CHAPTER 1- SETS

a set is a well-defined collection of objects.

The following points may be noted:
(i) Objects, elements and members of a set are synonymous terms.
(ii) Sets are usually denoted by capital letters A, B, C, X, Y, Z, etc.
(iii) The elements of a set are represented by small letters $a, b, c, x, y, z,$ etc.

There are two methods of representing a set:
(i) Roster or tabular form
(ii) Set-builder form.

(i) In roster form, all the elements of a set are listed, the elements are being separated
by commas and are enclosed within braces { }.

In roster form, the order in which the elements are listed is not important

It may be noted that while writing the set in roster form an element is not generally repeated, i.e., all the elements are taken as distinct.

(ii) In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set

Example 1 Write the solution set of the equation $x^2 + x – 2 = 0$ in roster form.
Solution The given equation can be written as
$(x – 1) (x + 2) = 0$, i. e., $x = 1, – 2$
Therefore, the solution set of the given equation can be written in roster form as $\{1, – 2\}$.

Example 2 Write the set $\{x : x$ is a positive integer and $x^2 < 40\}$ in the roster form.

Solution The required numbers are 1, 2, 3, 4, 5, 6. So, the given set in the roster form
is {1, 2, 3, 4, 5, 6}.

Example 3 Write the set A = {1, 4, 9, 16, 25, . . . }in set-builder form.
Solution We may write the set A as
A = $\{x : x$ is the square of a natural number$\}$
Alternatively, we can write
$A = \{x : x = n^2, \text{ where } n \in \mathbf{N}\}$

Example 4 Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set-builder form.

Solution We see that each member in the given set has the numerator one less than
the denominator. Also, the numerator begin from 1 and do not exceed 6. Hence, in the
set-builder form the given set is

$$\left\{x: x = \frac{n}{n+1}, \text{where n is a natural number and } 1 \leq \text{n} \leq 6\right\}$$

Example 5 Match each of the set on the left described in the roster form with the same set on the right described in the set-builder form:

(i) {P, R, I, N, C, A, L} (a) { x : x is a positive integer and is a divisor of 18}
(ii) { 0 } (b) { x : x is an integer and x2 – 9 = 0}
(iii) {1, 2, 3, 6, 9, 18} (c) {x : x is an integer and x + 1= 1}
(iv) {3, –3} (d) {x : x is a letter of the word PRINCIPAL}

Solution Since in (d), there are 9 letters in the word PRINCIPAL and two letters P and I are repeated, so (i) matches (d). Similarly, (ii) matches (c) as $x + 1 = 1$ implies $x = 0$. Also, 1, 2 ,3, 6, 9, 18 are all divisors of 18 and so (iii) matches (a). Finally, $x^2 – 9 = 0$ implies x = 3, –3 and so (iv) matches (b).

. **Example 6:** Write the following sets in roster form:
(i) A = {x : x is an integer and $–3 \leq x < 7$}
(ii) B = {x : x is a natural number less than 6}
(iii) C = {x : x is a two-digit natural number such that the sum of its digits is 8}
(iv) D = {x : x is a prime number which is divisor of 60}
(v) E = The set of all letters in the word TRIGONOMETRY
(vi) F = The set of all letters in the word BETTER

Solution:
A = {–2, –1, 0, 1, 2, 3, 4, 5, 6}
B = {1, 2, 3, 4, 5}
C = {17, 26, 35, 44, 53, 62, 71, 80} (i.e 1+7=8, 2+6=8 …..)
D = {2, 3, 5}.
E = {T, R, I, G, O, N, M, E, Y}
F = {B, E, T, R}
.

