin x \, dx = dt$ 1

$= -\pi\int_1^{-1} \frac{dt}{1+t^2}$

$= \pi[\tan^{-1} t]_{-1}^{1} = \frac{\pi^2}{2}$ $1\frac{1}{2}$

$\Rightarrow I = \frac{\pi^2}{4}$ $\frac{1}{2}$

OR

$$I = \int_0^{3/2} |x \sin \pi x|\, dx$$

$$= \int_0^1 x \sin \pi x \cdot dx - \int_1^{3/2} x \sin \pi x\, dx \qquad 1\frac{1}{2}$$

$$= \left[-x\frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\right]_0^1 - \left[-\frac{x \cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\right]_1^{3/2} \qquad 1\frac{1}{2}$$

$$= \frac{2}{\pi} + \frac{1}{\pi^2} \qquad 1$$

14. $x^2 - y^2 = C(x^2 + y^2)^2 \Rightarrow 2x - 2yy' = 2C(x^2 + y^2)(2x + 2yy')$ 1

$\Rightarrow \quad (x - yy') = \dfrac{x^2 - y^2}{y^2 + x^2}(2x + 2yy') \Rightarrow (y^2 + x^2)(x - yy') = (x^2 - y^2)(2x + 2yy')$ 1

$\Rightarrow \quad [-2y(x^2 - y^2) - y(y^2 + x^2)]\dfrac{dy}{dx} = 2x(x^2 - y^2) - x(y^2 + x^2)$ 1

$\Rightarrow \quad (y^3 - 3x^2y)\dfrac{dy}{dx} = (x^3 - 3xy^2)$

$\Rightarrow \quad (y^3 - 3x^2y)dy = (x^3 - 3xy^2)dx$ 1

Hence $(x^2 - y^2) = C(x^2 + y^2)^2$ is the solution of given differential equation.

15. $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = c_2 - c_3$ 1

(a) $c_1 = 1, \ c_2 = 2$

$[\vec{a}\ \vec{b}\ \vec{c}] = 2 - c_3$ 1

$\because \ \vec{a}, \vec{b}, \vec{c}$ are coplanar $[\vec{a}\ \vec{b}\ \vec{c}] = 0 \Rightarrow c_3 = 2$ 1

(b) $c_2 = -1, \ c_3 = 1$

$[\vec{a}\ \vec{b}\ \vec{c}] = c_2 - c_3 = -2 \neq 0$

$\Rightarrow$ No value of c_1 can make $\vec{a}, \vec{b}, \vec{c}$ coplanar 1

16. Let H_1 be the event that 6 appears on throwing a die

H_2 be the event that 6 does not appear on throwing a die

E be the event that he reports it is six 1

$$P(H_1) = \frac{1}{6}, P(H_2) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$P(E/H_1) = \frac{4}{5}, P(E/H_2) = \frac{1}{5}$$ 1

$$P(H_1/E) = \frac{P(H_1)\cdot P(E/H_1)}{P(H_1)\cdot P(E/H_1) + P(H_2)P(E/H_2)}$$ $\frac{1}{2}$

$$= \frac{4}{9}$$ $\frac{1}{2}$

Relevant value: Yes, Truthness leads to more respect in society. 1

17. Let $\frac{1}{2}\cos^{-1}\frac{a}{b} = x$ $\frac{1}{2}$

$$\text{LHS} = \tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right) = \frac{1+\tan x}{1-\tan x} + \frac{1-\tan x}{1+\tan x}$$ $1\frac{1}{2}$

$$= \frac{2(1+\tan^2 x)}{1-\tan^2 x} = \frac{2}{\cos 2x}$$ 1

$$= \frac{2b}{a} = \text{RHS}$$ 1

18. $$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= 3(x+y)\begin{vmatrix} 1 & x+y & x+2y \\ 1 & x & x+y \\ 1 & x+2y & x \end{vmatrix}$$ 1

$R_1 \to R_1 - R_2, R_3 \to R_3 - R_2$

$$= 3(x+y)\begin{vmatrix} 0 & y & y \\ 1 & x & x+y \\ 0 & 2y & -y \end{vmatrix}$$ 1+1

$$= -3(x+y)(-y^2 - 2y^2) = 9y^2(x+y)$$ 1

OR

Let $D = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ $\frac{1}{2}$

$CD = AB \Rightarrow \begin{bmatrix} 2x+5z & 2y+5w \\ 3x+8z & 3y+8w \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ 1+1

$2x + 5z = 3,\ 3x + 8z = 43;\ 2y + 5w = 0,\ 3y + 8w = 22.$

Solving, we get $x = -191,\ y = -110,\ z = 77,\ w = 44$ 1

$\therefore\ D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$ $\frac{1}{2}$

19. $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

$y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ 1

$u = (\sin x)^x$

$\Rightarrow\ \log u = x \log \sin x$ $\frac{1}{2}$

$\Rightarrow\ \frac{du}{dx} = (\sin x)^x\,[x \cot x + \log \sin x]$ 1

$v = \sin^{-1}\sqrt{x}$

$\Rightarrow\ \frac{dv}{dx} = \frac{1}{2\sqrt{x - x^2}}$ 1

$\therefore\ \frac{dy}{dx} = (\sin x)^x[x \cot x + \log \sin x] + \frac{1}{2\sqrt{x - x^2}}$ $\frac{1}{2}$

