NUMBERS

Natural numbers	**1 , 2 , 3 , 4 , 5 , 6 , . . .**
Even numbers	**2 , 4 , 6 , 8, 10 , 12 , . . .**
Odd numbers	**1 , 3 , 5 , 7 , 9 , 11 , . . .**

Prime numbers

Numbers (other than 1) which are not divisible by the numbers other than 1 and the number itself are called prime numbers

Example : 2 ,3 , 5 , 7 , 11 , 13 , 17 , 19 , 23 , 29 , 31 , 37 , . , . , . , .

Fractions

Fractions are the number of parts .

Example : $\frac{1}{2}$, $\frac{3}{5}$, $\frac{5}{4}$, $\frac{10}{7}$, . . .

Negative numbers

Numbers written with a minus sign are called negative numbers .

Positive numbers

Numbers which are not negative are called positive numbers .

NOTE : 0 is neither positive nor negative .

NUMBERS AND ALGEBRA

Algebra

Mathematical shorthand of writing number related facts using letters is called *algebra*.

Concept 1

Multiplying two numbers by a number separately and adding give the same result as multiplying their sum by the number .

That is , $xz + yz = (x + y)z$, for all numbers x, y, z

Activity

Try these problems

a) $36 \times 28 + 64 \times 28$ **b)** $125 \times 436 + 875 \times 436$

Answer

a) $36 \times 28 + 64 \times 28 = (36 + 64)28 = 100 \times 28 = 2800$

b) $125 \times 436 + 875 \times 436 = (125 + 875)436 = 1000 \times 436 = 436000$

More activity

Try these problems

a) $36 \times 18 + 14 \times 18$ **b)** $185 \times 122 + 215 \times 122$

Concept 2

Multiplying two numbers by a number separately and subtracting give the same result as multiplying their difference by the number .

That is , $xz - yz = (x - y)z$, for all numbers x, y, z

Activity

Try these problems

a) $58 \times 76 - 48 \times 76$ **b)** $239 \times 397 - 139 \times 397$

Answer

a) $58 \times 76 - 48 \times 76 = (58 - 48)76 = 10 \times 76 = 760$

b) $239 \times 397 - 139 \times 397 = (239 - 139)397 = 100 \times 397 = 39700$

More activity

Try these problems

a) $96 \times 39 - 76 \times 39$ **b)** $316 \times 125 - 116 \times 125$

Concept 3

Algebraic expressions

General form indicating arithmetical operations using letters are called algebraic expressions .

NOTE :

When we use algebra to state general properties of numbers ,we should also specify the type of numbers indicated by the letter .

In algebra , natural numbers are usually denoted by n . (any letter can be used)

Activity

Adding 10 repeatedly to 1 .

a) Find the numbers .

b) Find the algebraic expression of such numbers .

Answer

a) $11 , 21 , 31 , 41 , 51 , \ldots$

b) Algebraic expression = $1 + 10n$

More activity

a) Find the algebraic expression for the numbers got by adding 5 repeatedly to 1 .

b) Find the algebraic expression for the numbers got by adding 5 repeatedly to 4 .

c) Add the numbers in the same position of the first two patterns . Why do we get only multiples of 5 ?

Agebraic forms

1) Numbers of the form $2n$ are even .

2) Numbers of the form $2n - 1$ are odd .

3) Numbers of the form $5n$ are the multiples of 5 .

4) Numbers of the form $10n + 1$ are the numbers got by adding 1 to the multiples of 10 .

4) Numbers of the form $3n - 2$ are the numbers which give a remainder 1 when divided by 3

That is ,

1) Algebraic form of even numbers = $2n$

2) Algebraic form of odd numbers = $2n - 1$

3) Algebraic form of multiples of 5 = $5n$

4) Algebraic form of the numbers got by adding 1 to the multiples of 10 = $10n+1$

5) Algebraic form of the numbers which give a remainder 1 when divided by 3 = $3n - 2$

NEGATIVE NUMBERS – OPERATIONS

Concept 1

For positive numbers , the larger subtracted from the smaller is the negative of the smaller subtracted from the larger .

For any positive numbers x , y if $x < y$ then

$$x - y = -(y - x)$$

Activity

Try these problems

a) 4 – 5 **b)** 12 – 20 **c)** 521 – 743

Answer

a) 4 – 5 = –1

b) 12 – 20 = –8

c) 521 – 743 = –222

More activity

Try these problems .

a) 7 – 9 **b)** 37 – 95 **c)** 135 – 627

Concept 2

Adding to the negative of a positive number , a second positive number means subtracting the first number from the second number .

For any two positive numbers x and y

$$-x + y = y - x$$

Activity

Try these problems .

a) $-2 + 8$ **b)** $-95 + 20$ **c)** $-675 + 520$

Answer

a) $-2 + 8 = 6$

b) $-95 + 20 = -75$

c) $-675 + 520 = -155$

More activity

Try these problems .

a) $-9 + 6$ **b)** $-45 + 80$ **c)** $-326 + 792$

Concept 3

Subtracting a positive number from the negative of a positive number , we get the negative of the sum of these positive numbers .

For any positive numbers x and y

$$-x - y = -(x + y)$$

Activity

Try these problems .

a) $-8 - 5$ **b)** $-62 - 38$ **c)** $-372 - 251$

Answer

a) $-8 - 5 = -13$

b) $-62 - 38 = -100$

c) $-372 - 251 = -623$

More activity

Try these problems .

a) $-7 - 9$ **b)** $-18 - 73$ **c)** $-267 - 679$

Concept 4

Adding the negative of a positive number means subtracting that positive number

Activity

Try these problems .

a) $5 + (-3)$ **b)** $-15 + (-32)$ **c)** $715 + (-936)$

Answer

a) $5 + (-3) = 5 - 3 = 2$

b) $-15 + (-32) =- 15 - 32 = -47$

c) $715 + (-936) = 715 - 936 = -221$

More activity

Try these problems .

a) $4 + (-9)$ **b)** $-75 + (-19)$ **c)** $621 + (-384)$

Concept 5

Subtracting the negative of a positive number means adding that positive number

Activity

Try these problems .

a) $8 - (-6)$ **b)** $-63 - (-15)$ **c)** $-531 - (-856)$

Answer

a) $8 - (-6) = 8 + 6 = 14$

b) $-63 - (-15) = -63 + 15 = -48$

c) $-531 - (-856) = -531 + 856 = 325$

More activity

Try these problems .

a) $1 - (-9)$ **b)** $-78 - (-56)$ **c)** $-267 - (-598)$

Concept 6

The negative of the negative of a number is that number itself

That is , $-(-x) = x$, for any number x

,

Concept 7

The product of a positive number and the negative of a positive number means , the negative of the product of these positive numbers .

For any positive numbers x and y

$$-(x)\,y = x\,(-y) = -\,x\,y$$

Activity

Try these problems .

a) $7 \times (-5)$ **b)** -15×12 **c)** $125 \times (-426)$

Answer

a) $7 \times (-5) = -35$

b) $-15 \times 12 = -180$

c) $125 \times (-426) = -53250$

More activity

Try these problems .

a) $(-8) \times 9$ **b)** $25 \times (-46)$ **c)** $235 \times (-124)$

Concept 8

The product of the negatives of two positive numbers means the product of these positive numbers .

For any two positive numbers x , y

$$(-x)(-y) = xy$$

Activity

Try these problems .

a) $(-4)\times(-7)$ **b)** $(-36)\times(-15)$ **c)** $(-345)\times(-152)$

Answer

a) $(-4)\times(-7) = 28$

b) $(-36)\times(-15) = 540$

c) $(-345)\times(-152) = 52440$

More activity

Try these problems .

a) $(-6)\times(-9)$ **b)** $(-75)\times(-28)$ **c)** $(-495)\times(-364)$

NEGATIVE DIVISION

As in the case of positive numbers , division is the inverse of multiplication for negative numbers .

NOTE :

In algebra ,we usually write $x \div y$ **as** $\frac{x}{y}$ **.So , in the equation** $z = \frac{x}{y}$

$x = -6$, $y = 2$ **gives** $z = \frac{-6}{2} = -3$

$x = 6$, $y = -2$ **gives** $z = \frac{6}{-2} = -3$

$x = -6$, $y = -2$ **gives** $z = \frac{-6}{-2} = 3$

More activity

In the equation $z = \frac{x}{y}$ **, take x as the** x , y **as the numbers given below and calculate the number** z **.**

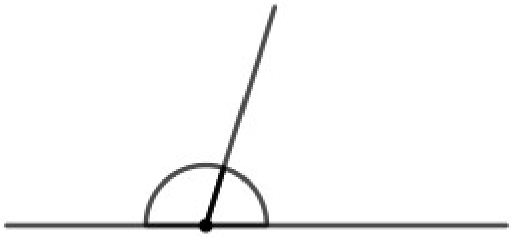
a) $x = -20$, $y = 5$

b) $x = 48$, $y = -6$

c) $x = -63$, $y = -9$

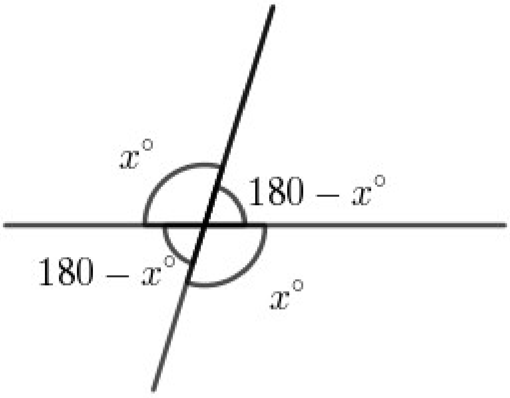
PARALLEL LINES

Linear pair



When two lines meet , the sum of the angles on either side is 180^0 .Two such angles , made by two lines meeting is called a linear pair .

Opposite angles



Of the four angles formed by two lines crossing each other , the opposite angles are equal

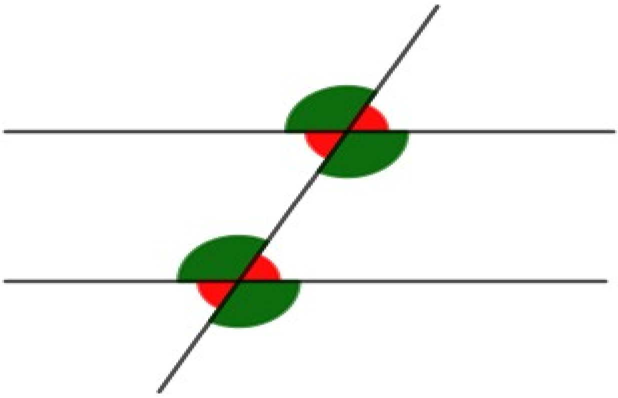
Parallel lines

Lines which are at the same distance everywhere , and do not meet anywhere , are called parallel .

Parallel lines make equal angles with any other line

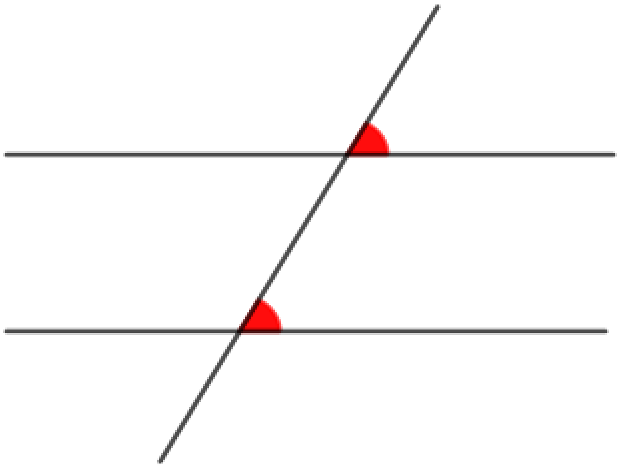
PARALLE LINES AND ANGLES

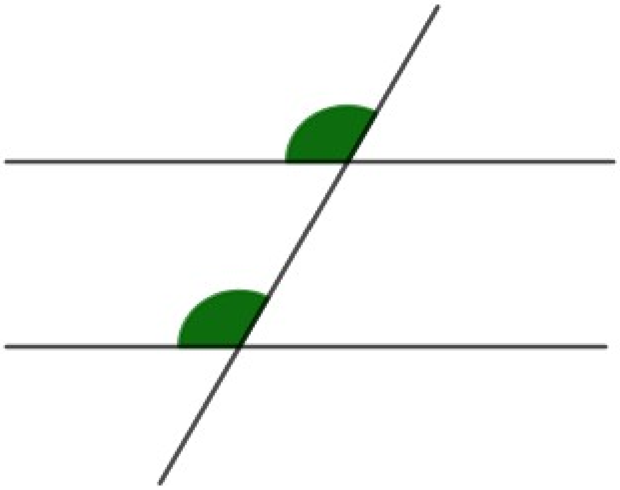
When a line cuts across a pair of parallel lines , eight angles are formed .

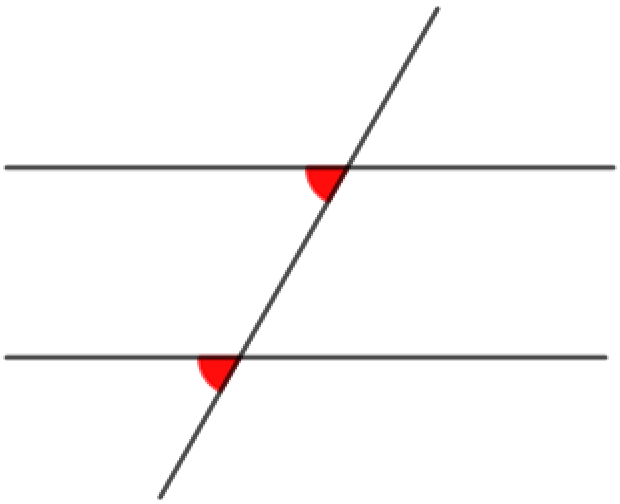


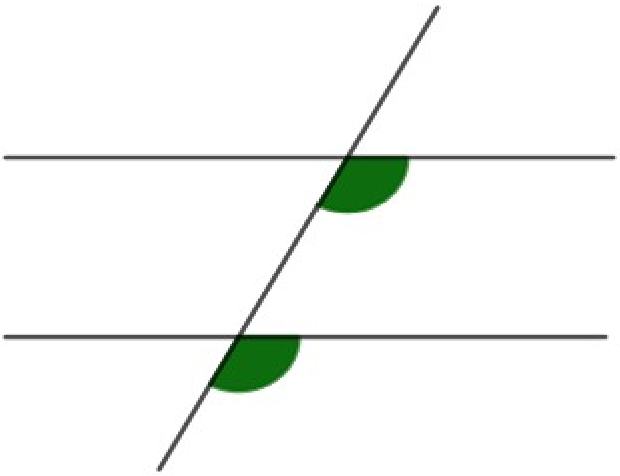
We can pair one angle at the bottom with one at top in several different ways . Some such pairs are equal , others are supplementary .