Example 7: Write the following sets in the set-builder form:
(i) (3, 6, 9, 12} (ii) {2,4,8,16,32} (iii) {5, 25, 125, 625}
(iv) {2, 4, 6, . . .} (v) {1,4,9, . . .,100}

Solution:

(i) $\{3, 6, 9, 12\} = \{x: x = 3n, n \in \text{N and } 1 \le n \le 4\}$

(ii) {2, 4, 8, 16, 32}
It can be seen that $2 = 2^1$, $4 = 2^2$, $8 = 2^3$, $16 = 2^4$, and $32 = 2^5$.
$\therefore \{2, 4, 8, 16, 32\} = \{x: x = 2^n, n \in \text{N and } 1 \le n \le 5\}$

(iii) {5, 25, 125, 625}
It can be seen that $5 = 5^1$, $25 = 5^2$, $125 = 5^3$, and $625 = 5^4$.
$\therefore \{5, 25, 125, 625\} = \{x: x = 5^n, n \in \text{N and } 1 \le n \le 4\}$
(iv) {2, 4, 6 …}

It is a set of all even natural numbers.
∴ {2, 4, 6 …} = {x: x is an even natural number}
(v) {1, 4, 9 … 100}

It can be seen that $1 = 1^2$, $4 = 2^2$, $9 = 3^2$ …$100 = 10^2$.
$\therefore \{1, 4, 9\ldots 100\} = \{x: x = n^2, n \in \text{N and } 1 \le n \le 10\}$
Example 8:
List all the elements of the following sets :
(i) A = {x : x is an odd natural number}
(ii) B = {x : x is an integer, $-\frac{1}{2} < x < \frac{9}{2}$}
(iii) C = {x : x is an integer, $x^2 \le 4$}
(iv) D = {x : x is a letter in the word "LOYAL"}
(v) E = {x : x is a month of a year not having 31 days}
(vi) F = {x : x is a consonant in the English alphabet which precedes k }.
Solution:
(i) A = {x: x is an odd natural number} = {1, 3, 5, 7, 9 …}
(ii) B = {x : x is an integer, $-\frac{1}{2} < x < \frac{9}{2}$}, x is between -0.5(- ½) to 4.5($\frac{9}{2}$)
So B = {0,1,2,3,4}
(iii) C = {x: x is an integer; $x^2 \le 4$ }
It can be seen that
$(-1)^2 = 1 \le 4$;
$(-2)^2 = 4 \le 4$;
$(-3)^2 = 9 > 4$
$0^2 = 0 \le 4$
$1^2 = 1 \le 4$ $\quad 2^2 = 4 \le 4$ $\quad 3^2 = 9 > 4$ $\quad$ so, C = {–2, –1, 0, 1, 2}

(iv) D = (x: x is a letter in the word "LOYAL") = {L, O, Y, A}
(v) E = {x: x is a month of a year not having 31 days}
= {February, April, June, September, November}

(vi) F = {x: x is a consonant in the English alphabet which precedes k}
= {b, c, d, f, g, h, j} (consonant: except a, e, i, o, u in alphabet)

Example 8

Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

(i) {1, 2, 3, 6}	(a) {x : x is a prime number and a divisor of 6}
(ii) {2, 3}	(b) {x : x is an odd natural number less than 10}
(iii) {M,A,T,H,E,I,C,S}	(c) {x : x is natural number and divisor of 6}
(iv) {1, 3, 5, 7, 9}	(d) {x : x is a letter of the word MATHEMATICS}.

Solution:

(i) {1, 2, 3, 6}	(c) {x : x is natural number and divisor of 6}
(ii) {2, 3}	(a) {x : x is a prime number and a divisor of 6}
(iii) {M,A,T,H,E,I,C,S}	(d) {x : x is a letter of the word MATHEMATICS}.
(iv) {1, 3, 5, 7, 9}	(b) {x : x is an odd natural number less than 10}

PYQ AND EXPECTED QUESTIONS

1. Consider the sets A and B given by (MARCH-2010)
A = {x: x is a natural number, $2 \leq x \leq 6$ }
B = {x: x is a prime number $x \leq 7$ }
Write A, B and C in the roster form

ANS: A = {2,3,4,5,6}; B = {2,3, 5,7}

2. Consider the sets A and B given by (MARCH-2011)
A = {x: x is a prime number <10}
B = {y. x is a natural number which divides 12}
Write A and B in the roster form.

ANS:
A = (2,3,5,7}; B = {1,2,3,4,6,12}

3.

Let A = {x: x is an integer $\frac{1}{2}<x<\frac{7}{2}$ } and B = {2,3,4} (MARCH-2012)
Write A in the roster form.