OR

$x^m \cdot y^n = (x + y)^{m + n}$

$\Rightarrow$ m log x + n log y = (m + n) log (x + y) 1

$$\Rightarrow \frac{m}{x} + \frac{n}{y} \cdot \frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$$ 1

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} \quad ...(i)$$ 1

$$\frac{d^2y}{dx^2} = \frac{x\frac{dy}{dx} - y}{x^2} = 0 \quad ...(ii) \text{ (using (i))}$$ 1

20.

x_i	p_i	p_ix_i	$p_i\, x_i^2$
0	2q	0	0
1	q	q	q
2	p	2p	4p
3	p	3p	9p

2

$\Sigma p_i = 1 \Rightarrow 3q + 2p = 1$ ...(1) $\frac{1}{2}$

$\Sigma p_i x_i^2 = 2\Sigma p_i x_i \Rightarrow q = 3p$ ...(2) $\frac{1}{2}$

from (I) and (2), $p = \frac{1}{11}$ 1

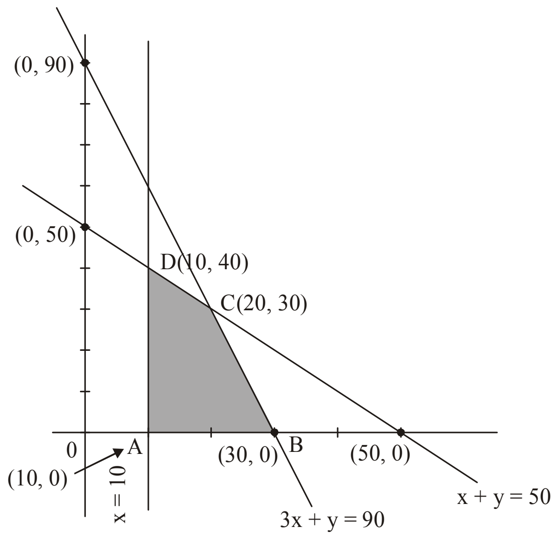
21. $\overrightarrow{AB} = \hat{i} - 3\hat{j} + \hat{k}, \overrightarrow{AC} = 3\hat{i} + 3\hat{j} - 4\hat{k}$ 1

Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$ 1

$$= \frac{1}{2} \text{ magnitude of } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$$

$$= \frac{1}{2}|9\hat{i} + 7\hat{j} + 12\hat{k}| = \frac{\sqrt{274}}{2} \text{ Sq.units}$$ 1+1

22.



Correct graph of 3 lines $1\frac{1}{2}$

Correct shade of 3 lines $1\frac{1}{2}$

$$Z|_{A(10,\ 0)} = 40$$

$$Z|_{B(30,\ 0)} = 120$$

$$Z|_{C(20,\ 30)} = 110$$

$$Z|_{D(10,\ 40)} = 80$$

Maximum value of Z = 120 at (30, 0) 1

23. $\int \frac{2x\,dx}{(x^2+1)(x^4+4)} = \int \frac{dy}{(y+1)(y^2+4)}$ [put $x^2 = y \Rightarrow 2x\,dx = dy$] $\frac{1}{2}$

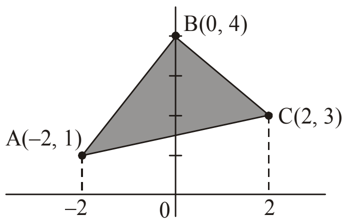
$$\frac{1}{(y+1)(y^2+4)} = \frac{1}{5(y+1)} + \frac{\frac{1}{5} - \frac{1}{5}y}{y^2+4}$$ $1\frac{1}{2}$

$$\therefore \int \frac{dy}{(y+1)(y^2+4)} = \frac{1}{5}\log|y+1| + \frac{1}{10}\tan^{-1}\frac{y}{2} - \frac{1}{10}\log(y^2+4) + C$$ $1\frac{1}{2}$

$$= \frac{1}{5}\log(x^2+1) + \frac{1}{10}\tan^{-1}\frac{x^2}{2} - \frac{1}{10}\log(x^4+4) + C$$ $\frac{1}{2}$

SECTION D

24.



Equation of AB: $y = \frac{3}{2}x + 4$ Correct Figure: 1

Equation of BC; $y = 4 - \frac{x}{2}$

Equation of AC; $y = \frac{1}{2}x + 2$ $1\frac{1}{2}$

Required area $= \int_{-2}^{0}\left(\frac{3}{2}x + 4\right)dx + \int_{0}^{2}\left(4 - \frac{x}{2}\right)dx - \int_{-2}^{2}\left(\frac{1}{2}x + 2\right)dx$ 1

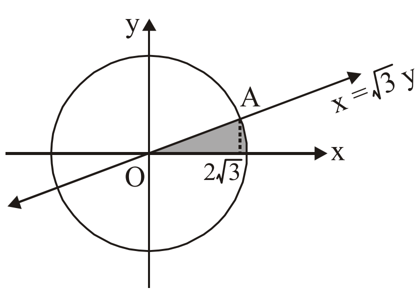
$= \left[\frac{3x^2}{4} + 4x\right]_{-2}^{0} + \left[4x - \frac{x^2}{4}\right]_{0}^{2} - \left[\frac{x^2}{4} + 2x\right]_{-2}^{2}$ $1\frac{1}{2}$

$= 5 + 7 - 8$ 1

$= 4$ sq.units

OR

Note: In this problem, two regions are possible instead of a unique one, so full 6 marks may be given for finding the area of either region correctly.