Corresponding angles





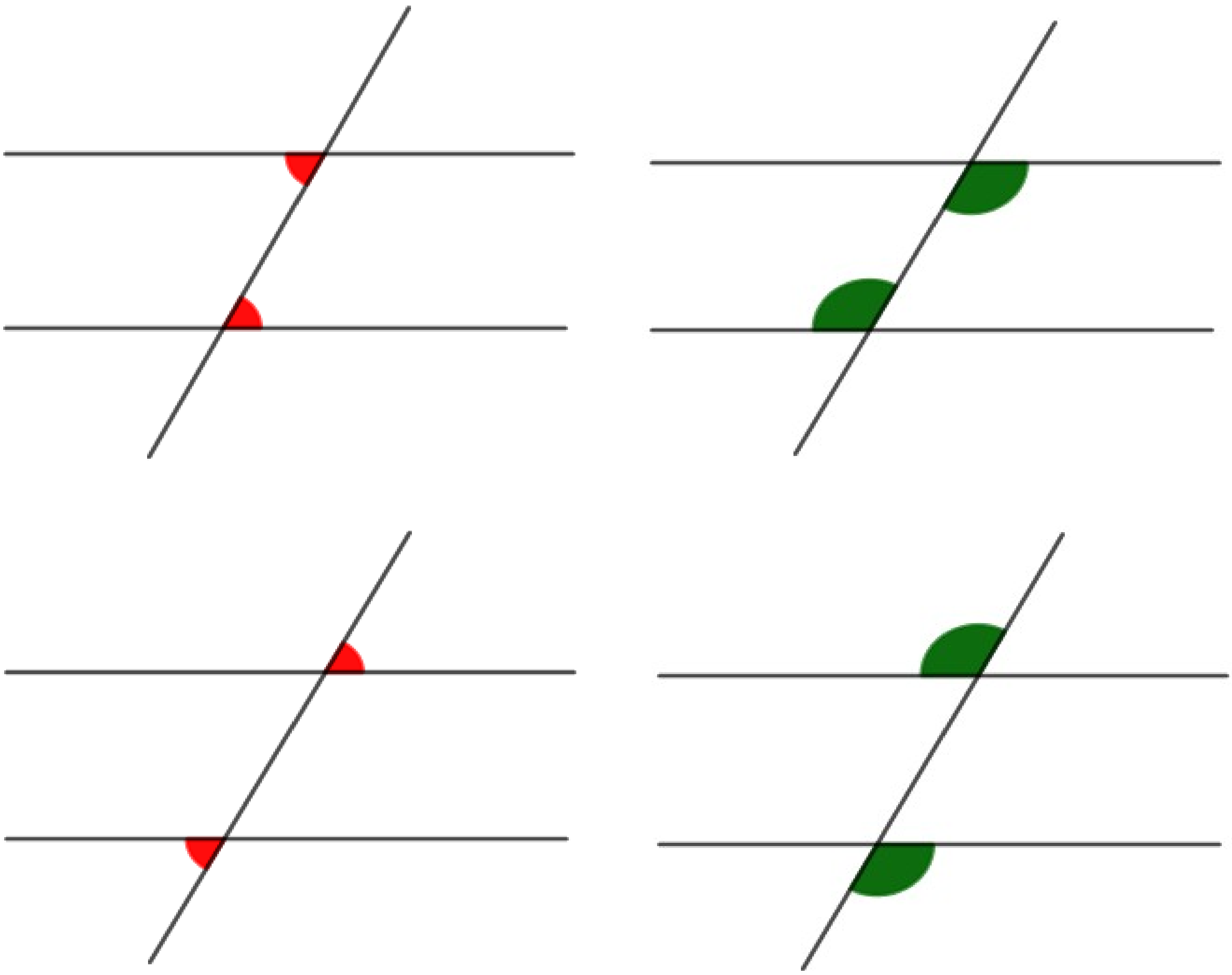




When a line cuts across a pair of parallel lines , there are four pairs of angles which are at similar positions at the bottom and the top . Angles in each such pairing , done according to similar positions are called corresponding angles .

Corresponding angles are equal

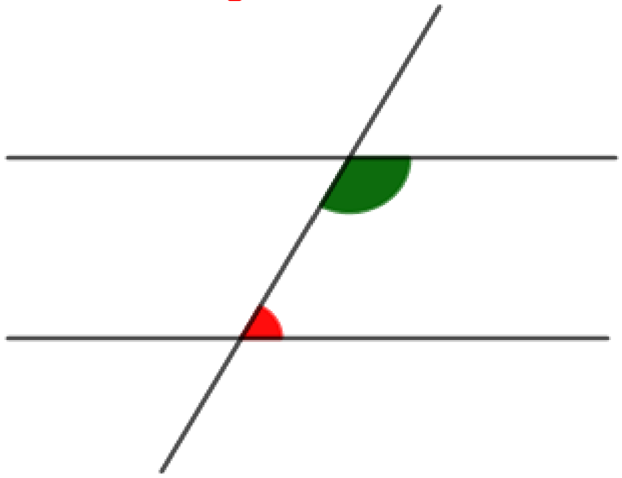
Alternate angles

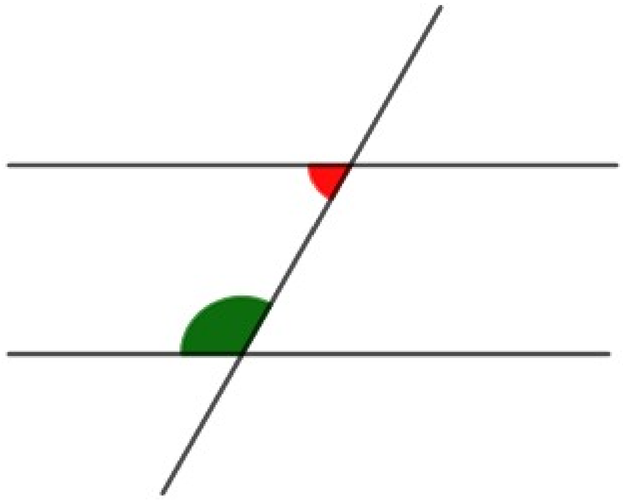


When a line cuts across a pair of parallel lines , we can pair the equal angles in four ways with the position quite opposite . Angles in such pairing ,done with reverse positions are called alternate angles .

Alternate angles are equal

Co-interior angles

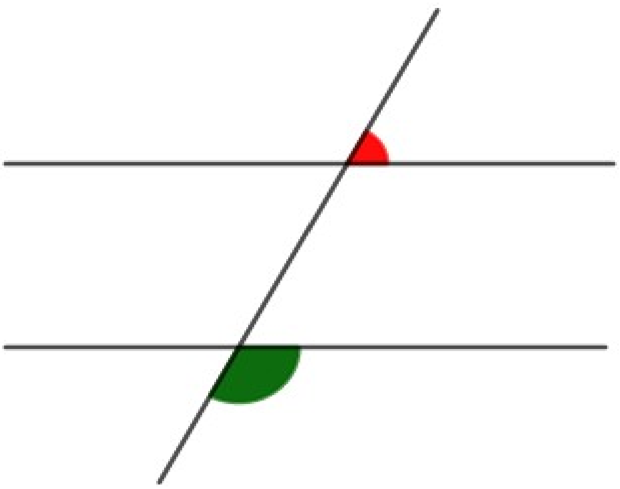


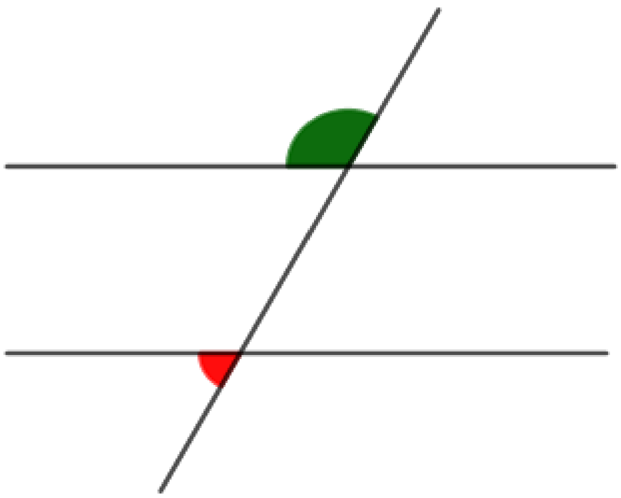


When a line cuts across a pair of parallel lines , there are two pairs of supplementary angles on the either side of the slanted line between the parallel lines . The angles in each of these two pairs are called co-interior angles .

Co-interior angles are supplementary

Co-exterior angles





When a line cuts across a pair of parallel lines , there are two pairs of supplementary angles on the either side of the slanted line not between the parallel lines . The angles in each of these two pairs are called co-exterior angles .

Co-exterior angles are supplementary

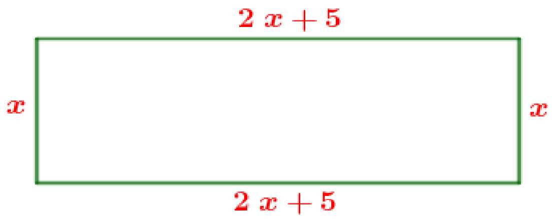
EQUATIONS

Activity

The length of a rectangle is 5 centimetres more than twice of its breadth and its perimeter is 34 centimetres . Find its area .

Answer

If breadth = x **then length =** $2x + 5$



$x + 2x + 5 + x + 2x + 5 = 34$

$6x + 10 = 34$

$6x = 24$

$x = \frac{24}{6} = 4$

Breadth = $x = 4\ cm$

Length $= 2x + 5 = 2 \times 4 + 5 = 8 + 5 = 13 cm$

Area = length x breadth = $13 \times 4 = 52\ sq.cm$

More activity

In a calendar , a square of four numbers is marked . The sum of the numbers is 80 .

What are the numbers ?

IDENTITIES

Concept 1

To multiply a sum of positive numbers by a sum of positive numbers , multiply each number in the second sum by each number in the first sum and add .

$$(x + y)(u + v) = xu + xv + yu + yv \quad ,$$

for any our positive numbers x, y , u , v

Activity

Look at these .

$$1 \times 4 = (2 \times 3) - 2$$

$$2 \times 5 = (3 \times 4) - 2$$

$$3 \times 6 = (4 \times 5) - 2$$

$$4 \times 7 = (5 \times 6) - 2$$

a) Write the next two lines in this pattern .

b) If we take four consecutive natural numbers , what is the relation between the product of the first and the last , and the product of the middle two ?

c) Explain the general principle using algebra .

Answer

a)

$$5 \times 8 = (6 \times 7) - 2$$

$$6 \times 9 = (7 \times 8) - 2$$

b) If we take four consecutive natural numbers , the product of the first and the last is 2 less than the product of the middle two.

c) If we take four consecutive natural numbers are x , $x+1$, $x+2$, $x+3$ **, then**

$$x(x+3) = (x+1)(x+2) - 2 \ .$$

$x(x+3) = x^2 + 3x$

$(x+1)(x+2) = x \times x + x \times 2 + 1 \times x + 1 \times 2 = x^2 + 2x + x + 2 = x^2 + 3x + 2$

Thus $x(x+3) = (x+1)(x+2) - 2$.

More activity

a) Mark four numbers forming a square in a calendar . Add the product of the diagonal pair and find the difference of these products .

b) Is it same for all the squares of four numbers ?

c) Explain why this is so , using algebra .

Concept 2

The square of sum of two positive numbers is the sum of the squares of the two numbers and twice their product .

For any two positive numbers x and y ,

$$(x + y)^2 = x^2 + y^2 + 2xy$$

Activity

Look at these .

$$1 \times 3 = 2^2 - 1$$

$$2 \times 4 = 3^2 - 1$$

$$3 \times 5 = 4^2 - 1$$

$$4 \times 6 = 5^2 - 1$$

a) Write the next two lines in this pattern .

b) If we take three consecutive natural numbers ,what is the relation between the product of the first and the last , and the middle number ?

c) Explain the general principle using algebra .

Answer

a)

$$7 \times 5 = 6^2 - 1$$

$$8 \times 6 = 7^2 - 1$$

b) If we take three consecutive natural numbers , the product of the first and the last is 1 less than the square of the middle number .

c) If we take the three consecutive natural numbers are x , $x+1$, $x+2$ **, then**

$$x(x+2) = (x+1)^2 - 1$$

$$x(x+2) = x^2 + 2x$$

$$(x+1)^2 - = x^2 + 1^2 + 2 \times x \times 1 - = x^2 + 2x + 1$$

Thus $x(x+2) = (x+1)^2 - 1$ **.**

More activity

Look at these .

$$1^2 + (4 \times 2) = 3^2$$

$$2^2 + (4 \times 3) = 4^2$$

$$3^2 + (4 \times 4) = 5^2$$

$$4^2 + (4 \times 5) = 6^2$$

a) Write the next two lines in this pattern .

b) Write the general principle of the above pattern ?

c) Explain the general principle using algebra .

Concept 3

The square of the difference of two positive numbers is twice their product subtracted from the sum of their squares .

For all positive numbers x , y with x > y ,

$$(x - y)^2 = x^2 + y^2 - 2xy$$

Activity

Find the squares of the following numbers

a) 49 **b)** 98

Answer

a) $49^2 = (50-1)^2 = 50^2 + 1^2 - 2 \times 50 \times 1 = 2500 + 1 - 100 = 2401$

b) $98^2 = (100-2)^2 = 100^2 + 2^2 - 2 \times 100 \times 2 = 10000 + 4 - 400 = 99604$

More activity

Find the squares of the following numbers

a) 47 **b)** 99

Concept 4

The product of the sum and the difference of two positive numbers is the difference of their squares .