ANS:
A = {1,2,3} since x lies between 0.5 (½) and 3.5 ($\frac{7}{2}$)

4. Let A = {x: x ∈ W, x<5} and B = (x : x is a prime number less than 5}
and U = {x: x is an integer $0 \le x \le 6$} (MARCH-2015)
Write A,B in roster form.

ANS: A= {0,1,2,3,4}; B = {2,3} (W is the set of whole numbers)

5. A: {x : x is a prime number $x \le 6$} (IMP-2015)
Represent A in Roster form

ANS: A ={2,3,5}

6. Write the following sets in the roaster form.
(i) A = {x | x is a positive integer less than 10 and $2^x - 1$ is an odd number}
(ii) C = {$x : x^2 + 7x - 8 = 0, x \in \mathbf{R}$}

ANS:
(i) $2^x - 1$ is always an odd number for all positive integral values of x.
In particular, $2^x - 1$ is an odd number for $x = 1, 2, ..., 9$.
Thus, A = {1, 2, 3, 4, 5, 6, 7, 8, 9}.
(ii) $x^2 + 7x - 8 = 0$ or $(x + 8)(x - 1) = 0$ giving $x = -8$ or $x = 1$
Thus, C = {– 8, 1}

7. Write the following sets in the roaster from
(i) A = {$x : x \in \mathbf{R}, 2x + 11 = 15$} (ii) B = {$x \mid x^2 = x, x \in \mathbf{R}$}
(iii) C = {x | x is a positive factor of a prime number p}
ANS:
(i) {2} (ii) {0, 1} (iii) {1, p}

Subsets

A set A is said to be a subset of a set B if every element of A is also an element of B.

In other words, $A \subset B$ if whenever $a \in A$, then $a \in B$. It is often convenient to use the symbol "$\Rightarrow$" which means *implies*. Using this symbol, we can write the definiton
of *subset* as follows:

$A \subset B$ if $a \in A \Rightarrow a \in B$

We read the above statement as

"*A is a subset of B if a is an element of A implies that a is also an element of B***".**

If A is not a subset of B, we write $A \not\subset B$.

ϕ *is a subset of every set***.**

Let A and B be two sets. If $A \subset B$ and $A \neq B$, then A is called a *proper subset of* B and B is called *superset* of A.

If a set A has only one element, we call it a *singleton set*. Thus,{ a } is a singleton set.

Example 9 Consider the sets

ϕ, A = { 1, 3 }, B = {1, 5, 9}, C = {1, 3, 5, 7, 9}.

Insert the symbol $\subset$ or $\not\subset$ between each of the following pair of sets:

(i) ϕ **. . .** B (ii) A . . . B (iii) A . . . C (iv) B . . . C

Solution

(i) $\phi \subset B$ as ϕ is a subset of every set.

(ii) $A \not\subset B$ as $3 \in A$ and $3 \notin B$

(iii) $A \subset C$ as $1, 3 \in A$ also belongs to C

(iv) $B \subset C$ as each element of B is also an element of C.

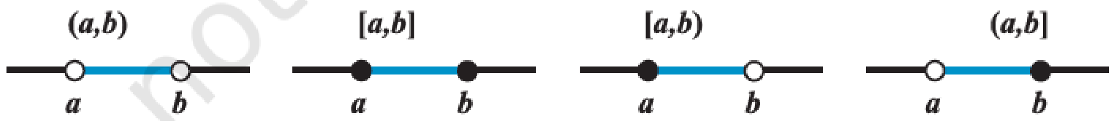
Note: $\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q}$, $\mathbf{Q} \subset \mathbf{R}$, $\mathbf{T} \subset \mathbf{R}$, $\mathbf{N} \not\subset \mathbf{T}$.

N : the set of all natural numbers
Z : the set of all integers
Q : the set of all rational numbers
R : the set of real numbers
Z+ : the set of positive integers
Q+ : the set of positive rational numbers, and
R+ : the set of positive real numbers.

Intervals as subsets of R

Let $a, b \in$ R and $a < b$. Then the set of real numbers $\{ y : a < y < b\}$ is called an *open interval* and is denoted by (a, b). All the points between a and b belong to the open interval (a, b) but a, b themselves do not belong to this interval. The interval which contains the end points also is called *closed interval* and is denoted by $[a, b]$. Thus $[a, b] = \{x : a \le x \le b\}$

We can also have intervals closed at one end and open at the other, i.e.,
$[a, b) = \{x : a \le x < b\}$ is an *open interval* from a to b, including a but excluding b.
$(a, b] = \{ x : a < x \le b \}$ is an *open interval* from a to b including b but excluding a.