Correct Figure 1

x-coordinate of points of intersection is $x = \pm 2\sqrt{3}$ 1

Required area

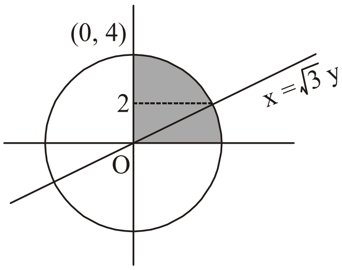
$= \int_{0}^{2\sqrt{3}} \frac{x}{\sqrt{3}} \cdot dx + \int_{2\sqrt{3}}^{4} \sqrt{4^2 - x^2}\, dx$ $1\frac{1}{2}$

$= \left[\frac{x^2}{2\sqrt{3}}\right]_{0}^{2\sqrt{3}} + \left[\frac{x\sqrt{16 - x^2}}{2} + 8\sin^{-1}\frac{x}{4}\right]_{2\sqrt{3}}^{4}$ $1\frac{1}{2}$

$= 2\sqrt{3} + 8\left(\frac{\pi}{2} - \frac{\pi}{3}\right) - 2\sqrt{3}$

$= \frac{4\pi}{3}$ sq.units 1

Alternate Solution



Correct figure 1

y-coordinate of point of intersection is y = 2 1

Required Area

$$= \sqrt{3}\int_0^2 y\,dx + \int_2^4 \sqrt{(4)^2 - y^2}\,dy \qquad 1\frac{1}{2}$$

$$= \sqrt{3}\left[\frac{y^2}{2}\right]_0^2 + \left[\frac{y\sqrt{16-y^2}}{2} + 8\sin^{-1}\frac{y}{4}\right]_2^4 \qquad 1\frac{1}{2}$$

$$= 2\sqrt{3} + 4\pi - 2\sqrt{3} - \frac{4\pi}{3}$$

$$= \frac{8\pi}{3}\text{ sq.units} \qquad 1$$

25. The given equation can be written as

$$\frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x} \qquad 1$$

$$\text{I.F.} = e^{\int \frac{1}{x}dx} = e^{\log x} = x \qquad 1$$

$\therefore$ Solution is

$$y \cdot x = \int (x\cos x + \sin x)dx + c \qquad 1$$

$$\Rightarrow yx = x \sin x + c \qquad 1$$

$$\text{or } y = \sin x + \frac{c}{x}$$

when $x = \frac{\pi}{2}, y = 1$, we get $c = 0$ 1

Required solution is $y = \sin x$ 1

26. Equation of family of planes

$\vec{r}\cdot[(2\hat{i}-3\hat{j}+4\hat{k})+\lambda(\hat{i}-\hat{j})] = 1-4\lambda$ 1

$\Rightarrow \vec{r}\cdot[(2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}] = 1-4\lambda$ ...(i) 1

plane (i) is perpendicular to

$\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+8 = 0$

$2(2+\lambda)-1(-3-\lambda)+1(4) = 0 \Rightarrow \lambda = -\frac{11}{3}$ 1+1

Substituting $\lambda = -\frac{11}{3}$ in equation (i), we get

$\vec{r}\cdot\left(-\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+4\hat{k}\right) = \frac{47}{3}$

$\Rightarrow \boxed{\vec{r}\cdot(-5\hat{i}+2\hat{j}+12\hat{k}) = 47}$ (vector equation)

or $\boxed{-5x+2y+12z-47 = 0}$ (cartesian equation)

} (ii) 1

Line $\frac{x-1}{1} = \frac{y-2}{1/2} = \frac{z-4}{1/3}$ lies on the plane

$\because$ (i) Point P(1, 2, 4) satisfies equation (ii) $\frac{1}{2}$

and $a_1a_2 + b_1b_2 + c_1c_2 = -5 + 1 + 4 = 0$ $\frac{1}{2}$

$\Rightarrow$ Line is perpendicular to the normal of plane $\therefore$ Plane contains the given line

OR

Equation of line L_1 passing through (1, 2, –4) is

$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$ 1

L_2: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$

L_3: $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

$\because\ L_1 \perp L_2 \Rightarrow 3a - 16b + 7c = 0$ 1

$L_1 \perp L_3 \Rightarrow 3a + 8b - 5c = 0$ 1

Solving, we get

$\frac{a}{24} = \frac{b}{36} = \frac{c}{72} \Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6}$ 1

$\therefore$ Required cartesian equation of line

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ 1

Vector equation

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ 1

27. Clearly $f^{-1}(y) = g(y): [-5, \infty) \to R_+$ and,

$fog(y) = f\left(\frac{\sqrt{y+6}-1}{3}\right) = 9\left(\frac{\sqrt{y+6}-1}{3}\right)^2 + 6\left(\frac{\sqrt{y+6}-1}{3}\right) - 5 = y$ 2

and $(gof)(x) = g(9x^2 + 6x - 5) = \frac{\sqrt{9x^2+6x+1}-1}{3} = x$ 2

$\therefore\ g = f^{-1}$ 1

(i) $f^{-1}(10) = \frac{\sqrt{16}-1}{3} = 1$ $\frac{1}{2}$

(ii) $f^{-1}(y) = \frac{4}{3} \Rightarrow y = 19$ $\frac{1}{2}$

OR

Note: Some short comings have been observed in this question which makes the question unsolvable.