For any two positive numbers x, y with x > y ,

$$(x+y)(x-y) = x^2 - y^2$$

Activity

Look at these .

$$3^2 - 1^2 = 4 \times 2$$

$$4^2 - 2^2 = 4 \times 3$$

$$5^2 - 3^2 = 4 \times 4$$

$$6^2 - 4^2 = 4 \times 5$$

a) Write the next two lines in this pattern .

b) If we take three consecutive natural numbers , what is the relation between the difference of the squares of first and the last , and the middle number ?

c) Explain the general principle using algebra .

Answer

a)

$$7^2 - 5^2 = 4 \times 6$$

$$8^2 - 6^2 = 4 \times 7$$

b) If we take three consecutive natural numbers , the difference of the squares of the first and last numbers is four times the middle number .

c) If we take the three natural numbers are x , $x+1$, $x+2$ **, then**

$$(x+2)^2 - x^2 = 4(x+1)$$

$$(x+2)^2 - x^2 = (x+2+x)(x+2-x) = (2x+2)2 = 4x+4$$

$$4(x+1) = 4x+4$$

Thus $(x+2)^2 - x^2 = 4(x+1)$ **.**

More activity

a) Mark four numbers forming a square in a calendar . Add the squares of the diagonal pair and find the difference of these sums .

b) Is it same for all the squares of four numbers ?

c) Explain why this is so , using algebra .

PAIRS OF EQUATIONS

Concept

If two informations of two measures are given , we can find them by forming a pair of equations with two letters .

Activity

The price of 3 pencils and 4 pens is 55 rupees . The price of 5 pencils and 2 pens is 45 rupees . What is the price of a pen ? And the price of the note book ?

Answer

Take , the price of a pen = x and the price of a pencil = y , then

$$3x + 4y = 55 \qquad (1)$$

$$5x + 2y = 45 \qquad (2)$$

(1) x 5 ==> $15x + 20y = 275$ **(3)**

(2) x 3 ==> $15x + 6y = 135$ **(4)**

(3) - (4) ==> $15x + 20y = 275$ -

$$15x + 6y = 135$$

$$0 + 14y = 140$$

$$y = \frac{140}{14} = 10$$

$$3x + 4 \times 10 = 55 \implies 3x + 40 = 55 \implies 3x = 15 \implies x = \frac{15}{3} = 5$$

Price of a pencil = x = 5 Rs , Price of a pen = y = 10 Rs

More activity

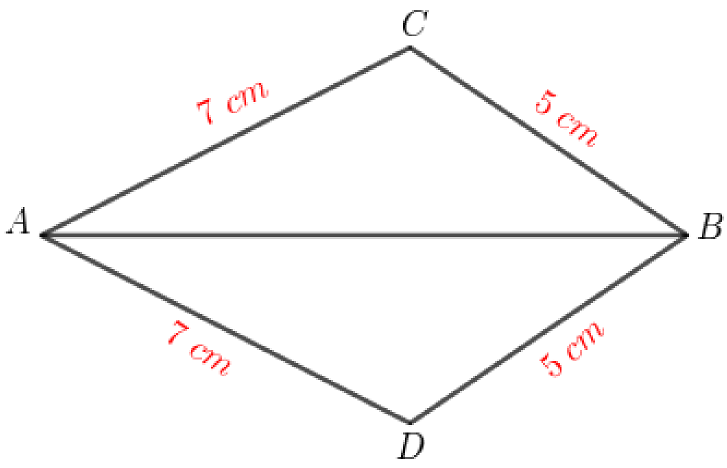
4 small buckets and 7 large buckets of water make 62 litres . 6 small buckets and 5 large buckets make only 60 litres . How much water can each bucket hold ?

EQUAL TRIANGLES

Concept -1

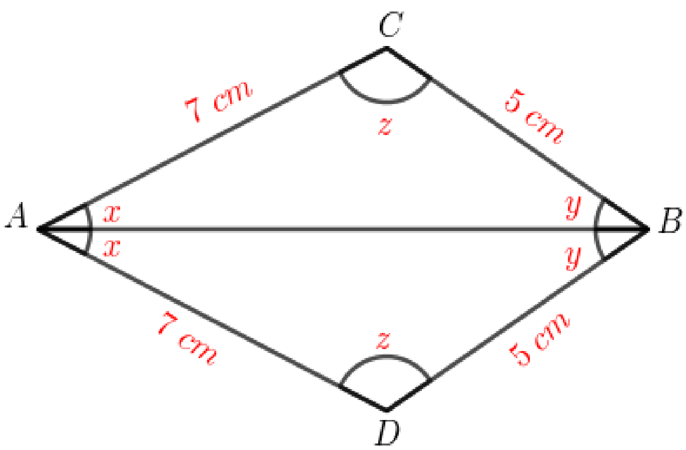
If the sides of a triangle are equal to the sides of another triangle , then the angles of the triangles are also equal.

Activity



Find the equal angles of triangles ABC and ABD .

Answer



In triangles ABC and ABD ,

$AC = AD$, $BC = BD$

$AD = AD$ **(common side)**

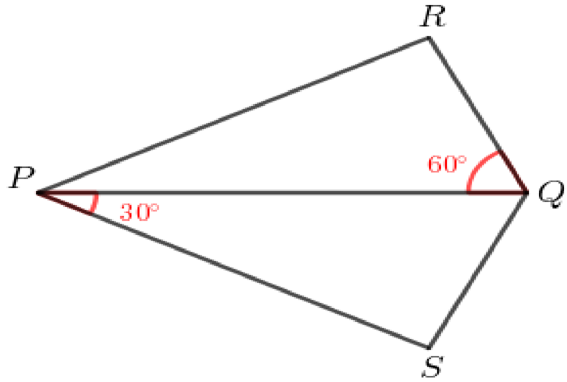
(If the sides of a triangle are equal to the sides of another triangle , then the angles of the triangles are also equal.)

$\angle BAC = \angle BAD$

$\angle ABC = \angle ABD$

$\angle ACB = \angle ADB$

More activity



In quadrilateral PSQR , PR = PS , QR = QS

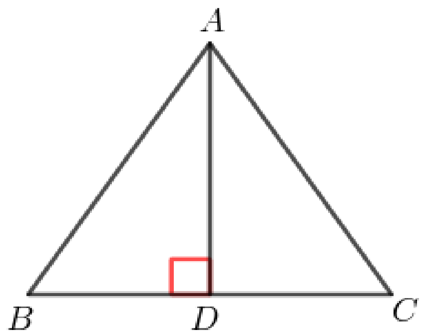
< SPQ = 30^0 , < PQR = 60^0

Find the measures of all the angles of the quadrilateral .

Concept -2

If two sides of a triangle and the angle made by them are equal to two sides of another triangle and the angle made by them , then the third sides of the triangles are also equal and the other two angles are also equal .

Activity



In the figure , D is the midpoint of BC . $\angle BDA = 90^0$

a) Prove that $AB = AC$ **.**

b) Prove that AD **is the bisector of** $\angle BAC$ **.**

Answer

In triangles ADB and ADC ,

$BD = CD$ **(D is the midpoint of BC)**

$AD = AD$ **(common side)**

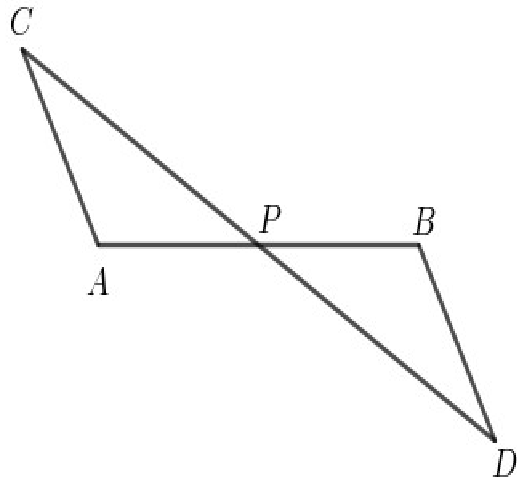
$\angle ADB = \angle ADC = \angle 90^0$

a) $AB = AC$ **(If two sides of a triangle and the angle made by them are equal to two sides of another triangle and the angle made by them , then the third sides of the triangles are also equal and the other two angles are also equal .)**

b) $\angle BAD = \angle CAD$

AD is the bisector of ∠ BAC .

More activity



In the figure , P is the midpoint of AB and CD .

a) Which angle has the same measure as that of ∠ APC ?

b) Prove that AC = BD .

c) Prove that BD is parallel to AC .

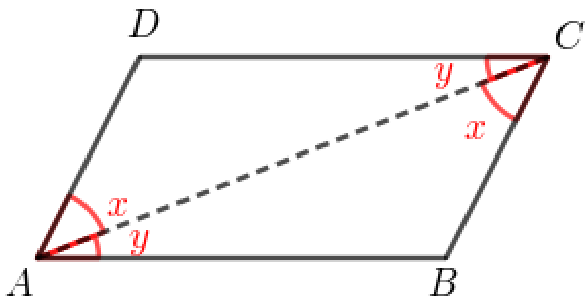
Concept -3

If one side of a triangle and the angles at its ends are equal to one side of another triangle and the angles at its ends , then the third angles are also equal and the sides opposite equal angles are equal .

Activity

In any parallelogram , opposite sides are equal .

Answer



In the figure , ABCD is a parallelogram .

That is , side AB is parallel to the side CD ,

and AD is parallel to BC .

Join AC .

In triangles ACD and ABC ,

$AC = AC$ **(common side)**

$\angle CAD = \angle ACB$ **(alternate angles are equal)**

$\angle ACD = \angle BAC$

So $AB = CD$, $AD = BC$ **(If one side of a triangle and the angles at its ends are equal to one side of another triangle and the angles at its ends , then the third angles are also equal and the sides opposite equal angles are equal)**

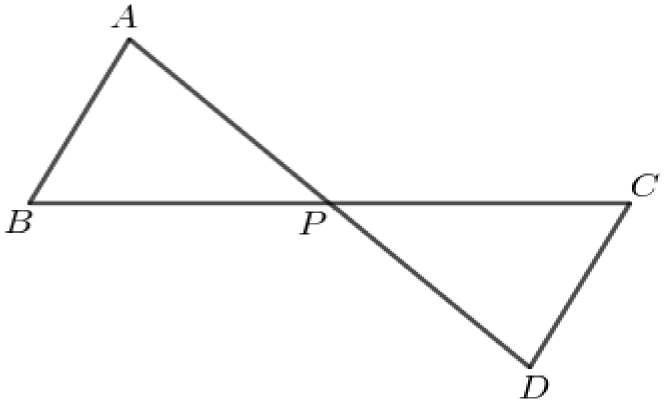
More activity

In the figure P is the midpoint of BC .

AB and CD are parallel .

a) Prove that AB = CD .

b) P is the midpoint of AD .



ISOSCELES TRIANGLES

Concept - 4

A triangle with two sides equal , is called an isosceles triangle .

Concept - 5

Equilateral triangle

A triangle with all three sides equal , is called an equilateral triangle .

Concept - 6

In any isosceles triangle , the perpendicular from the point joining equal sides to the opposite side bisects the angle at this point and the side opposite

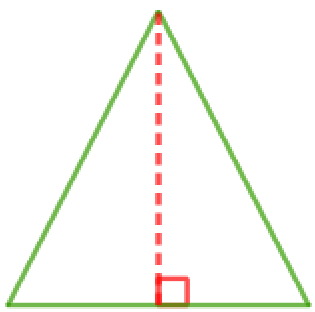
A line dividing a line or an angle into two equal parts is called a bisector

NOTE : The relations between the base and the third vertex of an isosceles triangle can be put in three different ways .

1) The perpendicular from the third vertex bisects the base .

2) The line joining the third vertex and the midpoint of the base is perpendicular to the base .

3) The third vertex is on the perpendicular bisector of the base .



POLYGONS

We can divide a polygon into triangles by drawing maximum number of diagonals from any one of its vertices . Hence we can find the sum of the angles of this polygon .

We can draw a maximum of $n - 3$ diagonals from a vertex of an n- sided polygon

If we draw maximum number of diagonals from a vertex of an n - sided polygon , we get $n - 2$ triangles .

The sum of the angles of an n-sided polygon is $(n-2) \times 180^0$

Polygon	Number of sides	Sum of angles
Triangle	3	180^0
Quadrilateral	4	$2 \times 180 = 360^0$
Pentagon	5	$3 \times 180 = 540^0$
Hexagon	6	$4 \times 180 = 720^0$
Heptagon	7	$5 \times 180 = 900^0$
Octagon	8	$6 \times 180 = 1080^0$
Nonagon	9	$7 \times 180 = 1260^0$
Decagon	10	$8 \times 180 = 1440^0$

Activity

What is the sum of the angles of a 12 sided polygon ?

Answer

Sum of angles = $10 \times 180 = 1800^0$

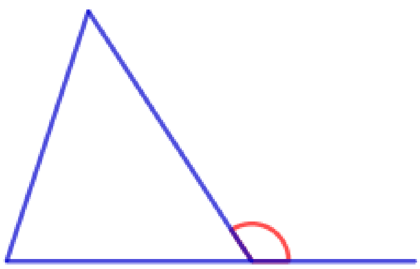
More activity

The sum of the angles of a polygon is 3600^0 . How many sides does it have ?

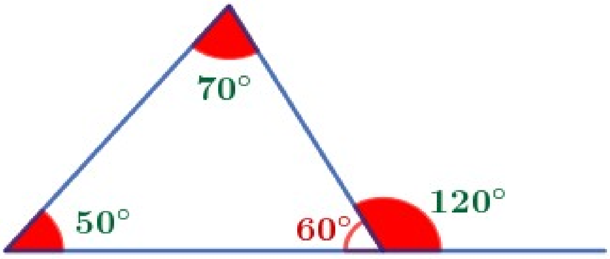
Angle around a point is 360^0

Outer angle of a triangle

If we extend one side of a triangle , we get a new angle outside the triangle . This angle is called an outer angle (exterior angle) .



In any triangle , the outer angle at a vertex is equal to the sum of the inner angles at the other two vertices .



The sum of inner and outer angle at a vertex of a polygon is 180^0

The sum of the outer angles of any polygon is 360^0

Activity

All angles in an 18 sided polygon are equal . How much is each outer angle ?

Answer

Sum of the outer angles = 360^0

An outer angle = $\frac{360}{18} = 20^0$

More activity

In a polygon with all angles equal , one outer angle is 12^0 .How many sides does it have ?