The number $(b – a)$ is called the *length of any of the intervals* (a, b), $[a, b]$, $[a, b)$ or $(a, b]$.

Example 10

Write the following as intervals:
(i) $\{x : x \in$ R, $– 4 < x \le 6\}$ (ii) $\{x : x \in$ R, $– 12 < x < –10\}$
(iii) $\{x : x \in$ R, $0 \le x < 7\}$ (iv) $\{x : x \in$ R, $3 \le x \le 4\}$

Solution

(i) $\{x: x \in$ R, $–4 < x \le 6\} = (–4, 6]$

(ii) $\{x: x \in$ R, $–12 < x < –10\} = (–12, –10)$

(iii) $\{x: x \in$ R, $0 \le x < 7\} = [0, 7)$

(iv) $\{x: x \in$ R, $3 \le x \le 4\} = [3, 4]$

Example 11

Write the following intervals in set-builder form :
(i) $(– 3, 0)$ (ii) $[6 , 12]$ (iii) $(6, 12]$ (iv) $[–23, 5)$

Solution

(i) $(–3, 0) = \{x: x \in$ R, $–3 < x < 0\}$
(ii) $[6, 12] = \{x: x \in$ R, $6 \le x \le 12\}$
(iii) $(6, 12] =\{x: x \in$ R, $6 < x \le 12\}$

(iv) $[–23, 5) = \{x: x \in$ R, $–23 \le x < 5\}$

Operations on Sets

Union of sets Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements
being taken only once. The symbol '$\cup$' is used to denote the *union*. *Symbolically, we*
write $A \cup B$ *and usually read as 'A union B'*.

Example 12 Let A = { 2, 4, 6, 8} and B = { 6, 8, 10, 12}. Find $A \cup B$.

Solution We have $A \cup B$ = { 2, 4, 6, 8, 10, 12}

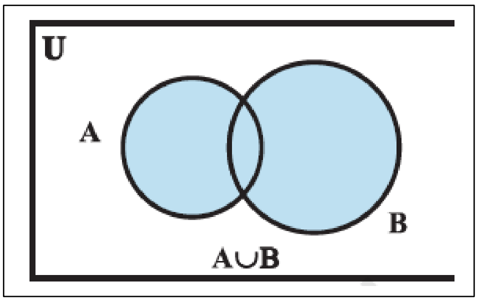
Note that the common elements 6 and 8 have been taken only once while writing
$A \cup B$.

Example 13 Let A = { *a, e, i, o, u* } and B = { *a, i, u* }. Show that $A \cup B = A$

Solution We have, $A \cup B$ = { *a, e, i, o, u* } = A.

This example illustrates that union of sets A and its subset B is the set A itself, i.e., if $B \subset A$, then $A \cup B = A$.

The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both). In symbols, we write.
$A \cup B = \{ x : x \in A \text{ or } x \in B \}$



Some Properties of the Operation of Union

(i) $A \cup B = B \cup A$ (Commutative law)
(ii) $(A \cup B) \cup C = A \cup (B \cup C)$
(Associative law)
(iii) $A \cup \phi = A$ (Law of identity element, ϕ is the identity of $\cup$)
(iv) $A \cup A = A$ (Idempotent law)
(v) $U \cup A = U$ (Law of U)

Intersection of sets The intersection of sets A and B is the set of all elements which are common to both A and B. The symbol '$\cap$' is used to denote the *intersection*.
The intersection of two sets A and B is the set of all those elements which belong to
both A and B. Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

Example 15 Consider the sets A and B of Example 12. Find $A \cap B$.