So, 6 marks may be given for a genuine attempt.

$a * b = a - b + ab \ \forall a, b \in A = Q - [1]$

$b * a = b - a + ba$

$(a * b) \neq b * a \Rightarrow *$ is not commutative. $1\frac{1}{2}$

$(a * b) * c = (a - b + ab) * c$

$= a - b - c + ab + ac - bc + abc$

$a * (b * c) = a * (b - c + bc)$

$= a - b + c + ab - ac - bc + abc$

$(a * b) * c \neq a * (b * c)$ $1\frac{1}{2}$

$\Rightarrow$ * is not associative.

Existence of identity

$a * e = a - e + ae = a$ $\qquad$ $e * a = e - a + ea = a$

$\Rightarrow e(a - 1) = 0$ $\qquad$ $\Rightarrow e(1 + a) = 2a$

$\Rightarrow e = 0$ $\qquad$ $\Rightarrow e = \frac{2a}{1+a}$ $1\frac{1}{2}$

$\because$ e is not unique

$\therefore$ No idendity element exists.

$a * b = e = b * a$

$\therefore$ No identity element exists. $1\frac{1}{2}$

$\Rightarrow$ Inverse element does not exist.

28. Let side of square base be x cm and height of the box be y cm.

$x^2y = 1024 \Rightarrow y = \frac{1024}{x^2}$ 1

cost of the box. $C = 5 \times 2x^2 + 2.5 \times 4xy$

$= 10x^2 + \frac{10240}{x}$ 1

$\Rightarrow \frac{dC}{dx} = 20x - \frac{10240}{x^2}$ 1

$\frac{dC}{dx} = 0 \Rightarrow x = 8$ 1

$$\frac{d^2C}{dx^2} = 20 + \frac{20480}{x^3}$$

$$\left.\frac{d^2C}{dx^2}\right|_{x=8} > 0 \Rightarrow C \text{ is minimum at } x = 8 \text{ cm}$$ 1

∴ Minimum cost C = ₹ 1920

29. Here |A| = 1200 1

Co-factors are

$$\left.\begin{array}{lll} C_{11} = 75, & C_{21} = 150, = & C_{31} = 75 \\ C_{12} = 110, & C_{22} = -100, & C_{32} = 30 \\ C_{13} = 72, & C_{23} = 0, & C_{33} = -24 \end{array}\right\}$$ 2

$$A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$ $\frac{1}{2}$

Given equation in matrix from is:

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

$\Rightarrow$ A X = B

$\Rightarrow X = A^{-1}B$ 1

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{3} \\ \frac{1}{5} \end{bmatrix}$$ 1

$\Rightarrow x = 2,\ y = -3,\ z = 5$ $\frac{1}{2}$

QUESTION PAPER CODE 65/1/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\cos^2 90° + \cos^2 60° + \cos^2 \gamma = 1$ $\frac{1}{2}$

$\cos\gamma = \pm\frac{\sqrt{3}}{2}, \gamma = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\frac{1}{2}$

2. $\int_2^3 3^x dx = \left[\frac{3^x}{\log 3}\right]_2^3 = \frac{18}{\log 3}$ $\frac{1}{2}+\frac{1}{2}$

3. $\lim_{x\to 0_-} f(x) = \lim_{x\to 0_-} \frac{kx}{|x|} = -k$ $\frac{1}{2}$

$k = -3$ $\frac{1}{2}$

4. $|A^{-1}| = \frac{1}{|A|} \Rightarrow k = -1$ 1

SECTION B

5. $P(E \cap F') = P(E) - P(E \cap F)$ 1

$= P(E) - P(E)\cdot P(F)$ $\frac{1}{2}$

$= P(E)[1 - P(F)]$

$= P(E)P(F')$ $\frac{1}{2}$

$\Rightarrow$ E and F' are independent events.

6. Let x necklaces and y bracelets are manufactured

L.P.P. is

Maximize profit, P = 100x + 300y $\frac{1}{2}$

subject to constraints

$x + y \leq 24$

$\frac{1}{2}x + y \leq 16$ or $x + 2y \leq 32$ $\quad \frac{1}{2} \times 3 = 1\frac{1}{2}$

$x, y, \geq 1$

7. $\int \frac{dx}{x^2 + 4x + 8} = \int \frac{dx}{(x+2)^2 + (2)^2}$ $\quad 1$

$= \frac{1}{2}\tan^{-1}\frac{x+2}{2} + C$ $\quad 1$

8. Equation of given line is $\frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35}$ $\quad \frac{1}{2}$

Its DR's $\left\langle \frac{1}{5}, -\frac{1}{7}, \frac{1}{35} \right\rangle$ or $\langle 7, -5, 1 \rangle$ $\quad \frac{1}{2}$

Equation of required line

$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$ $\quad 1$

9. $f(x) = 4x^3 - 18x^2 + 27x - 7$

$f'(x) = 12x^2 - 36x + 27$ $\quad \frac{1}{2}$

$= 3(2x - 3)^2 \geq 0 \ \forall x \in R$ $\quad 1$

$\Rightarrow$ f(x) is increasing on R $\quad \frac{1}{2}$

10. $V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{3}{4\pi r^2}$ $\quad 1$