Regular polygon

A polygon with equal sides and angles is called a regular polygon .

Activity

A regular polygon has 12 sides .

a) What is the measure of each outer angle ?

b) What is the measure of each inner angle ?

Answer

a) Sum of the outer angles = 360^0

An outer angle = $\frac{360}{12} = 30^0$

b) An inner angle = $180 - 30 = 150^0$

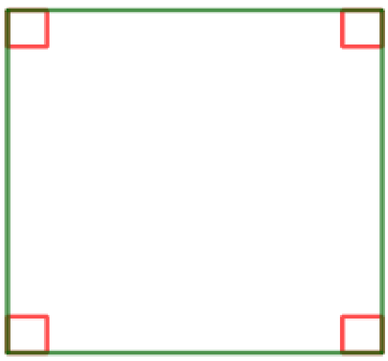
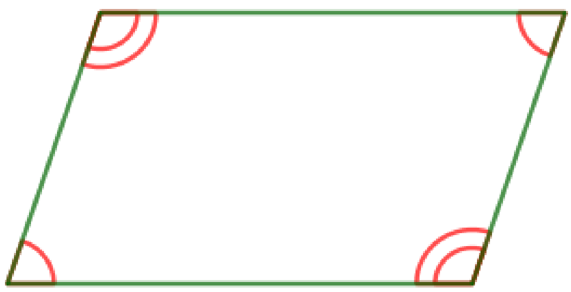
More activity

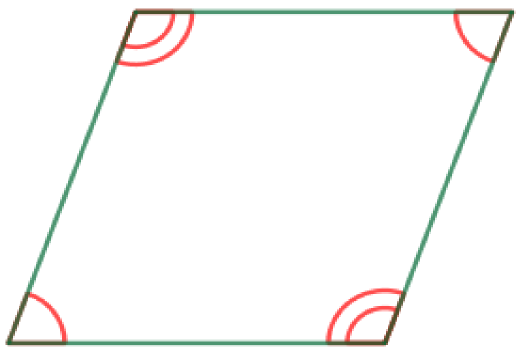
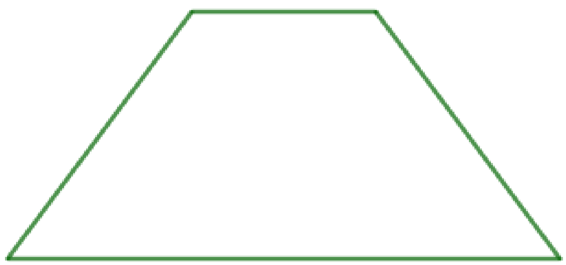
Each angle of a regular polygon is 170^0 .

a) What is the measure of each outer angle ?

b) How many sides does it have ?

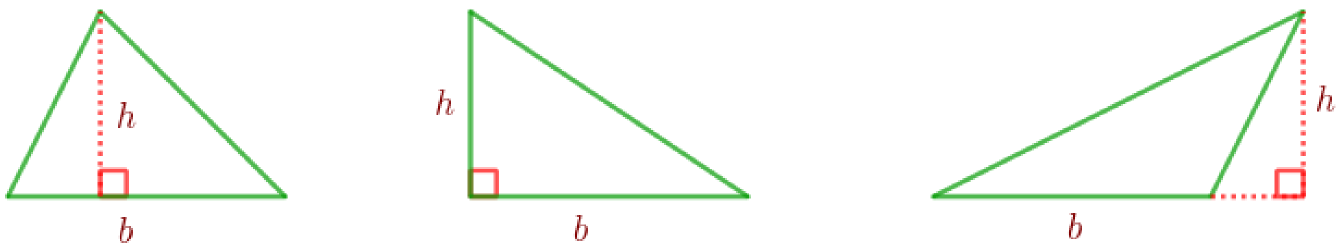
QUADRILATERALS

Polygon	Features
Rectangle 	Opposite sides equal
	Opposite sides parallel
	All angles right
	Diagonals equal
	Diagonals bisect each other
Square 	All sides equal
	Opposite sides parallel
	All angles right
	Diagonals equal
	Diagonals bisect each other
	Diagonals perpendicular to each other
Parallelogram 	Opposite sides equal
	Opposite sides parallel
	Opposite angles equal
	Diagonals bisect each other
	Sum of angles on the same side 180^0

<table>
<tr><td rowspan="6"><u>Rhombus</u>
</td><td>All sides equal</td></tr>
<tr><td>Opposite sides parallel</td></tr>
<tr><td>Opposite angles equal</td></tr>
<tr><td>Diagonals bisect each other</td></tr>
<tr><td>Diagonals perpendicular to each other</td></tr>
<tr><td>Sum of angles on the same side 180^0</td></tr>
<tr><td rowspan="2"><u>Trapezium</u>
</td><td>Only one pair of opposite sides parallel</td></tr>
<tr><td>Sum of the angles on each of non-parallel sides 180^0 .</td></tr>
<tr><td rowspan="5"><u>Isosceles trapezium</u>
</td><td>Only one pair of opposite sides parallel</td></tr>
<tr><td>Non - parallel sides equal</td></tr>
<tr><td>Diagonals equal</td></tr>
<tr><td>Angles on each parallel sides equal</td></tr>
<tr><td>Sum of angles on the same side 180^0 .</td></tr>
</table>

AREA

Area of a Triangle



The area of a triangle is half the product of one side with the perpendicular from the opposite side .

Area of the triangle = $\frac{1}{2}$ x a side x perpendicular from the opposite side

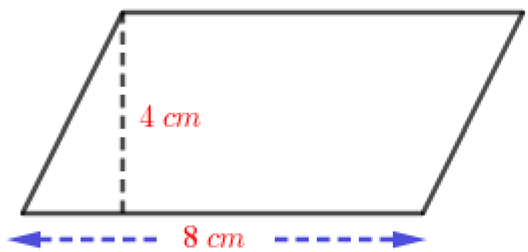
Figure	Perimeter	Area
Equilateral triangle	3 x side	$\frac{\sqrt{3} \times side^2}{4}$
Rectangle	2 x (length + breadth)	length x breadth
Square	4 x side	side x side

Area of a Parallelogram

The area of a parallelogram is the product one side with the distance to the opposite side

Activity

Calculate the area of the parallelogram in the figure .



Answer

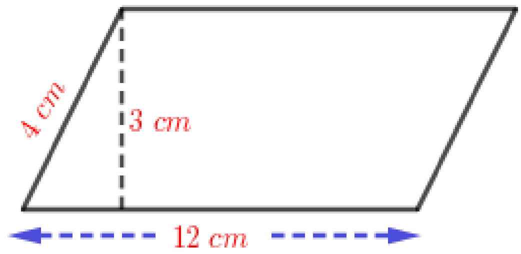
Area of the parallelogram = a side x distance to the opposite side

= 8 x 4 = 32 square centimetres

More activity

a) Calculate the area of the parallelogram in the figure

b) What is the distance between the parallel sides of length 4 centimetres .



Area of a Rhombus

The area of a rhombus is half the product of the diagonals .

Activity

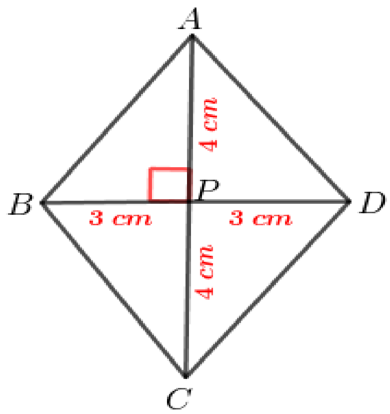
The length of the diagonals of a rhombus are 6 centimetres and 8 centimetres .

a) Calculate its area .

b) What is the measure of the angle between the diagonals ?

c) Compute the length of the side .

Answer



a) Area of the rhombus = $\frac{1}{2}$ x **product of the diagonals .**

= $\frac{1}{2} \times 6 \times 8$ **= 24 square centimetres**

b) Angle between the diagonals = 90^0

c) In right triangle APB ,

$Base^2 + Altitude^2 = Hypotenuse^2 \implies BP^2 + AP^2 = AB^2$

$\implies 3^2 + 4^2 = AB^2 \implies 9 + 16 = AB^2$

$\implies AB^2 = 25 \implies AB = \sqrt{25} = 5\ cm$

Side of the rhombus = 5 centimetres .

More activity

The area of a rhombus is 96 square centimetres and the length of one of its diagonal is 16 centimetres .

a) What is the length of its second diagonal ?

b) What is the measure of the angle between the diagonals ?

c) Compute the length of the side .

d) Compute the distance between the parallel sides .

Area of a trapezium

The area of a trapezium is half the product of the parallel sides and the distance between them .

Activity

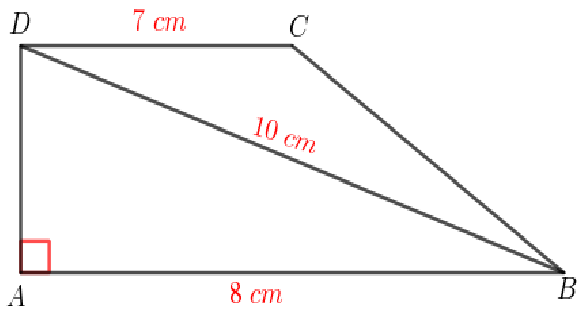
The length of the parallel sides of a trapezium are 12 centimetres , 8 centimetres and the distance between them is 10 centimetres . What is its area ?

Answer

a) Area of the trapezium = $\frac{1}{2}$ x **sum of parallel sides** x **distance between them**

$= \frac{1}{2} \times (12 + 8) \times 10 = \frac{1}{2} \times 20 \times 10$ **= 100 square centimetres**

More activity



In the figure , ABCD is a trapezium .

$\angle A = 90^0$, AB = 8 centimetres ,

BD = 10 centimetres and , DC = 7 centimetres

a) What is the length of AD ?

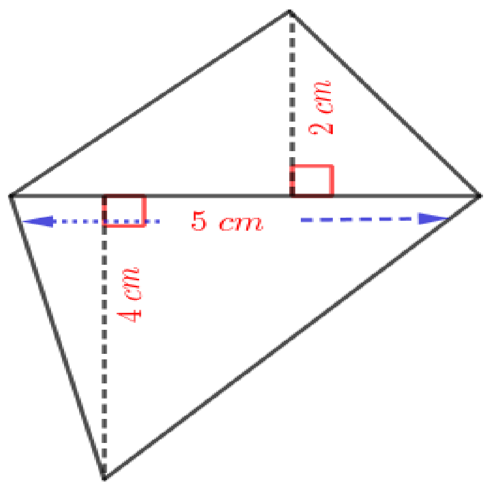
b) Compute the area of the trapezium .

Area of a Quadrilateral

The area of a quadrilateral is half the product of a diagonal and the sum of the perpendicular distances from the opposite vertices to this diagonal .

Activity

Compute the area of the quadrilateral shown in the figure .



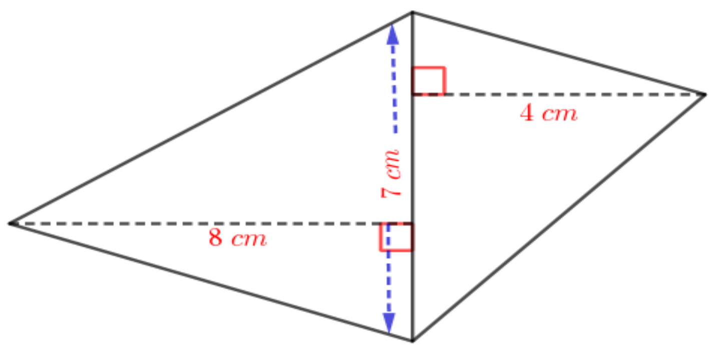
Answer

Area of the quadrilateral = $\frac{1}{2}$ x a diagonal x sum of the perpendicular distances from the opposite vertices to this diagonal

$= \frac{1}{2} \times 5 \times (4 + 2) = \frac{1}{2} \times 5 \times 6 = 15$ square centimetres .

More activity

Compute the area of the quadrilateral shown in the figure .



RATIO

Concept 1

If two quantities are in the ratio $a : b$, then there is a quantity x such that the first is ax and the second is bx

Activity

The length and breadth of a rectangle are in the ratio 5 : 4 and its perimeter is 54 centi- metres . Calculate the length and breath .

Answer

Length $= 5x$

Breadth $= 4x$

Perimeter = 54 ==> $2 \times 5x + 2 \times 4x = 54$

$10x + 8x = 54$

$18x = 54$ ==> $x = \frac{54}{18} = 3$

Length $= 5x = 5 \times 3 = 15\ cm$

Breadth $= 4x = 4 \times 3 = 12\ cm$

More activity

In a regular polygon , the ratio of the inner and outer angle is 7 : 2 .

a) What is each inner angle ?

b) What is each outer angle ?

c) How many sides does the polygon have ?

Concept 2

If three quantities are in the ratio $a : b : c$, then there is a quantity x such that the first is ax , the second bx and the third is cx .

Activity

The outer angles of a triangle are in the ratio 3 : 4 : 5 .

a) What is the sum of the outer angles ?

b) What is each outer angle ?

Answer

a) Sum of outer angles = 360^0

b) If we take the outer angles are $3x$, $4x$, $5x$

$$3x + 4x + 5x = 360 \implies 12x = 360 \implies x = \frac{360}{12} = 15$$

Outer angles = 3×15 , 4×15 , 5×15 = 45^0 , 60^0 , 75^0

More activity

The length , breadth ,and height of a rectangular block are in the ratio 5 : 3 : 6 and its volume is 2160 cubic centimetres . Calculate the length , breadth and height .

NEW NUMBERS

The square of any fraction is not 2

The diagonal of a square of side 1 cannot be expressed as a fraction .

Concept 1

We can not express all the lengths using fractions

We have to make a new kind of number to denote measures which cannot be expressed as fractions .

Concept 2

If x is a positive number , in some cases $\sqrt{x}$ would be a natural number or fraction , in some cases , we compute fractions whose squares get closer to x and write $\sqrt{x}$ in decimal form .

Concept 3

To get fractions approximately equal to the sum (or difference) of square roots of positi - ve numbers which are not perfect squares , we add approximately equal fractions of each

Activity

The length of the perpendicular sides of a right triangle are $\sqrt{3}$ centimetres and $\sqrt{2}$ centimetres .

a) Calculate the length of its hypotenuse .

b) Calculate the perimeter of the triangle .