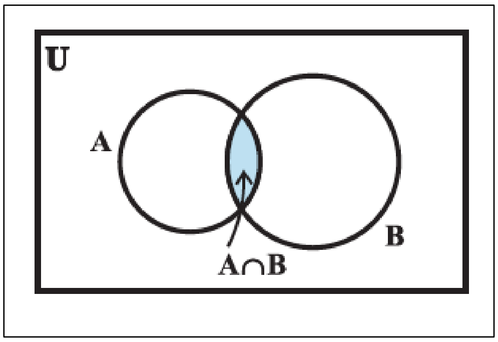
Solution We see that 6, 8 are the only elements which are common to both A and B.
Hence $A \cap B = \{ 6, 8 \}$.

Example 16 Consider the sets X and Y of Example 14. Find $X \cap Y$.

Solution We see that element 'Geeta' is the only element common to both. Hence,
$X \cap Y = \{\text{Geeta}\}$.

Example 17 Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{ 2, 3, 5, 7 \}$. Find $A \cap B$ and
hence show that $A \cap B = B$.

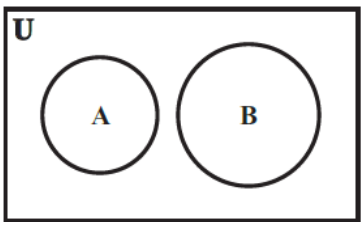
Solution We have $A \cap B = \{ 2, 3, 5, 7 \} = B$. We note that $B \subset A$ and that $A \cap B = B$.



The intersection of two sets A and B
is the set of all those elements which belong to both A and B. Symbolically, we write
$A \cap B = \{x : x \in A \text{ and } x \in B\}$

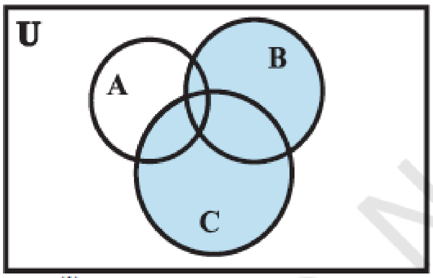
If A and B are two sets such that $A \cap B = \phi$, then A and B are called *disjoint sets*.

Disjoint sets

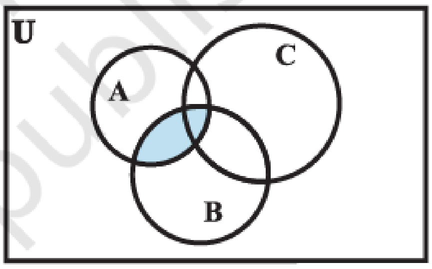


Some Properties of Operation of Intersection

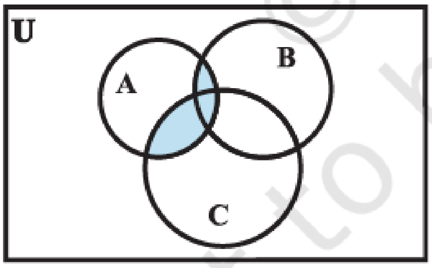
(i) $A \cap B = B \cap A$ (Commutative law).
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).
(iii) $\phi \cap A = \phi$, $U \cap A = A$ (Law of ϕ and U).
(iv) $A \cap A = A$ (Idempotent law)
(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) i. e., $\cap$ distributes over $\cup$



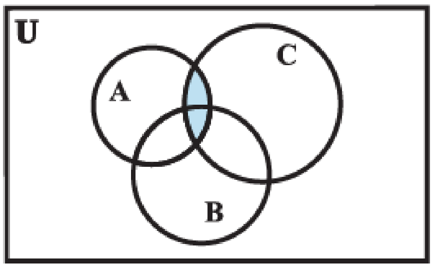
(i) (B∪C)



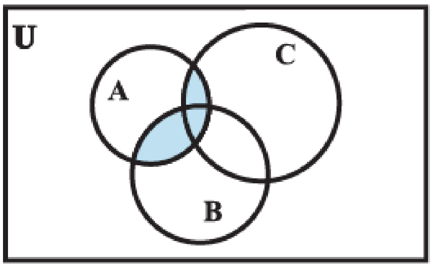
(iii) (A∩B)



(ii) A∩(B∪C)



(iv) (A∩C)



(v) (A∩B) ∪ (A∩C)