$S = 4\pi r^2$

$\Rightarrow \frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$ $\quad \frac{1}{2}$

$\Rightarrow \left.\frac{dS}{dt}\right|_{r=2} = 3 cm^2/s$ $\quad \frac{1}{2}$

11. Let $A = [a_{ij}]_{n \times n}$ be skew symmetric matrix

A is skew symmetric

$\therefore \quad A = -A'$ 1

$\Rightarrow \quad a_{ij} = -a_{ji} \ \forall \ i, j$

For diagonal elements i = j,

$\Rightarrow \quad 2a_{ii} = 0$

$\Rightarrow \quad a_{ii} = 0 \Rightarrow$ diagonal elements are zero. 1

12. $y = \sin^{-1}\left(6x\sqrt{1-9x^2}\right), -\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$

put $3x = \sin\theta = \Rightarrow \theta = \sin^{-1} 3x$ $\frac{1}{2}$

$y = \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\sin^{-1} 3x$ $\frac{1}{2}$

A =

$\therefore \quad \frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$ 1

SECTION C

13. $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = c_2 - c_3$ 1

(a) $c_1 = 1, \ c_2 = 2$

$[\vec{a}\ \vec{b}\ \vec{c}] = 2 - c_3$ 1

$\because \ \vec{a}, \vec{b}, \vec{c}$ are coplanar $[\vec{a}\ \vec{b}\ \vec{c}] = 0 \Rightarrow c_3 = 2$ 1

(b) $c_2 = -1, \ c_3 = 1$

$[\vec{a}\ \vec{b}\ \vec{c}] = c_2 - c_3 = -2 \neq 0$

$\Rightarrow$ No value of c_1 can make $\vec{a}, \vec{b}, \vec{c}$ coplanar 1

14. $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}\cdot\vec{b} = 0 = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a}$ ...(i) 1

Let α, β and γ be the angles made by $(\vec{a}+\vec{b}+\vec{c})$ with $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively

$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a} = |\vec{a}+\vec{b}+\vec{c}|\,|\vec{a}|\cos\alpha$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$$

Similarly, $\beta = \cos^{-1}\left(\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$ and $\gamma = \cos^{-1}\left(\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$ 1

using (i), we get $\alpha = \beta = \gamma$ $\frac{1}{2}$

Now $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$ 1

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 3|\vec{a}|^2$ (using (i))

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}|\vec{a}|$

$A = A'$

$\therefore \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \beta = \gamma$ $\frac{1}{2}$

15.

x	P(x)
0	p
1	p
2	k
3	k

$\Sigma p(x) = 1 \Rightarrow 2p + 2k = 1 \Rightarrow k = \frac{1}{2} - p$ 1

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	p	0	0
1	p	p	p
2	$\frac{1}{2} - p$	$1 - 2p$	$2 - 4p$
3	$\frac{1}{2} - p$	$\frac{3}{2} - 3p$	$\frac{9}{2} - 9p$
		$\frac{5}{2} - 4p$	$\frac{13}{2} - 12p$

2

As per problem, $\Sigma\ p_i x_i^2 = 2\Sigma\ p_i x_i$

$\Rightarrow \quad p = \frac{3}{8}$ 1

16. Let H_1 be the event that 6 appears on throwing a die

H_2 be the event that 6 does not appear on throwing a die

E be the event that he reports it is six 1

$P(H_1) = \frac{1}{6}, P(H_2) = 1 - \frac{1}{6} = \frac{5}{6}$

$P(E/H_1) = \frac{4}{5}, P(E/H_2) = \frac{1}{5}$ 1

$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) P(E/H_2)}$ $\frac{1}{2}$

$= \frac{4}{9}$ $\frac{1}{2}$

Relevant value: Yes, Truthfulness leads to more respect in society. 1

17. $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$= 3(x+y)\begin{vmatrix} 1 & x+y & x+2y \\ 1 & x & x+y \\ 1 & x+2y & x \end{vmatrix}$ 1

$R_1 \to R_1 - R_2, R_3 \to R_3 - R_2$

$= 3(x+y)\begin{vmatrix} 0 & y & y \\ 1 & x & x+y \\ 0 & 2y & -y \end{vmatrix}$ 1+1

$= -3(x + y)(-y^2 - 2y^2) = 9y^2(x + y)$ 1

OR

Let $D = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ $\frac{1}{2}$

$CD = AB \Rightarrow \begin{bmatrix} 2x+5z & 2y+5w \\ 3x+8z & 3y+8w \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ 1+1

$2x + 5z = 3,\ 3x + 8z = 43;\ 2y + 5w = 0,\ 3y + 8w = 22$

Solving, we get $x = -191,\ y = -110,\ z = 77,\ w = 44$ 1

$\therefore\ D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$ $\frac{1}{2}$

18. $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

$y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ 1

$u = (\sin x)^x$

$\Rightarrow \log u = x \log \sin x$ $\frac{1}{2}$

$\Rightarrow \frac{du}{dx} = (\sin x)^x [x \cot x + \log \sin x]$ 1

$v = \sin^{-1}\sqrt{x}$

$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{x - x^2}}$ 1

$\therefore \frac{dy}{dx} = (\sin x)^x [x \cot x + \log \sin x] + \frac{1}{2\sqrt{x - x^2}}$ $\frac{1}{2}$