(**Hint** : $\sqrt{2} = 1.41$, $\sqrt{3} = 1.73$, $\sqrt{5} = 2.23$)

Answer

a) $Base^2 + Altitude^2 = Hypotenuse^2 \implies (\sqrt{3})^2 + (\sqrt{2})^2 = Hypotenuse^2$

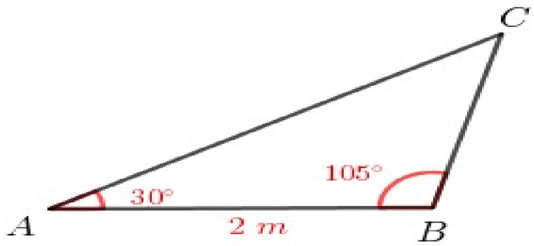
$\implies 3 + 2 = Hypotenuse^2 \implies Hypotenuse^2 = 5 \implies Hypotenuse = \sqrt{5}\ cm$

b) Perimeter = $\sqrt{3} + \sqrt{2} + \sqrt{5} = 1.73 + 1.41 + 2.23 = 5.37$ സെ.മീ

More activity

In the figure , in triangle ABC

AB = 2 metres , ∠ A = 30^0 , ∠ B = 105^0



a) What is the altitude of an equilateral triangle of side 2 metres ?

b) Calculate the perimeter of triangle ABC .

(Hints : Draw a perpendicular from B to AC . $\sqrt{2} = 1.41$, $\sqrt{3} = 1.73$ **)**

Concept 4

For any positive numbers x and y , $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$

Activity

Length of a rectangle is $\sqrt{5}$ centimetres and its breadth is $\sqrt{2}$ centimetres . Find its area .

Answer

Area = $\sqrt{5} \times \sqrt{2} = \sqrt{10}\ sq.cm$

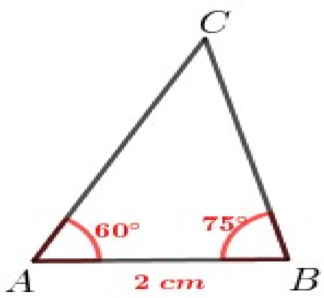
More activity

In the figure , in triangle ABC , AB = 2 centimetres . ∠ A = 60^0 , ∠ B = 75^0

a) Find the length of the perpendicular from B to AC .

b) What is the length of AC ?

c) Calculate the area of the triangle .



Concept 5

For any positive numbers x , y the product $\sqrt{x} \times \sqrt{y} = \sqrt{z}$

can be written as the divisions $\frac{\sqrt{z}}{\sqrt{x}} = \sqrt{y}$, $\frac{\sqrt{z}}{\sqrt{y}} = \sqrt{x}$

Activity

Try these problems .

a) $\frac{\sqrt{12}}{\sqrt{6}}$ **b)** $\frac{\sqrt{24}}{\sqrt{8}}$

Answer

a) $\frac{\sqrt{12}}{\sqrt{6}} = \sqrt{2}$ **b)** $\frac{\sqrt{24}}{\sqrt{8}} = \sqrt{3}$

More activity

Try these problems .

a) $\frac{\sqrt{42}}{\sqrt{7}}$ **b)** $\frac{\sqrt{90}}{\sqrt{18}}$

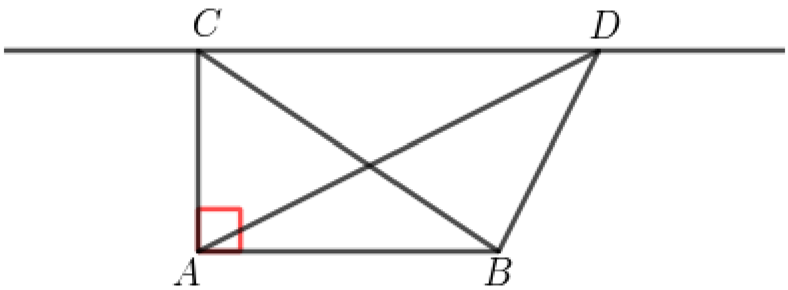
PARALLEL LINES - 2

Concept -1

All angles with the same base and area have their third vertices on a line parallel to the base .

Conversely , all triangles with the same base and the third vertex on a line parallel to the base have the same area .

Activity



In the figure $< A = 90^0$. D is a point on a line through C parallel to AB .

AB = 4 centimetres , BC = 5 centimetres .

a) What is the length of AC ?

b) Find the area of triangle ABC .

c) Find the area of triangle ABD .

Answer

a) In right triangle ABC ,

$$Base^2 + Altitude^2 = hypotenuse^2 \implies AB^2 + AC^2 = BC^2 \implies 4^2 + AC^2 = 5^2$$

$$\implies 16 + AC^2 = 25 \implies AC^2 = 25 - 16 = 9 \implies AC = \sqrt{9} = 3\ cm$$

b) Area of triangle ABC $= \frac{1}{2} \times a\ side \times altitude\ to\ that\ side = \frac{1}{2} \times AB \times AC$

$$= \frac{1}{2} \times 4 \times 3 = \textbf{6 square centimetres .}$$

c) Area of triangle ABD = Area of triangle ABC = 6 square centimetres .

More activity

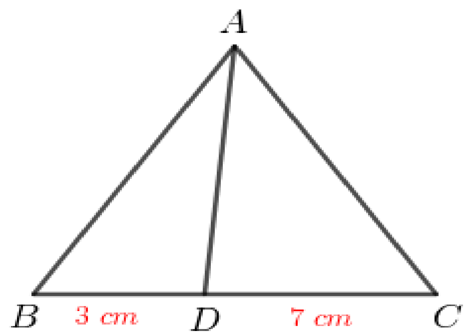
Draw a circle and a triangle with one vertex at the centre of the circle and the other two on the circle .Draw another triangle of the same area with all three vertices on the circle .

Concept -2

A line from the vertex of a triangle divides the length of the opposite side and the area of the triangle in the same ratio .

Activity

In the figure , area of triangle ABC is 100 square centimetres BD = 3 centimetres and DC= 7 centimetres



a) Find the ratio of the areas of the triangles ABD and ADC .

b) Find the area of triangle ABD .

c) Find the area of triangle ADC

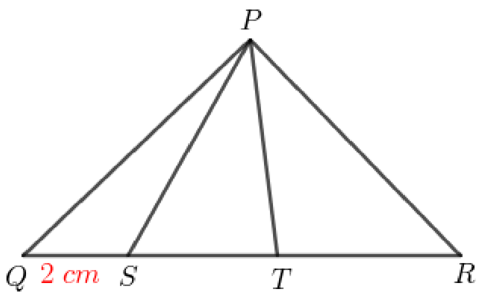
Answer

a) Ratio of the areas of triangles ABD and ADC $= BD : DC = 3 : 7$

b) Area of triangle ABD $= \frac{3}{10} \times 100 = 30\ sq.cm$

c) Area of triangle ADC $= \frac{7}{10} \times 100 = 70\ sq.cm$

More activity



In the figure , area of triangle PQS is 10 square - centimetres , area of triangle PST is 40 square -

centimetres and area of triangle PTR is 50 square centimetres . QS = 2 centimetres

a) $QS : ST : TR = ...: ..... : ......$

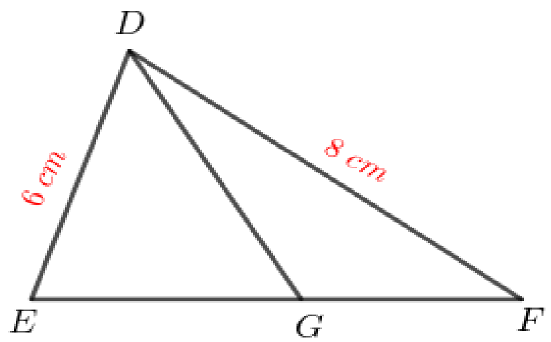
b) What is the length of ST ?

c) What is the length of QR ?

Concept -3

In any triangle , the bisector of an angle divides the opposite side in the ratio of the sides of the angle .

Activity



In the figure , DG is the bisector of < EDF .

DE = 6 centimetres , DF = 8 centimetres .

Area of triangle DEG is 30 square centimetres .

a) $EG : GF = ... : ....$

b) What is the ratio of the areas of the triangles DEG and DGF ?

c) Find the area of triangle DGF .

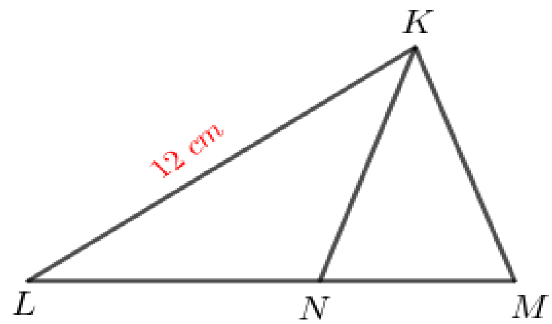
Answer

a) $EG : GF = 6 : 8$

b) Ratio of the areas of triangles DEG and DGF = $EG : GF = 6 : 8$

c) Area of triangle DGF = $\frac{8}{6} \times 30 = 40\ sq.cm$

More activity



In the figure KN is the bisector of < LKM .

.

KL = 12 centimetres , area of triangle KLN is 30 square centimetres and area of triangle KLM is 20 square centimetres .

a) $LN : NM = \ldots\ldots : \ldots\ldots$

b) What is the length of KM ?

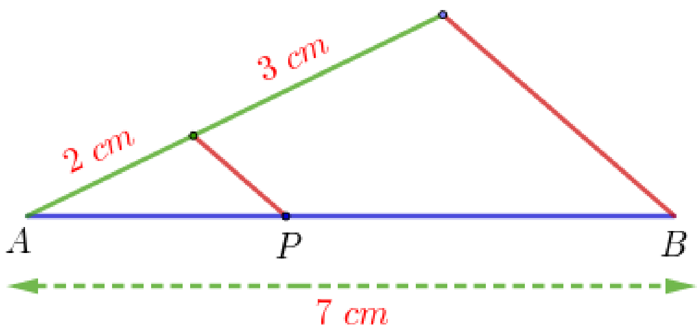
Concept -4

Three or more parallel lines cut any two lines in the same ratio .

Activity

Draw a 7 centimetres long line and divide it in the ratio 2 : 3

Answer



$AP : PB = 2 : 3$

(Draw a line 7 centimetres long. Draw a line 5 centimetres long from one end and divide it into 2 centimetres and 3 centimetres .Now join the ends of the lines and draw a line parallel to it through the point of division of the shorter lines to cut longer line in the ratio 2:3)

More activity

Draw a 11 centimetres long line and divide it in the ratio 2 : 3 : 4 .

Concept - 5

If three or more parallel lines cut a line into equal parts , they will cut any any line into equal parts .

More activity

Divide a 8 centimetres long line into three equal parts .

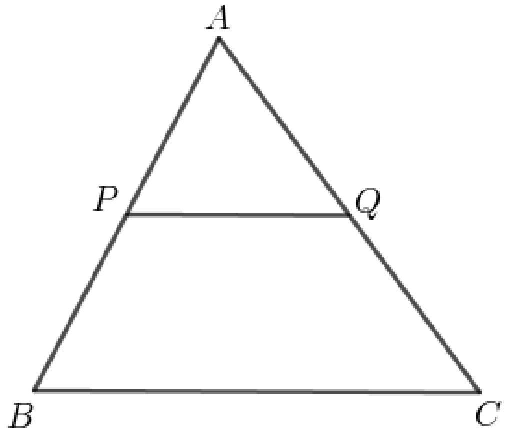
Concept - 6

In any triangle , a line drawn parallel to a side cuts the other two sides in the same ratio .

Activity

Prove that a line drawn through the midpoint of a side of a triangle parallel to another side passing through the midpoint of the third side .

Answer



In the figure P is the midpoint of AB and PQ is parallel to BC .

==> $\frac{AP}{PB} = \frac{AQ}{QC}$

$\frac{AQ}{QC} = 1 \quad (AP = PB)$

==> $AQ = QC$

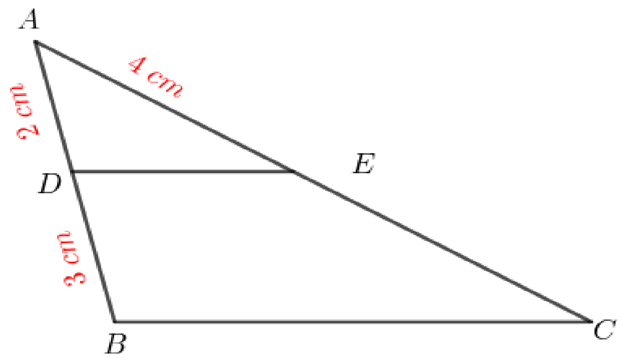
That is , Q is the midpoint of AC .

More activity

In the figure , in triangle ABC , DE is parallel to BC

AD = 2 centimetres , DB = 3 centimetres ,

AE = 4 centimetres .



a) $AE : EC = ... : .....$

b) What is the length of EC ?

Concept - 7

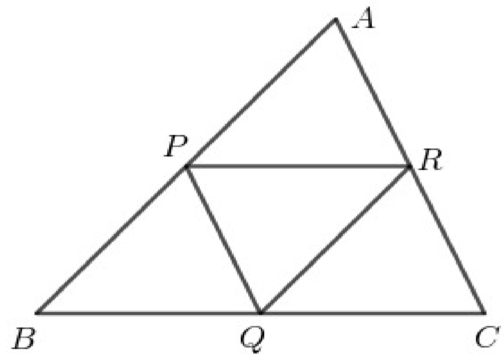
A line which divides two sides of a triangle in the same ratio is parallel to the third side .

Concept - 8

The length of the line joining the midpoints of two sides of a triangle is half the length of the third side

Activity

In the figure , P, Q and R are the midpoints of the sides of triangle ABC . BC = 10 centimetres .



a) What is the length of PR ?

b) Prove that BQRP is a parallelogram .

c) Find two more parallelograms from the figure .

Answer

a) $PR = \frac{BC}{2} = \frac{10}{2} = 5\,centimetres$

b) PR = BQ and PR is parallel to BQ .