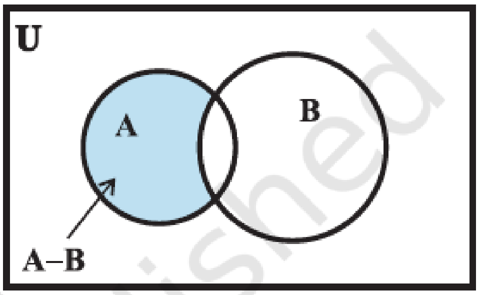
Difference of sets

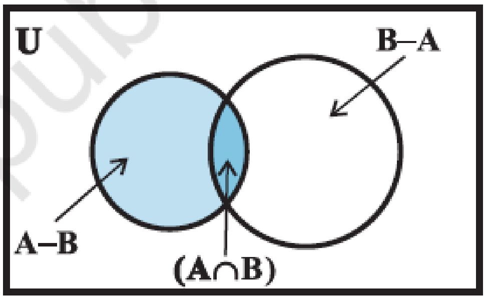
The difference of the sets A and B in this order is the set
of elements which belong to A but not to B. Symbolically, we write A – B and read as
" A minus B".

Example 18 Let A = { 1, 2, 3, 4, 5, 6}, B = { 2, 4, 6, 8 }. Find A – B and B – A.

Solution We have, A – B = { 1, 3, 5 }, since the elements 1, 3, 5 belong to A but
not to B and B – A = { 8 }, since the element 8 belongs to B and not to A.
We note that A – B ≠ B – A.

Remark The sets A – B, A ∩ B and B – A are
mutually disjoint sets, i.e., the intersection of any of
these two sets is the null set





Example 19: Let A = { a, b }, B = {a, b, c}. Is A ⊂ B ? What is A ∪ B ?

Solution

Here, A = {a, b} and B = {a, b, c} Yes, A ⊂ B. A ∪ B = {a, b, c} = B

Example 20:

If A and B are two sets such that A ⊂ B, then what is A ∪ B ?

Solution

If A and B are two sets such that A ⊂ B, then A ∪ B = B.

Example 21:

If A = { 3, 5, 7, 9, 11 }, B = {7, 9, 11, 13}, C = {11, 13, 15}and D = {15, 17}; find

(i) A ∩ (B ∪ C) (ii) A ∩ D (iii) A ∩ (B ∪ D) (iv) (A ∩ B) ∩ (B ∪ C)

Solution

(i) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) = {7, 9, 11} {11} = {7, 9, 11}

(ii) A ∩ D = Φ

(iii) A ∩ (B ∪ D) = (A ∩ B) ∪ (A ∩ D) = {7, 9, 11} ∪ Φ = {7, 9, 11}

(iv) (A ∩ B) ∩ (B ∪ C) = {7, 9, 11} ∩ {7, 9, 11, 13, 15} = {7, 9, 11}

Example 22:

If X= { a, b, c, d } and Y = { f, b, d, g}, find

(i) X – Y (ii) Y – X (iii) X ∩ Y

Solution

(i) X – Y = {a, c}

(ii) Y – X = {f, g}

(iii) X ∩ Y = {b, d}

Complement of a Set

Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A′ to denote the complement of A with respect to U. Thus,

A′ = {x : x ∈ U and x ∉ A }. Obviously A′ = U – A

If A is a subset of the universal set U, then its complement A′ is also a subset of U.

Example 23 Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 3, 5, 7, 9}. Find A′.

Solution We note that 2, 4, 6, 8, 10 are the only elements of U which do not belong to A. Hence A′ = { 2, 4, 6, 8,10 }.

De Morgan's laws.

The complement of the union of two sets is the intersection of their complements and the complement of the intersection of two sets is the union of their complements. These are called *De Morgan's laws*.