OR

$x^m \cdot y^n = (x + y)^{m + n}$

$\Rightarrow m \log x + n \log y = (m + n) \log (x + y)$ 1

$\Rightarrow \frac{m}{x} + \frac{n}{y} \cdot \frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$ 1

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} \qquad ...(i)$$ 1

$$\frac{d^2y}{dx^2} = \frac{x\frac{dy}{dx} - y}{x^2} = 0 \qquad ...(ii) \ \text{(using (i))}$$ 1

19. $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

$= \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx$ 1

$\Rightarrow 2I = \pi\int_0^{\pi} \frac{\sin x \, dx}{1+\cos^2 x}$

Put $\cos x = t$ and $-\sin x \, dx = dt$ 1

$= -\pi\int_1^{-1} \frac{dt}{1+t^2}$

$= \pi[\tan^{-1} t]_{-1}^{1} = \frac{\pi^2}{2}$ $1\frac{1}{2}$

$\Rightarrow I = \frac{\pi^2}{4}$ $\frac{1}{2}$

OR

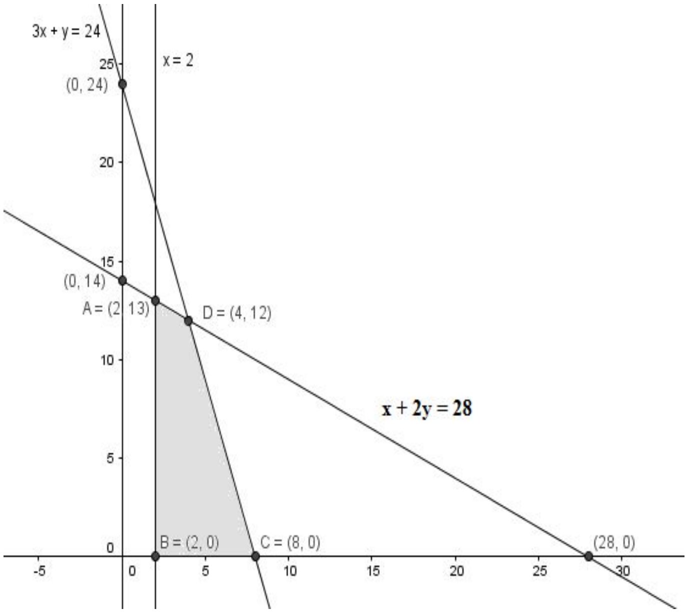
$I = \int_0^{3/2} | x \sin \pi x | dx$

$= \int_0^{1} x \sin \pi x \cdot dx - \int_1^{3/2} x \sin \pi x \, dx$ $1\frac{1}{2}$

$= \left[-x\frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\right]_0^1 - \left[-\frac{x \cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\right]_1^{3/2}$ $1\frac{1}{2}$

$= \frac{2}{\pi} + \frac{1}{\pi^2}$ 1

20.



Correct graph of 3 lines $1\frac{1}{2}$

Correct shade of 3 lines $1\frac{1}{2}$

$Z = 20x + 10y$

$Z|_{A(2,\ 13)} = 170$

$Z|_{B(2,\ 0)} = 40$

$Z|_{D(4,\ 12)} = 200$

$Z|_{C(8,\ 0)} = 160$

Maximum value of Z = 200 at x = 4, y = 12 1

21. $x^2 - y^2 = cx \Rightarrow \dfrac{x^2 - y^2}{x} = c$ 1

$$\Rightarrow \frac{x\left(2x - 2y\dfrac{dy}{dx}\right) - (x^2 - y^2)}{x^2} = 0$$ 1

$$\Rightarrow 2x^2 - 2xy\frac{dy}{dx} - x^2 + y^2 = 0$$ 1

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$ 1

Hence proved.

22. $\displaystyle\int \frac{(3\sin x - 2)\cos x}{13 - \cos^2 x - 7\sin x}dx = \int \frac{(3\sin x - 2)\cos x}{\sin^2 x - 7\sin x + 12}dx$ 1

put sin x = y, cos x dx = dy

$$= \int \frac{(3y - 2)dy}{y^2 - 7y + 12}$$ $\frac{1}{2}$

$$= \int \frac{(3y - 2)dy}{(y - 4)(y - 3)}$$

$$= \int \left(\frac{10}{y-4} - \frac{7}{y-3} \right) dy$$ 1

$$= 10 \log | y - 4 | - 7 \log | y - 3 | + C$$ 1

$$= 10 \log | \sin x - 4 | - 7 \log | \sin x - 3 | + C$$ $\frac{1}{2}$

23. $\cos (\tan^{-1} x) = \sin\left(\cot^{-1} \frac{3}{4} \right)$

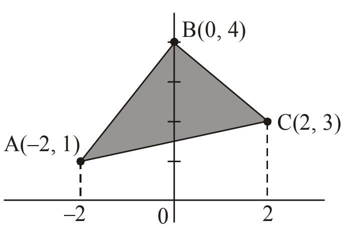
$$\Rightarrow \cos\left(\cos^{-1} \frac{1}{\sqrt{1+x^2}} \right) = \sin\left(\sin^{-1} \frac{4}{5} \right)$$ 1+1

$$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$$ 1

$$\Rightarrow x = \pm \frac{3}{4}$$ 1

SECTION D

24. Equation of AB: $y = \frac{3}{2}x + 4$ Correct Figure: 1



Equation of BC; $y = 4 - \frac{x}{2}$

Equation of AC; $y = \frac{1}{2}x + 2$ $1\frac{1}{2}$

Required area $= \int_{-2}^{0} \left(\frac{3}{2}x + 4 \right) dx + \int_{0}^{2} \left(4 - \frac{x}{2} \right) dx - \int_{-2}^{2} \left(\frac{1}{2}x + 2 \right) dx$ 1