So BQRP is a parallelogram . (since a pair of opposite sides are equal and parallel)

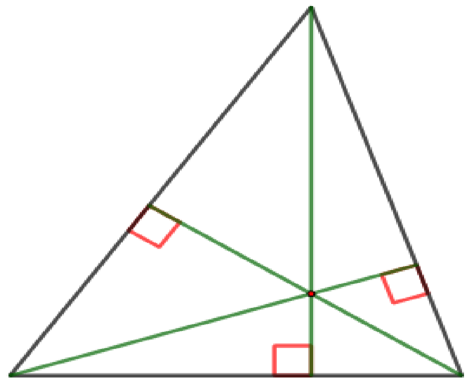
c) QCRP , APQR

More activity

Prove that the quadrilateral formed by joining the midpoints of the sides of a quadrila - teral is a parallelogram .

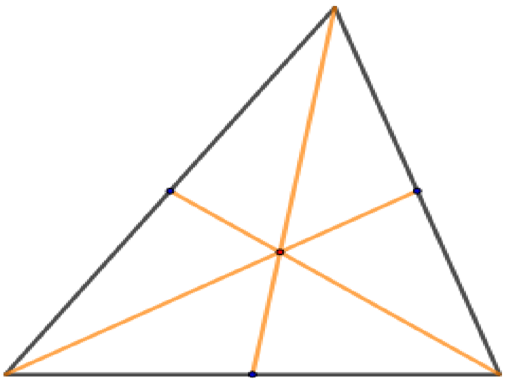
Concept - 9

In any triangle , ll the perpendiculars from the vertices to the opposite sides passes through a single point .



Concept - 10

The lines joining the vertices of a triangle to the midpoint of the opposite sides passes through a single point



Median of a triangle

A line joining a vertex of a triangle to the midpoint of the opposite side is called a median of the triangle

Concept - 11

In any triangle , all the medians intersect at a single point and that point divides each median in the ratio 2 : 1 , measured from the vertex .

SIMILAR TRIANGLES

Concept - 1

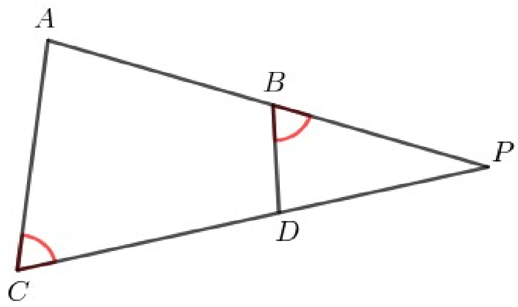
The sides of triangles with the same angles , taken in the order of size , are in the same ratio.

Activity

In the figure two lines AB and CD are extended to meet at P . $\angle ACP = \angle DBP$.

a) Prove that the angles of the triangles APC and BPD are the same ?

b) Prove that $PA \times PB = PC \times PD$.



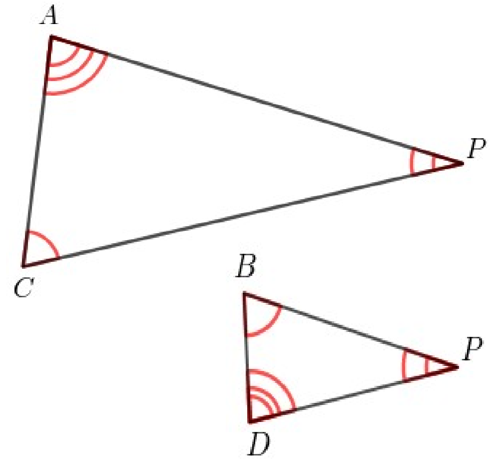
Answer

a) $\angle ACP = \angle DBP$.

$\angle APC = \angle BPD$ **(common angle)**

==> Third angles of these triangles are same .

$\angle CAP = \angle BDP$



b) $\frac{PA}{PD} = \frac{PC}{PB} = \frac{AC}{BD}$

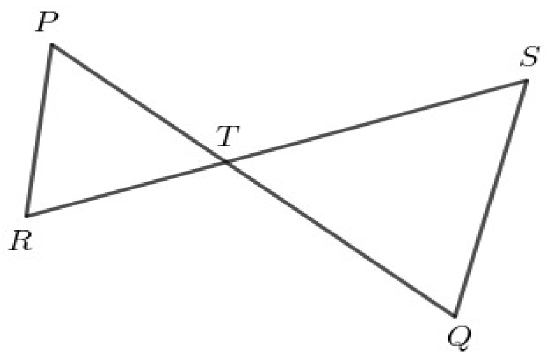
$\frac{PA}{PD} = \frac{PC}{PB}$ ==> $PA \times PB = PC \times PD$

More activity

In the figure , two lines PQ and RS intersect at T .

a) Prove that the angles of the triangles PRT and QTS are the same ?

b) Prove that $TP \times TQ = TR \times TS$.

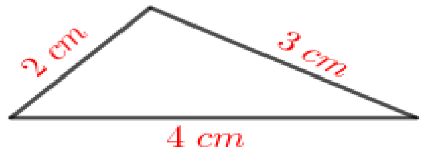


Concept - 2

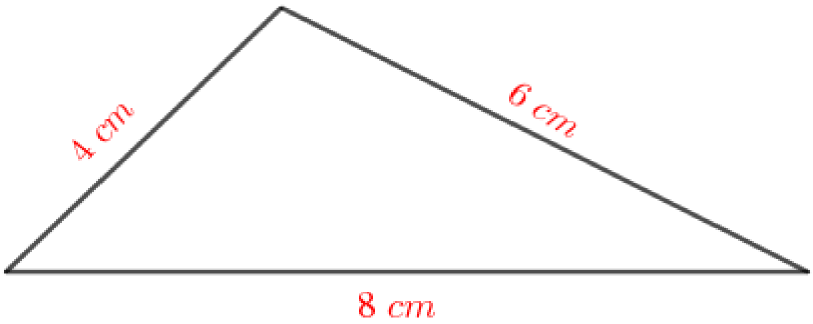
If the sides of two triangles are scaled by the same factor , then their angles are the same

Activity

Draw a triangle of angles the same as those of the triangle shown the figure and sides scaled by 2 .



Answer



More activity

a) Draw a triangle of sides 3 , 4 , 6 centimetres .

b) Draw a triangle of angles the same as those of this triangle and sides scaled by $1\frac{1}{2}$ **.**

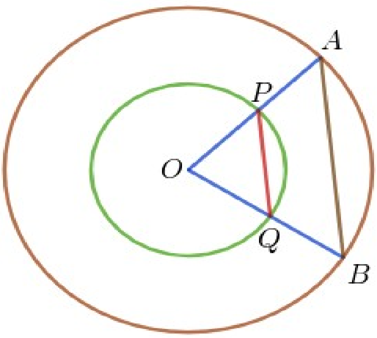
Concept - 3

In triangles with two sides scaled by the same factor and the angle between them the same , the third side are also scaled by the same factor .

Activity

The picture shows two circles centred at O and two triangles formed by joining the centre to the points of intersection of the circles with two radii of the larger circle .

Prove that the triangles OPQ and OAB are similar .



Answer

Take the radius of the smaller circle is r and the radius of the larger circle is R

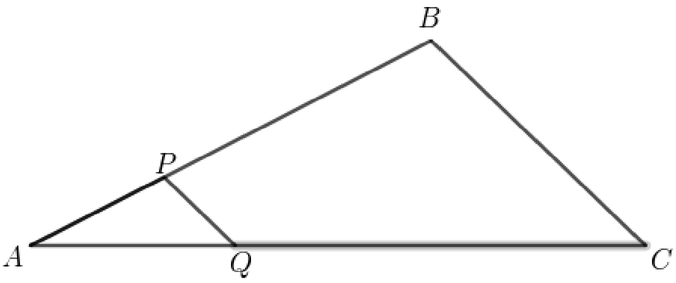
$$\frac{OP}{OA} = \frac{r}{R}$$

$$\frac{OQ}{OB} = \frac{r}{R} \quad ==> \quad \frac{OP}{OA} = \frac{OQ}{OB}$$

$\angle POQ = \angle AOB$ **(common angle)**

==> Triangles OPQ and OAB are similar .

More activity



In the figure AB = 6 centimetres ,

AP = 2 centimetres , AQ = 3centimetres ,

AC = 9 centimetres .

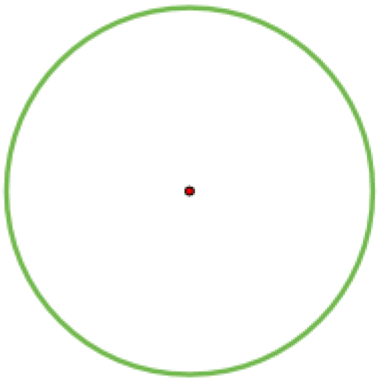
Prove that the angles of the triangles APQ and ABC are the same .

NOTE :

For two triangles to be similar , they have to be related in one of the following ways .

- **Having the same angles .**
- **Having scaled by the same factor .**
- **Having two sides scaled by the same factor and the angles between them equal.**

CIRCLES

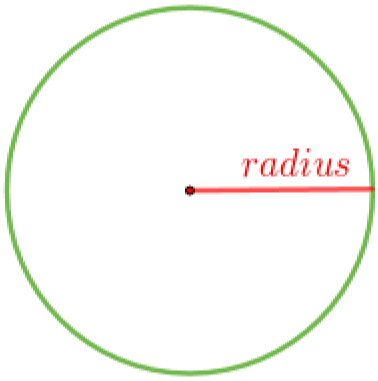


Centre of a circle

The fixed point at which we fix the pointed end of the compass to draw a circle is called the centre of the circle .

Radius of a circle

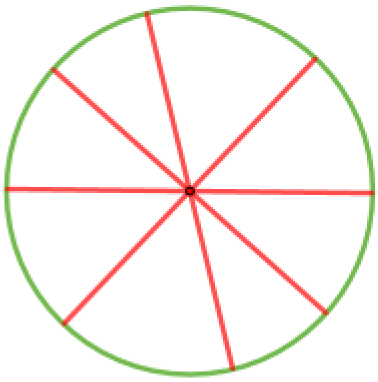
The distance from the centre to the circle is called the radius of the circle .



NOTE :

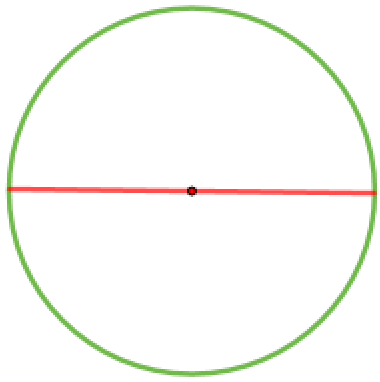
So many radii can be drawn in a circle .

Radii of a circle are equal .



Diameter of a circle

Diameter of a circle is the line joining two points on the circle through its centre .

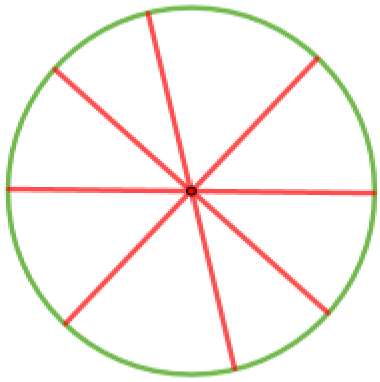


NOTE :, Length of such a line is is also called diameter .

So many diameters can be drawn in a circle .

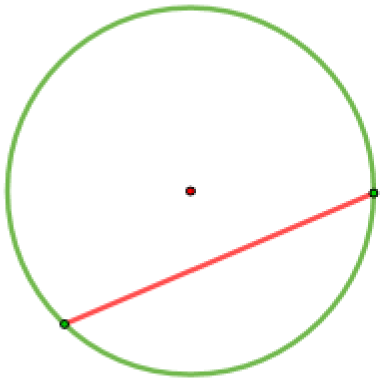
Diameter is twice the radius

(Radius is half the diameter)



Chord

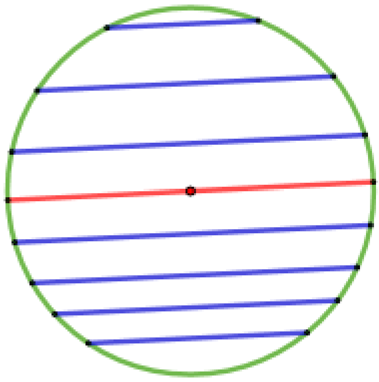
A line joining two points on a circle is called a chord



NOTE :

So many chords can be drawn in a circle .

The longest chord of a circle is its diameter .



Concept - 1

The perpendicular from the centre of a circle to a chord bisects the chord

Concept - 2

The line joining the centre of a circle and the midpoint of a chord is perpendicular to the chord .

Concept - 3

Chords at the same distance from the centre are of the same length

Concept -4

Equal chords are at equal distances from the centre .

Concept - 5

Length of Chords

In a circle , the square of half a chord is the difference of the squares of the radius and the perpendicular from the centre to the chord.

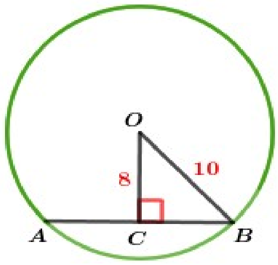
Activity

In a circle , a chord 8 centimetres away from the centre is 10 centimetres long . Compute the length of the chord .

Answer

In right triangle BOC ,

$$Base^2 + Altitude^2 = Hypotenuse^2 \implies BC^2 + OC^2 = OB^2$$



$$\implies BC^2 + 8^2 = 10^2 \implies BC^2 + 64 = 100$$

$$\implies BC^2 = 100 - 64 = 36 \implies BC = \sqrt{36} = 6\ cm$$

Length of the chord = $2 \times BC = 2 \times 6 = 12\ cm$

More activity

In a circle of radius 5 centimetres , two parallel chords of lengths 6 and 8 centimetres are drawn on either side of the a diameter .

a) What is the distance between the parallel chords ?

b) If parallel chords are drawn on the same side of the diameter , what would be the distance between them ?

