6. LINEAR INEQUALITIES

A statement involving the symbols ‘>’, ‘<’, ‘ ≥’, ‘≤’ is called an inequality. For example $5 > 3$, $x \leq 4$, $x + y \geq 9$.
(i) Inequalities which do not involve variables are called numerical inequalities. For example $3 < 8$, $5 \geq 2$.
(ii) Inequalities which involve variables are called literal inequalities. For example, $x > 3$, $y \leq 5$, $x - y \geq 0$.
(iii) An inequality may contain more than one variable and it can be linear, quadratic or cubic etc. For eaxmple, $3x - 2 < 0$ is a linear inequality in one variable, $2x + 3y \geq 4$ is a linear inequality in two variables and $x^2 + 3x + 2 < 0$ is a quadratic inequality in one variable.
(iv) Inequalities involving the symbol ‘>’ or ‘<’ are called **strict inequalities**. For example, $3x - y > 5$, $x < 3$.
(v) Inequalities involving the symbol ‘≥’ or ‘≤’ are called **slack inequalities**. For example, $3x - y \geq 5$, $x \leq 5$.

Solution of an inequality

(i) The value(s) of the variable(s) which makes the inequality a true statement is called its **solutions**. The set of all solutions of an inequality is called the **solution set** of the inequality. For example, $x - 1 \geq 0$, has infinite number of solutions as all real values greater than or equal to one make it a true statement. The inequality $x^2 + 1 < 0$ has no solution in **R** as no real value of x makes it a true statement.

To solve an inequality we can

(i) Add (or subtract) the same quantity to (from) both sides without changing the sign of inequality.
(ii) Multiply (or divide) both sides by the same positive quantity without changing the sign of inequality. However, if both sides of inequality are multiplied (or divided) by the same negative quantity the sign of inequality is reversed, i.e., ‘>’ changes
into ‘<’ and vice versa.

Representation of solution of linear inequality in one variable on a number line

To represent the solution of a linear inequality in one variable on a number line, we use the following conventions:
(i) If the inequality involves ‘≥’ or ‘≤’, we draw filled circle (•) on the number line to indicate that the number corresponding to the filled circle is included in the solution set.
(ii) If the inequality involves ‘>’ or ‘<’, we draw an open circle (O) on the number
number

line to indicate that the number corresponding to the open circle is excluded from the solution set.

Graphical representation of the solution of a linear inequality

(a) To represent the solution of a linear inequality in one or two variables graphically in a plane, we proceed as follows:
(i) If the inequality involves '≥' or '≤', we draw the graph of the line as a thick line to indicate that the points on this line are included in the solution set.
(ii) If the inequality involves '>' or '<', we draw the graph of the line as dotted line to indicate that the points on the line are excluded from the solution set.
(b) Solution of a linear inequality in one variable can be represented on number line as well as in the plane but the solution of a linear inequality in two variables of the type $ax + by > c$, $ax + by \geq c$, $ax + by < c$ or $ax + by \leq c$ $(a \neq 0, b \neq 0)$ can be represented in the plane only.
(c) Two or more inequalities taken together comprise a system of inequalities and the solutions of the system of inequalities are the solutions common to all the inequalities comprising the system.

Two important results

(a) If $a, b \in \mathbf{R}$ and $b \neq 0$, then
(i) $ab > 0$ or $\frac{a}{b} > 0 \Rightarrow$ a and b are of the same sign.
(ii) $ab < 0$ or $\frac{a}{b} < 0 \Rightarrow$ a and b are of opposite sign.

Example 9 Solve $3x + 2y > 6$ graphically.

$3x+2y = 6$

x	y
0	3
2	0

Dotted lines(---) are used when Inequalities are (>,<)
Dark lines (———) when Inequalities are (≥, ≤)

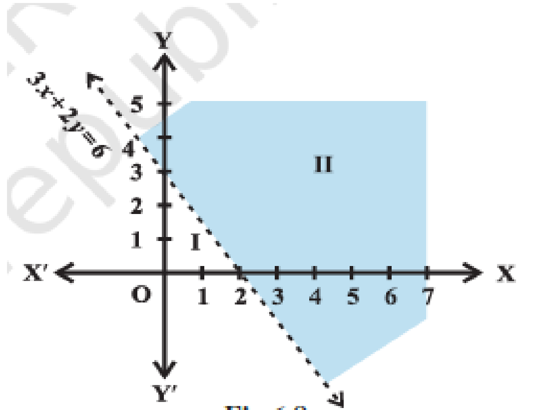


Fig 6.8

Example 10 Solve $3x - 6 \geq 0$ graphically in two dimensional plane

$3x - 6 = 0$

We select a point, say (0, 0) and substituting it in given inequality, we see that:

$3(0) - 6 \geq 0$ or $-6 \geq 0$ which is false.

Thus, the solution region is the shaded region on the right hand side of the line $x = 2$.