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Some Properties of Complement Sets

1. Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$

2. De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

3. Law of double complementation : $(A')' = A$

4. Laws of empty set and universal set $\phi' = U$ and $U' = \phi$.

Example 23

If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$. Verify that (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Solution

(i) $(A\cup B)'=\{2,3,4,5,6,7,8\}'=\{1,9\}$

$A'\cap B'=\{1,3,5,7,9\}\cap\{1,4,6,8,9\}=\{1,9\}$

$(A\cup B)'=A'\cap B'$

(ii) $(A\cap B)'=\{2\}'=\{1,3,4,5,6,7,8,9\}$

$A'\cup B'=\{1,3,5,7,9\}\cup\{1,4,6,8,9\}=\{1,3,4,5,6,7,8,9\}$

$(A\cap B)'=A'\cup B'$

Example 23

Fill in the blanks to make each of the following a true statement:

(i) $A \cup A' = \ldots$ (ii) $\phi' \cap A = \ldots$

(iii) $A \cap A' = \ldots$ (iv) $U' \cap A = \ldots$

Solution

(i) $A \cup A' = U$

(ii) $\Phi' \cap A = U \cap A = A$

$\therefore \Phi' \cap A = A$

(iii) $A \cap A' = \Phi$

(iv) $U' \cap A = \Phi \cap A = \Phi$

$\therefore U' \cap A = \Phi$

Practical Problems on Union and Intersection of Two Sets

Let A and B be finite sets. If $A \cap B = \phi$

(i) $n(A \cup B) = n(A) + n(B)$

(ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

If A, B and C are finite sets, then

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Example 24 If X and Y are two sets such that $X \cup Y$ has 50 elements, X has 28 elements and Y has 32 elements, how many elements does $X \cap Y$ have ?

Solution Given that

$n(X \cup Y) = 50$, $n(X) = 28$, $n(Y) = 32$,

$n(X \cap Y) = ?$

By using the formula

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$,

we find that

$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$

$= 28 + 32 - 50 = 10$

Example 25 In a school there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics ?

Solution Let M denote the set of teachers who teach mathematics and P denote the set of teachers who teach physics. In the statement of the problem, the word 'or' gives us a clue of union and the word 'and' gives us a clue of intersection. We, therefore, have

$n(M \cup P) = 20$, $n(M) = 12$ and $n(M \cap P) = 4$

We wish to determine $n(P)$. Using the result

$n(M \cup P) = n(M) + n(P) - n(M \cap P)$,

we obtain $20 = 12 + n(P) - 4$

Thus $n(P) = 12$

Hence 12 teachers teach physics

Example 26 In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football?

Solution Let X be the set of students who like to play cricket and Y be the set of students who like to play football. Then $X \cup Y$ is the set of students who like to play at least one game, and $X \cap Y$ is the set of students who like to play both games. Given $n(X) = 24$, $n(Y) = 16$, $n(X \cup Y) = 35$, $n(X \cap Y) = ?$

Using the formula $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$, we get

$35 = 24 + 16 - n(X \cap Y)$ Thus, $n(X \cap Y) = 5$ i.e., 5 students like to play both games.

Example 27 In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.

Solution Let U denote the set of surveyed students and A denote the set of students taking apple juice and B denote the set of students taking orange juice. Then

n (U) = 400, n (A) = 100, n (B) = 150 and n (A ∩ B) = 75.

Now n (A′ ∩ B′) = n (A ∪ B)′

= n (U) – n (A ∪ B)

= n (U) – n (A) – n (B) + n (A ∩ B)

= 400 – 100 – 150 + 75 = 225

Hence 225 students were taking neither apple juice nor orange juice.

Example 28

If X and Y are two sets such that n (X) = 17, n (Y) = 23 and n (X ∪ Y) = 38, find n (X ∩ Y).

Solution

It is given that:

$n(X)=17, n(Y)=23, n(X\cup Y)=38$

$n(X\cap Y)=?$

Using the formula

$n(X\cup Y)=n(X)+n(Y)-n(X\cap Y)$

$\therefore 38=17+23-n(X\cap Y)$

$\Rightarrow n(X\cap Y)=40-38=2$

$\therefore n(X\cap Y)=2$

Example 29

If X and Y are two sets such that X ∪ Y has 18 elements, X has 8 elements and Y has 15 elements; how many elements does X ∩ Y have?