POINTS AND CIRCLES

We can draw so many circles through a point

We can draw so many circles through two points

If three points are on a line , we cannot draw a circle passing through all these points

Concept - 7

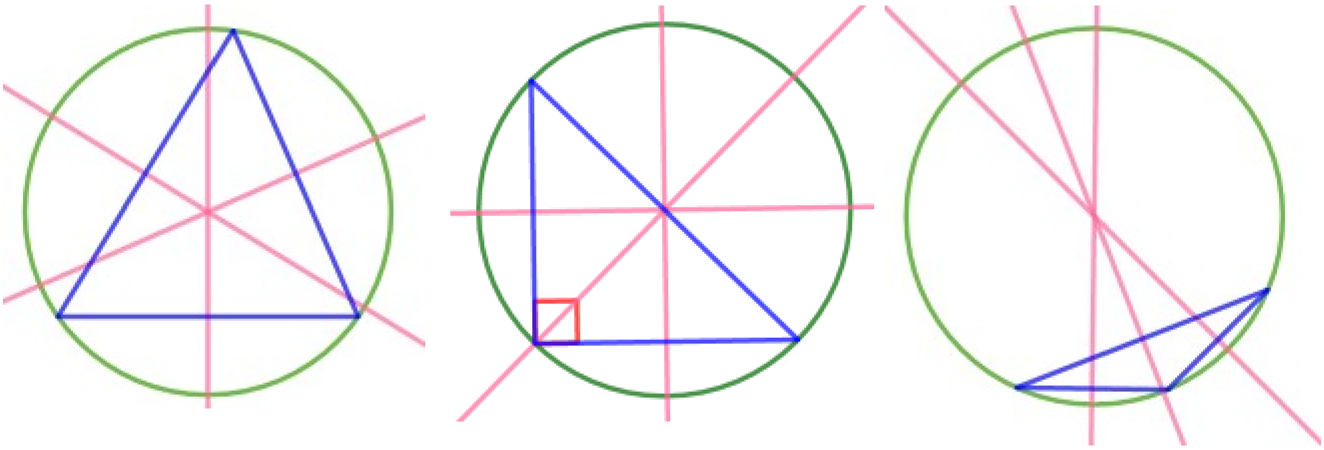
In any triangle , the perpendicular bisectors of all three sides intersect at a single point

If three points are not on a line , we can draw only one circle through these points

Circumcircle of a triangle

A circle passing through all three vertices of a triangle is called the circumcircle of the triangle .

The centre of the circumcircle of a triangle is the point of intersection of the perpendicular bisectors of the sides .

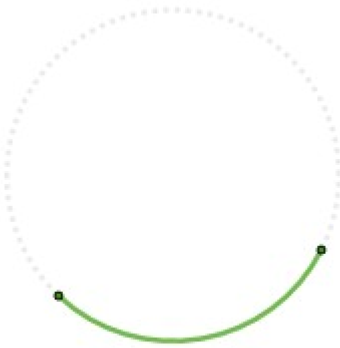


NOTE :

- **The circumcentre of an acute triangle lie inside the triangle .**
- **The circumcentre of a right triangle is the midpoint of its hypotenuse .**
- **The circumcentre of an obtuse triangle lie outside the triangle .**

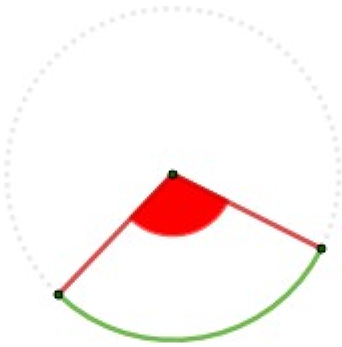
ARC

Any part of a circle between two points on it is called an arc .



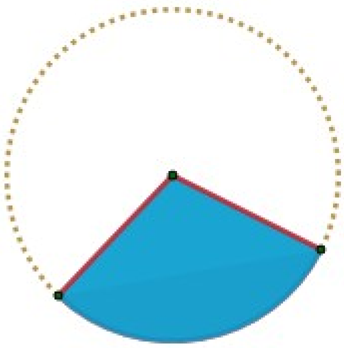
Central angle of an arc

The angle between two radii joining the ends of an arc to the centre of the circle is called the central angle of the arc .



Sector

An arc and the radii through its ends enclose a part of the area of the circle . Such a piece of a circle is called a sector



CIRCLE MEASURES

The perimeters of circles are in the same ratio as their diameters

Ratio of the perimeters of circles are same as the ratio of their diameters

NOTE :

For a circle ,

Ratio of the perimeters = Ratio of the diameters = Ratio of the radii

Concept 1

The perimeter of a circle is 2π times the radius .

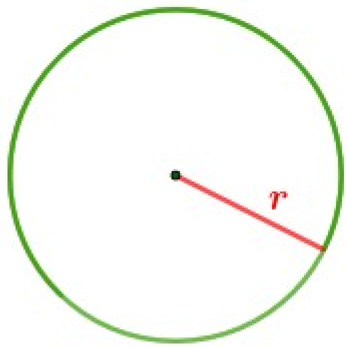
Concept 2

The area of a circle is π times square of the radius

In a circle of radius r ,

Perimeter = $2\pi r$

Area = πr^2



Activity

Radius of a circle is 5 centimetres .

a) Calculate its perimeter .

b) Calculate its area .

Answer

a) Perimeter = $2\pi r = 2\pi \times 5 = 10\pi\ cm$

b) Area = $\pi r^2 = \pi \times 5^2 = 25\pi\ sq.cm$

More activity

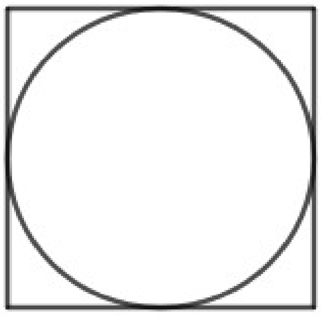
A circle is drawn inside a square as shown in the figure .

Length of the side of a square is 6 centimetres .

a) Find the radius of the circle .

b) Calculate the perimeter of the circle .

b) Calculate the area of the circle .



Concept 3

Length of an arc

The length of an arc is that fraction of the perimeter of the circle as the fraction of 360^0 that its central angle is .

Concept 4

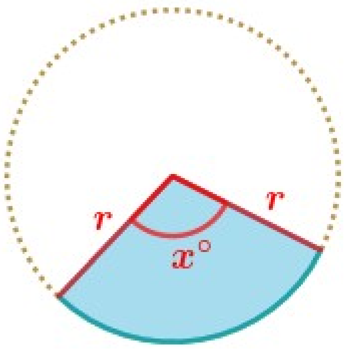
Area of a sector

The area of a sector is that fraction of the area of the circle as the fraction of 360^0 that its central angle is .

In a circle of radius r ,

a) the length of an arc of central angle x^0 is $2\pi r \times \frac{x}{360}$

b) the area of a sector of central angle x^0 is $\pi r^2 \times \frac{x}{360}$



Activity

In a circle of radius 6 centimetres ,

a) What is the length of an arc of central angle 120^0 ?

b) What is the area of a sector of central angle 120^0 ?

Answer

a) Length of the arc = $2\pi \times 6 \times \frac{120}{360} = 4\pi\ cm$

b) Area of the sector = $\pi \times 6^2 \times \frac{120}{360} = \pi \times 36 \times \frac{120}{360} = 12\pi\ sq.cm$

More activity

In a circle , the length of an arc of central angle 60^0 is $6\pi\ cm$

a) Calculate the perimeter of the circle .

b) Calculate the radius of the circle .

c) What is the area of a sector of central angle 60^0 ?

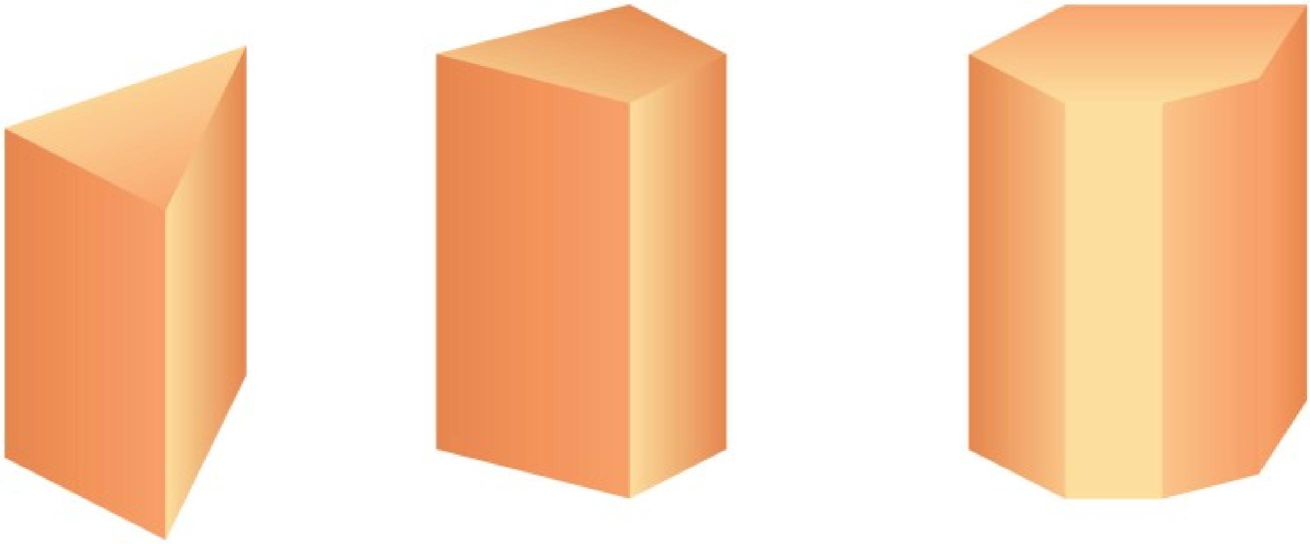
PRISMS

Solids

Geometrical objects having spreads and vertical heights are called solids (Three dimensional objects)

Prisms

Geometrical objects having two identical polygons and rectangles of the same height , with the polygons as opposite sides are called prisms .



Faces of a prism

The polygons and rectangles in a prism are called its faces .

The polygons on the top and bottom are called bases and the rectangles are called lateral faces .

Depending on the shape of the bases , they are named

Concept 1

Volume of a prism

The volume of any prism is the product of its base area and height .

Volume = Base area x Height

Activity

The base of a prism is a square of side 20 centimetres and its height 30 centimetres .

a) Calculate the volume of the prism .

b) How much litres of water can be contained in this prism ?

Answer

a) Volume = Base area x height $= \quad 20 \times 20 \times 30 = 12000\ cubic\ cm$

b) $= \dfrac{12000}{1000} = 12\ litres$

More activity

The base of a prism is an equilateral triangle of perimeter 12 centimetres and its height is 9 centimetres .

a) What is the length of its base edge ?

b) Calculate the volume of the prism .

Concept 2

Lateral surface area

The lateral surface area of any prism is the product of the base perimeter and height

Lateral surface area = Base perimeter x Height

NOTE :

For a closed prism , the total surface area can be calculated by adding the base areas to the lateral surface area .

Activity

The base of a prism is a rectangle of length 10 centimetres and breadth 8 centimetres and its height is 15 centimetres .

a) Calculate its base perimeter .

b) Calculate its lateral surface area .

c) Calculate its total surface area .

Answer

a) Base perimeter $= 2\times(10+8) = 2\times 18 = 36\ cm$

b) Lateral surface area = Base perimeter x Height = $36\times 15 = 540\ sq.cm$

c) Total surface area = Lateral surface area + 2 x Base area

$= 540 + 2\times 10\times 8 = 540 + 160 = 700\ sq.cm$

More activity

The base area of a square prism is 100 square centimetres and its height 8 centimetres .

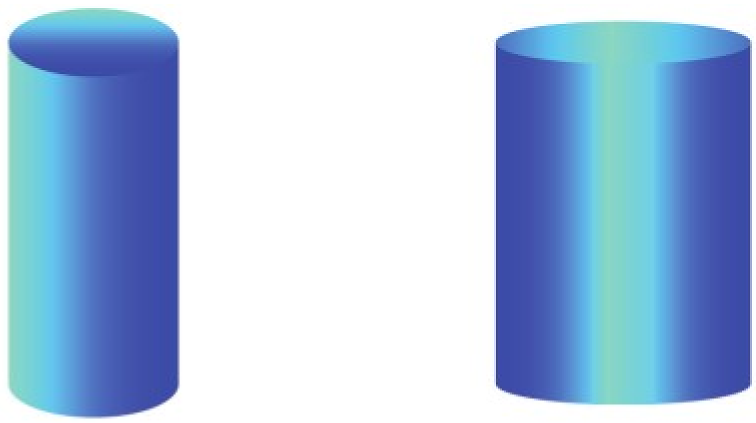
a) What is the length of its base edge ?

b) Calculate its lateral surface area .

c) Calculate its total surface area .

CYLINDER

Solids with circles at both ends and a smoothly curving surface all around are called cylinders .



Concept 3

Volume of a cylinder

The volume of a cylinder is the product of its base area and height .

Activity

The base perimeter of a cylinder is 18π centimetres and its height is 20 centimetres .

a) What is its base radius ?

b) Calculate the volume of the cylinder .

Answer

a) Base perimeter = 18π **cm** ==> $2\pi r = 18\pi$ ==> $r = \frac{18\pi}{2\pi} = 9\ cm$

b) Volume = Base area x Height = $\pi \times 9^2 \times 20 = 1620\pi\ cubic.cm$

More activity

The base of a rectangular block of wood is a square of side 10 centimetres and its height is 25 centimetres . A largest cylinder is carved out of this .

a) What are the base radius and height of the cylinder ?

b) Calculate the volume of the cylinder .

Concept 4

Curved surface area of a cylinder

The curved surface area of a cylinder is the product of the base perimeter and height .

NOTE :

For a closed cylinder , the total surface area can be calculated by adding the base areas to the curved surface area .

Activity

The base diameter of a solid cylinder is 10 centimetres and its height is 12 centimetres .

a) What is its base radius ?

b) Calculate its curved surface area .

c) Calculate its total surface area .

Answer

a) Base radius = $\frac{10}{2} = 5\ cm$

b) Curved surface area = Base perimeter x Height = $2\pi \times 5 \times 12 = 120\pi\ sq.cm$

c) Total surface area = Curved surface area + 2 x Base area

$= 120\pi + 2 \times \pi \times 5^2 = 120\pi + 50\pi = 170\pi\ sq.cm$

More activity

The inner diameter of a well is 3 metres and it is 8 metres deep .

a) What is the base perimeter of the well ?

b) Calculate the curved surface area of the inside of the well ?

c) What would be the cost cementing its inside at 400rupees per square metre ?