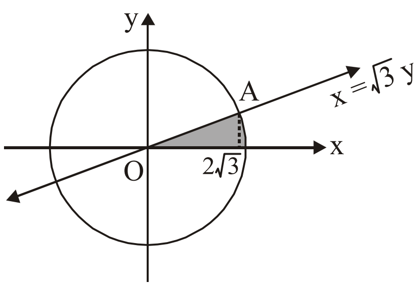
$$= \left[\frac{3x^2}{4} + 4x \right]_{-2}^{0} + \left[4x - \frac{x^2}{4} \right]_{0}^{2} - \left[\frac{x^2}{4} + 2x \right]_{-2}^{2}$$ $1\frac{1}{2}$

$= 5 + 7 - 8$ 1

$= 4$ sq.units

OR

Note: In this problem, two regions are possible instead of a unique, so full 6 marks may be given for finding the area of either region correctly.



Correct Figure 1

x-coordinate of points of intersection is $x = \pm 2\sqrt{3}$ 1

Required area

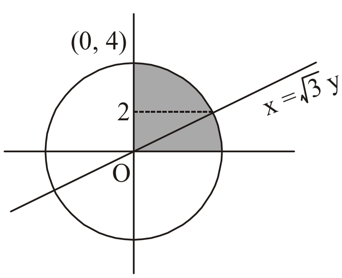
$$= \int_0^{2\sqrt{3}} \frac{x}{\sqrt{3}} \cdot dx + \int_{2\sqrt{3}}^{4} \sqrt{4^2 - x^2}\, dx \qquad 1\frac{1}{2}$$

$$= \left[\frac{x^2}{2\sqrt{3}}\right]_0^{2\sqrt{3}} + \left[\frac{x\sqrt{16-x^2}}{2} + 8\sin^{-1}\frac{x}{4}\right]_{2\sqrt{3}}^{4} \qquad 1\frac{1}{2}$$

$$= 2\sqrt{3} + 8\left(\frac{\pi}{2} - \frac{\pi}{3}\right) - 2\sqrt{3}$$

$$= \frac{4\pi}{3}\text{ sq.units} \qquad 1$$

Other Possible Solution



Correct figure 1

y-co-ordinate of point of intersection is y = 2 1

Required Area

$$= \sqrt{3}\int_0^2 y\, dx + \int_2^4 \sqrt{(4)^2 - y^2}\, dy \qquad 1\frac{1}{2}$$

$$= \sqrt{3}\left[\frac{y^2}{2}\right]_0^2 + \left[\frac{y\sqrt{16-y^2}}{2} + 8\sin^{-1}\frac{y}{4}\right]_2^4 \qquad 1\frac{1}{2}$$

$$= 2\sqrt{3} + 4\pi - 2\sqrt{3} - \frac{4\pi}{3}$$

$$= \frac{8\pi}{3}\text{ sq.units} \qquad 1$$

25. Equation of family of planes

$$\vec{r}\cdot[(2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(\hat{i} - \hat{j})] = 1 - 4\lambda \qquad 1$$

$\Rightarrow \vec{r}\cdot[(2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}] = 1-4\lambda$ ...(i) 1

plane (i) is perpendicular to

$\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+8 = 0$

$2(2+\lambda)-1(-3-\lambda)+1(4) = 0 \Rightarrow \lambda = -\frac{11}{3}$ 1+1

Substituting $\lambda = -\frac{11}{3}$ in equation (i), we get

$\vec{r}\cdot\left(-\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+4\hat{k}\right) = \frac{47}{3}$

$\Rightarrow \boxed{\vec{r}\cdot(-5\hat{i}+2\hat{j}+12\hat{k}) = 47}$ (vector equation)

or $\boxed{-5x+2y+12z-47 = 0}$ (cartesian equation)

} (ii) 1

Line $\frac{x-1}{1} = \frac{y-2}{1/2} = \frac{z-4}{1/3}$ lies on the plane

$\because$ (i) Point P(1, 2, 4) satisfies equation (ii) $\frac{1}{2}$

and $a_1a_2+b_1b_2+c_1c_2 = -5+1+4 = 0$ $\frac{1}{2}$

$\Rightarrow$ Line is perpendicular to the normal of plane $\therefore$ Plane contains the given line

OR

Equation of line L_1 passing through (1, 2, –4) is

$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$ 1

L_2: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$

L_3: $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

$\because L_1 \perp L_2 \Rightarrow 3a-16b+7c = 0$ 1

$L_1 \perp L_3 \Rightarrow 3a+8b-5c = 0$ 1

Solving, we get

$\frac{a}{24} = \frac{b}{36} = \frac{c}{72} \Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6}$ 1