Solution

It is given that,

$n(X\cup Y)=18, n(X)=8, n(Y)=15$

$n(X\cap Y)=?$

We know that,

$n(X\cup Y)=n(X)+n(Y)-n(X\cap Y)$

$\therefore 18=8+15-n(X\cap Y)$

$\Rightarrow n(X\cap Y)=23-18=5$

$\therefore n(X\cap Y)=5$

Example 30

In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Solution

Let H be the set of people who speak Hindi, and E be the set of people who speak English
∴ $n(H \cup E) = 400$, $n(H) = 250$, $n(E) = 200$ $n(H \cap E) = ?$
We know that: $n(H \cup E) = n(H) + n(E) - n(H \cap E)$
∴ $400 = 250 + 200 - n(H \cap E)$
⇒ $400 = 450 - n(H \cap E)$ ⇒ $n(H \cap E) = 450 - 400$
∴ $n(H \cap E) = 50$
Thus, 50 people can speak both Hindi and English.

Example 30

In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

Solution

Let C denote the set of people who like coffee, and T denote the set of people who like tea $n(C \cup T) = 70$, $n(C) = 37$, $n(T) = 52$ We know that:
$n(C \cup T) = n(C) + n(T) - n(C \cap T)$ ∴ $70 = 37 + 52 - n(C \cap T)$
⇒ $70 = 89 - n(C \cap T)$
⇒ $n(C \cap T) = 89 - 70 = 19$
Thus, 19 people like both coffee and tea.

Example 31

In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Solution

Let F be the set of people in the committee who speak French, and
S be the set of people in the committee who speak Spanish
∴ $n(F) = 50$, $n(S) = 20$, $n(S \cap F) = 10$
We know that: $n(S \cup F) = n(S) + n(F) - n(S \cap F)$
$= 20 + 50 - 10$
$= 70 - 10 = 60$
Thus, 60 people in the committee speak at least one of the two languages

PYQ AND EXPECTED QUESTIONS FOR PRACTICE

1.

a) If A = {2,3,4,5} and B = {4,5,6,7} then write:

i) A ∪B

ii) A ∩B

b) Which one of the following is equal to?

$\{x: x\epsilon R, 2 < x \leq 4\}$

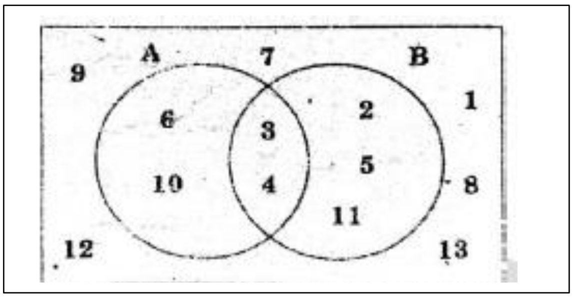
i) {2,3,4} ii) {3,4} iii) [2,4] iv) (2,4]

(2+1) IMPROVEMENT 2018

2. Consider a Venn diagram of the Universal set U = {1,2,3,4,5,6,7,8,9,10,11,12,13}

a) Write sets A, B in Roster form.

b) Verify **(A ∪ B)´ = A′ ∩ B′**



(1+2) [**MARCH 2018]**

3. If U={ 1,2,3,4,5,6,7,8}, A = {2,4,6,8} and B= {2,4,8}, then

a) Write A′ and B′

b) For the above sets A and B, prove that (A ∪ B)´ = A′ ∩ B′

c) Check whether, (A ∩ B)′ = A′ ∪ B′

(1+2+1) **IMPROVEMENT 2016**

4. Out of 25 members in an office 17 like to take tea, 16 like to take coffee. Assume that each takes at least one of the two drinks. How many like:

i) Both coffee and tea?

ii) Only tea and not coffee?

(2) IMPROVEMENT 2015

5. Let A = $\{x: x\epsilon W, x < 5\}$, and

B = $\{x: x \text{ } is \text{ } a \text{ } prime \text{ } number \text{ } less \text{ } than \text{ } 5\}$

U= $\{x: x \text{ } is \text{ } an \text{ } integer \text{ } 0 \leq x \leq 6\}$

a) Write A, B in roster form

b) Find (A-B) ∪ (B-A)

(1+2)MARCH 2015

6.

Given U = {1,2,3,4,5,6,7,8,9,10},A= {1,2,3,4,5} and B = {3,4,5,6}

i) Write A∪B

ii) Verify whether (A ∪ B)´ = A′ ∩ B′

iii) Verify whether n(A∪B) = n(A-B)+n(A∩B)+n(B – A)

(1+2+2)(IMP-2011)