PROPORTION

Equality of ratios is called proportion

Concept 1

Direct proportion

Let's take an independent quantity as x and the dependent quantity y . If y is always x multiplied by a fixed quantity k (which does not change with x) , then we can write the relation between them as $y = kx$ .Then the ratio of x to y remains unchanged as $1 : k$

That is , y changes proportional to x . This proportion is known as direct proportion .

Constant of proportionality

The fixed number occurring in the equation of proportional change is called the constant of proportionality .

Activity

a) Prove that the perimeter of a square varies proportional as the length of a side .

b) What is the constant of proportionality here ?

Answer

a) In any square ,the perimeter is 4 times the length of a side .

That is , in any square , the ratio of the length of a side to the perimeter is 1 : 4

That is , the length of a side and perimeter of a square are scaled by the same factor .

That is perimeter of a square varies proportionally as the the length of a square .

b) If the length of a side of a square is x and its perimeter is y , then y = 4 x

Constant of proportionality = 4

More activity

a) Prove that the perimeter of a circle varies proportional as the radius .

b) What is the constant of proportionality here ?

Concept 2

Inverse proportion

Let's take an independent quantity as x and the dependent quantity y . If y is always a quotient obtained by dividing a fixed quantity k by x (k does not change with x) , then we can write the relation between them as $y = \frac{k}{x}$. That is , y changes proportio - nal to the reciprocal of x . This proportion is known as inverse proportion .

Activity

For any regular polygon , we can draw a circle through all vertices .

a) Prove that for regular polygons , angle made by two adjacent vertices at the centre of the circle passing through all the vertices is inversely proportional to the number of sides

b) What is the the constant of proportionality here ?

Answer

Regular polygon	Number of sides	Angle made by two adjacent vertices at the centre of the circle .
Equilateral triangle	3	$\frac{360}{3} = 120^0$
Square	4	$\frac{360}{4} = 90^0$
Regular pentagon	5	$\frac{360}{5} = 72^0$
Regular hexagon	6	$\frac{360}{6} = 60^0$

If the number of sides is x and the measure of the angle made by two adjacent vertices

at the centre of the circle is y , then $y = \frac{360}{x}$

That is , $y = 360 \times \frac{1}{x}$

Here y is proportional to the reciprocal of x .

b) Constant of proportionality = 360

More activity

An object travelling from one one point to a point 100 metres away at a steady speed along a straight line .

a) Prove that the speed of the object is inversely proportional to the time taken to reach the destination .

b) What is the the constant of proportionality here ?

REAL NUMBERS

The natural numbers , tractions and their negatives with zero as well are collectively called *rational numbers* . All other umbers are called *irrational numbers*.

General form of a rational number

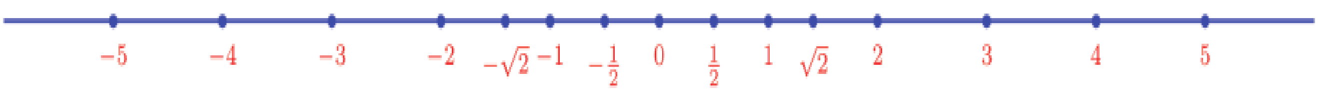
All rational numbers have a common form $\frac{x}{y}$ where x and y are natural numbers or their negatives with x possibly zero also .

Real numbers

The rational and irrational numbers together are called *real numbers*.

Number line

Mark a point on a line and denote it as zero . Mark one more point on this line . Taking the distance from the first point to the second as 1 (the unit of length) , we can write the distances to all other points on the right as numbers .To mark distances to all points , we need irrational numbers also . To mark the numbers to the left of the first point (0) , we use the the negative of the numbers to the right . Thus , all points on this line can be marked using real numbers . On the other hand , all real numbers can be seen as the points on this line . Such a line is called *number line* or *real line* .



For any two real numbers , the position of the larger number on the number line is to the right of the smaller number .

Concept 1

The distance between any two points on the number line is the smaller of the numbers denoting them subtracted from the larger .

Concept 2

The midpoint of two points on the number line is that point denoted by half the sum of the numbers denoting these points .

Activity

Find the distance between the two points on the number line , denoted by each pair of numbers given below .

a) $3 \ , \ -7$ **b)** $-9 \ , \ -1$

Answer

a) Distance $= \ 3 - (-7) = 3 + 7 = 10$

Midpoint $= \dfrac{3 + (-7)}{2} = \dfrac{-4}{2} = -2$

b) Distance $= \ -1 - (-9) = -1 + 9 = 8$

Midpoint $= \dfrac{-9 + (-1)}{2} = \dfrac{-10}{2} = -5$

More activity

Find the distance between the two points on the number line , denoted by each pair of numbers given below .

a) $-16 \ , \ 4$ **b)** $-11 \ , \ -25$

Concept 3

On the number line , the distance between the point denoted by 0 and point denoted by another number is the *absolute value* of that number .

$$|x| = \begin{cases} x, & \text{if } x > 0 \\ -x, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Concept 4

The distance between two points on the number line is the absolute value of the difference of the numbers denoting these points .

The distance between two points denoted by the numbers x and y on the number line is

$$|x - y|$$

Activity

Find x satisfying the equation , $|x-1| = |x-3|$

Answer

$$x = \frac{1+3}{2} = \frac{4}{2} = 2$$

More activity

Find x satisfying each of the equations below .

$a)\quad |x-2| = |x-8|$

$b)\quad |x+3| = |x+7|$

POLYNOMIALS

Look at the algebraic expressions given below

$$x^2 + 5x + 6$$

$$x^3 + 6x^2 + 11x + 6$$

$$49 - 9.8x$$

In all these , the only mathematical operations involved are multiplying different powers of the number x by various numbers and adding or subtracting them .A definite number is also sometimes added or subtracted . An algebraic expression involving such operations is called a *polynomial* .

Note :

x is a changing number .

Features of polynomials

- **Various powers of the changing number .**
- **All powers are natural numbers .**
- **Powers of the changing number are multiplied by definite numbers .**
- **Sum or difference of the products are found .**
- **A definite number is added or subtracted .**

Note :

Algebraic expressions involving the operations of taking reciprocal or square root of the changing number are not polynomials .

Degree of a polynomial

In a polynomial , we take powers of the changing numbers . The largest power used is called the *degree* of the polynomial .

General form of a polynomial

Based on the degree , we can write the general forms of the polynomials .

Polynomial	General form
First degree polynomial	$ax + b$
Second degree polynomial	$ax^2 + bx + c$
Third degree polynomial	$ax^3 + bx^2 + cx + d$

Here the letters a, b, c and d denote fixed numbers (definite numbers) .

They can be any sort of numbers (natural numbers , fractions , numbers which are not fractions or negative numbers) . They are called *coefficients* in a polynomial .

Activity

Find $p(0)$, $p(1)$ *and* $p(-1)$ **in the following polynomials .**

a) $p(x) = 2x + 3$ **b)** $p(x) = x^2 + 5x + 6$

Answer

a) $p(x) = 2x + 3$

$p(0) = 2 \times 0 + 3 = 0 + 3 = 3$

$p(1) = 2 \times 1 + 3 = 2 + 3 = 5$

$p(-1) = 2 \times (-1) + 3 = -2 + 3 = 1$

b) $p(x) = x^2 + 5x + 6$

$p(0) = 0^2 + 5 \times 0 + 6 = 0 + 0 + 6 = 6$

$p(1) = 1^2 + 5 \times 1 + 6 = 1 + 5 + 6 = 12$

$p(-1) = (-1)^2 + 5 \times (-1) + 6 = 1 - 5 + 6 = 2$

More activity

Find $p(0)$, $p(1)$ and $p(-2)$ **in the following polynomials .**

a) $p(x) = x^2 + 7x + 10$

b) $p(x) = x^2 - 2x - 8$

STATISTICS

Frequency table

Activity

The scores of 45 children in a test are given below .

8	7	6	3	8	8	7	7	6
7	9	7	6	8	7	2	6	7
10	6	7	3	9	5	4	5	4
4	4	5	8	10	8	8	9	7
7	6	8	8	7	4	5	9	8

Make a frequency table .

Answer

Score	Number of children (frequency)
2	1
3	2
4	5
5	4
6	6
7	11
8	10
9	4
10	2
Total	45

More activity

The number of members in 50 households of a village are listed below .

8	6	9	4	4	2	6	4	5	3
7	3	3	2	3	7	6	3	2	5
5	13	9	9	7	4	4	5	4	3
3	7	2	3	3	10	8	6	6	4
2	4	5	4	3	8	7	5	6	3

Make a frequency table .

Frequency table with classes

Activity

The runs that a batsman got in 50 one-day cricket matches are given below .

50	0	49	60	100	68	27	48	15	65
101	45	2	52	25	18	29	53	72	90
32	81	28	104	35	49	2	60	87	71
68	20	10	30	55	47	21	35	12	20
38	102	35	11	27	43	38	40	48	71

Make a frequency table .

Answer

Runs	Number of matches (frequency)
0 – 10	3
10 – 20	6
20 – 30	7
30 – 40	7
40 – 50	8
50 – 60	4
60 – 70	5
70 – 80	3
80 – 90	2
90 – 100	1
100 – 110	4

More activity

The weights of the members of the school Health club are given below .

38	39	41	59	48	48	38	58	50	55
39	40	41	49	32	43	45	53	37	44
51	52	33	46	55	36	45	47	43	33

Make a frequency table with the length of the class interval 5

(Hint : Take classes as 30 – 35 , 35 – 40 **, . . .)**

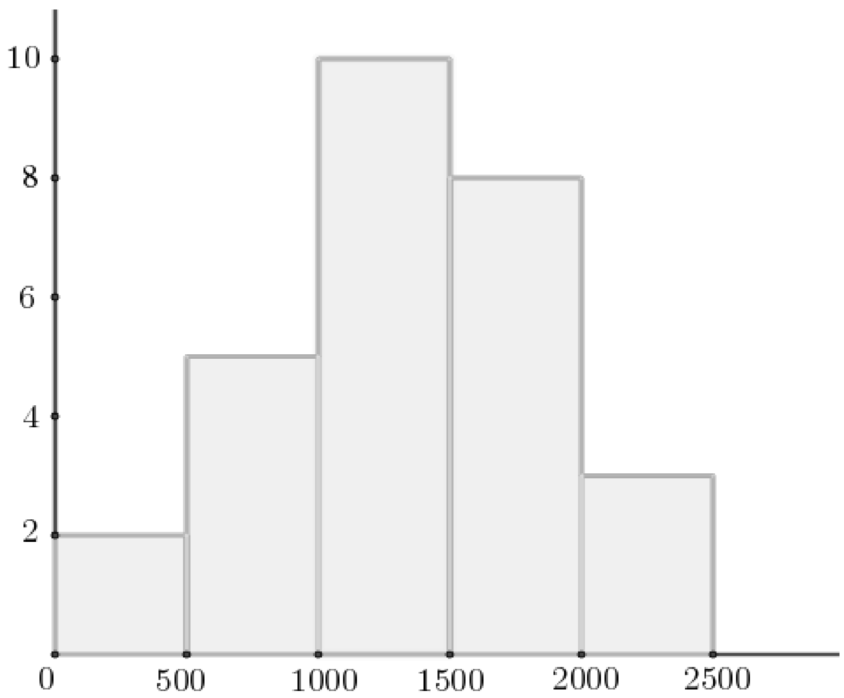
Histogram

Activity

The table below gives the amount of water 28 households use . Draw a histogram .

Amount of water (litres)	Number of households
0 – 500	2
500 – 1000	5
1000 – 1500	10
1500 – 2000	8
2000 – 2500	3

Answer



Note :

In the figure above , the classes are marked on the horizontal line and the frequencies on the vertical line .The width of each rectangle shows the length of the class interval and its height shows the frequency .Such a picture is called a *histogram.*

More activity

Detail of rainfall in June and July are given in the table below . Draw a histogram .

Rainfall (mm)	Number of days
0 – 10	1
10 – 20	4
20 – 30	7
30 – 40	9
40 – 50	15
50 – 60	10
60 – 70	9
70 – 80	6

ARITHMETIC MEAN

The average of any set of numbers between two fixed numbers is also between these two numbers

The number got by dividing the sum by the number (which we usually call average) called *arithmetic mean* or simply *mean* .

Activity

The table shows the labourers in a factory sorted according to their daily wages .

Daily wage (Rs)	Number of workers
300	2
350	4
400	6
450	4
500	4

What is the average daily wage in this factory ?

Answer

Daily wage (Rs)	Number of workers	Total wages (Rs)
300	2	$300 \times 2 = 600$
350	4	$350 \times 4 = 1400$
400	6	$400 \times 6 = 2400$
450	4	$4500 \times 4 = 1800$
500	4	$500 \times 4 = 2000$
Total	20	8200

Average daily wage = $\frac{8200}{20} = 410 \ Rs$

More activity

The table shows the children in a class ,sorted according to the marks they got for a maths test .

Mark	Number of children
2	1
3	2
4	5
5	4
6	6
7	11
8	10
9	4
10	2

Calculate the average mark of the class .

Activity

The table shows the labourers in a factory sorted according to their daily wages .

Daily wages (Rs)	Number of workers
300 – 400	8
400 – 500	6
500 – 600	14
600 – 700	10
700 – 800	7
800 – 900	5

What is the average daily wage in this factory ?

Answer

Daily wages (Rs)	**Number of workers**	**Mid value of the class**	**Total wages (Rs)**
$300 - 400$	8	$\frac{300+400}{2} = 350$	$350 \times 8 = 2800$
$400 - 500$	6	$\frac{400+500}{2} = 450$	$450 \times 6 = 2700$
$500 - 600$	14	$\frac{500+600}{2} = 550$	$550 \times 14 = 7700$
$600 - 700$	10	$\frac{600+700}{2} = 650$	$650 \times 10 = 6500$
$700 - 800$	7	$\frac{700+800}{2} = 850$	$750 \times 7 = 5250$
$800 - 900$	5	$\frac{800+900}{2} = 850$	$850 \times 5 = 4250$
Total	50		29200

Average daily wage = $\frac{29200}{50} = 584\ Rs$

More activity

The table below shows the children in a class , sorted according to their heights .

Height (cm)	**Number of children**
$148 - 152$	8
$152 - 156$	10
$156 - 160$	15
$160 - 164$	10
$164 - 168$	7

What is the mean height of a child in this class ?