$\therefore$ Required cartesian equation of line

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ 1

Vector equation

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ 1

26. Clearly $f^{-1}(y) = g(y)$: $[-5, \infty) \to R_+$ and,

$fog(y) = f\left(\frac{\sqrt{y+6}-1}{3}\right) = 9\left(\frac{\sqrt{y+6}-1}{3}\right)^2 + 6\left(\frac{\sqrt{y+6}-1}{3}\right) - 5 = y$ 2

and $(gof)(x) = g(9x^2 + 6x - 5) = \frac{\sqrt{9x^2 + 6x + 1} - 1}{3} = x$ 2

$\therefore\ g = f^{-1}$ 1

(i) $f^{-1}(10) = \frac{\sqrt{16}-1}{3} = 1$ $\frac{1}{2}$

(ii) $f^{-1}(y) = \frac{4}{3} \Rightarrow y = 19$ $\frac{1}{2}$

OR

Note: Some short comings have been observed in this question which makes the question unsolvable.

So, 6 marks may be given for a genuine attempt.

$a * b = a - b + ab \ \forall\ a, b \in A = Q - [1]$

$b * a = b - a + ba$

$(a * b) \neq b * a \Rightarrow *$ is not commutative. $1\frac{1}{2}$

$(a * b) * c = (a - b + ab) * c$

$= a - b - c + ab + ac - bc + abc$

$a * (b * c) = a * (b - c + bc)$

$= a - b + c + ab - ac - bc + abc$

$(a * b) * c \neq a * (b * c)$ $1\frac{1}{2}$

$\Rightarrow$ * is not associative.

Existence of identity

$a * e = a - e + ae = a$ $\qquad$ $e * a = e - a + ea = a$

$\Rightarrow e\,(a - 1) = 0$ $\qquad$ $\Rightarrow e(1 + a) = 2a$

$\Rightarrow e = 0$ $\qquad$ $\Rightarrow e = \frac{2a}{1+a}$ $1\frac{1}{2}$

$\because$ e is not unique

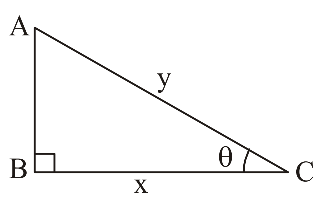
$\therefore$ No idendity element exists.

$a * b = e = b * a$

$\therefore$ No identity element exists. $1\frac{1}{2}$

$\Rightarrow$ Inverse element does not exist.

27.



Given x + y = k 1

$$\text{Area of } \Delta = \frac{1}{2} x \sqrt{y^2 - x^2}$$

$$\text{Let } Z = \frac{1}{4} x^2 (y^2 - x^2)$$

$$= \frac{1}{4} x^2 [(k - x)^2 - x^2]$$

$$= \frac{1}{4} [k^2 x^2 - 2kx^3] \qquad 1$$

$$\frac{dz}{dx} = \frac{1}{4}[2k^2 x - 6kx^2] = 0 \Rightarrow k - 3x = 0 \Rightarrow x = \frac{k}{3} \qquad 1\frac{1}{2}$$

$\Rightarrow x + y - 3x = 0$ or $y = 2x$

$$\frac{d^2 z}{dx^2} = \frac{1}{4}[2k^2 - 12kx] \qquad 1$$

$$\left.\frac{d^2z}{dx^2}\right|_{x=\frac{k}{3}} = \frac{1}{4}[2k^2 - 4k^2] = -\frac{k^2}{2} < 0$$

$\therefore$ Area will be maximum for 2x = y — 1

but $\frac{x}{y} = \cos\theta \Rightarrow \cos\theta = \frac{x}{2x} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ — $\frac{1}{2}$

28. $|A| = -16$ — 1

Co-factors are

$C_{11} = -4,\ C_{21} = 4,\ C_{31} = 4$

$C_{12} = -5,\ C_{22} = 1,\ C_{32} = -3$ — 2

$C_{13} = 7,\ C_{23} = -11,\ C_{33} = 1$

$$A^{-1} = \frac{-1}{16}\begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}$$ $\frac{1}{2}$

given equations can be written as

$A'X = C \Rightarrow X = \left(A^{-1}\right)' C$ — $\frac{1}{2}+1$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{16}\begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}\begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$ $\frac{1}{2}$

$\Rightarrow x = 1,\ y = 2,\ z = -3$ — $\frac{1}{2}$

29. Given equation can be written as

$\Rightarrow \frac{dy}{dx} + (\cot x)\,y = 2x + x^2 \cot x$ — 1

I.F. $= e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$ — 1

Solution is, $y \times \sin x = \int (2x \sin x + x^2 \cos x)\, dx$ — 1

$\Rightarrow \quad y \sin x = x^2 \sin x + C$ $\qquad 1\frac{1}{2}$

when $x = \frac{\pi}{2}$, $y = 0$, we get $c = \frac{-\pi^2}{4}$ $\qquad 1$

$\therefore$ Required solution is, $\quad 4y \sin x = 4x^2 \sin x - \pi^2$ $\qquad \frac{1}{2}$

or, $\quad y = x^2 - \pi^2/4 \text{ cosec } x$