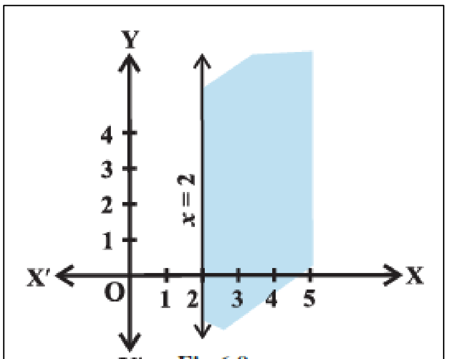


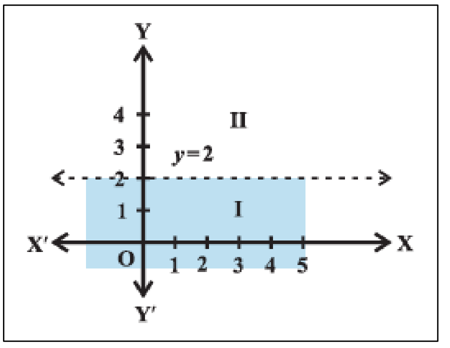
Fig 6.9

Example 11 Solve $y < 2$ graphically

$y = 2$ select a point, (0, 0) in lower half plane I and putting $y = 0$ in the given inequality, we see that

$1 \times 0 < 2$ or $0 < 2$ which is true.

Thus, the solution region is the shaded region below the line $y = 2$.



Solution of System of Linear Inequalities in Two Variables

Example 12 Solve the following system of linear inequalities graphically.

$x + y \geq 5$ ... (1)

$x - y \leq 3$ ... (2)

Solution

x+y = 5

x	y
0	5
5	0

x-y =3

x	y
0	-3
3	0

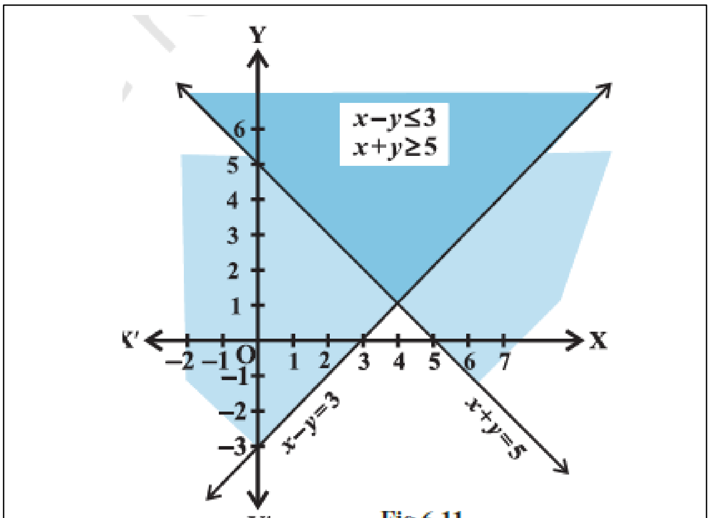


Fig 6.11

the double shaded region, common to the above two shaded regions is the required solution region of the given system of inequalities.

Example 13 Solve the following system of inequalities graphically

$5x + 4y \leq 40$ ... (1)

$x \geq 2$ ... (2)

$y \geq 3$ ... (3)

$5x + 4y = 40$

x	y
0	10
8	0

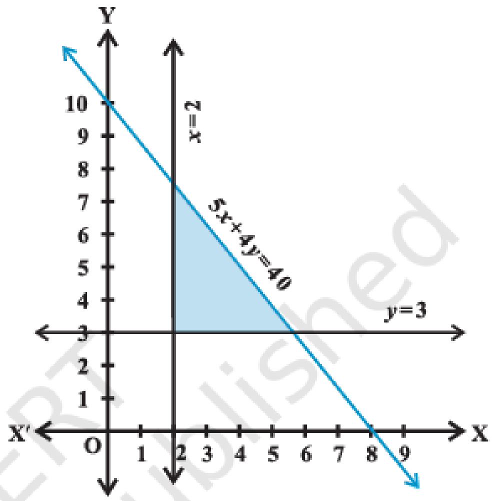


Fig 6.12

inequality (2) represents the shaded region right of line $x = 2$ but inequality (3) represents the shaded region above the line $y = 3$. Hence, shaded region (Fig 6.12) including all the point on the lines are also the solution of the given system of the linear inequalities

Example 14 Solve the following system of inequalities

$8x + 3y \leq 100$ ... (1)

$x \geq 0$ ... (2)

$y \geq 0$ ... (3)

$8x + 3y = 100$

x	y
0	33.3
12.5	0

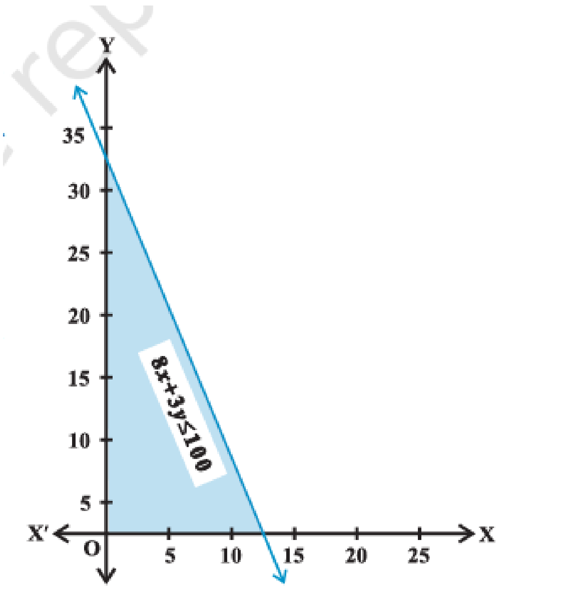


Fig 6.13

Note: $x \geq 0$, $y \geq 0$ and the solution region lie only in the first quadrant.

Example 15 Solve the following system of inequalities graphically

$x + 2y \leq 8$ ... (1)

$2x + y \leq 8$ ... (2)

$x > 0$ ... (3)

$y > 0$ ... (4)

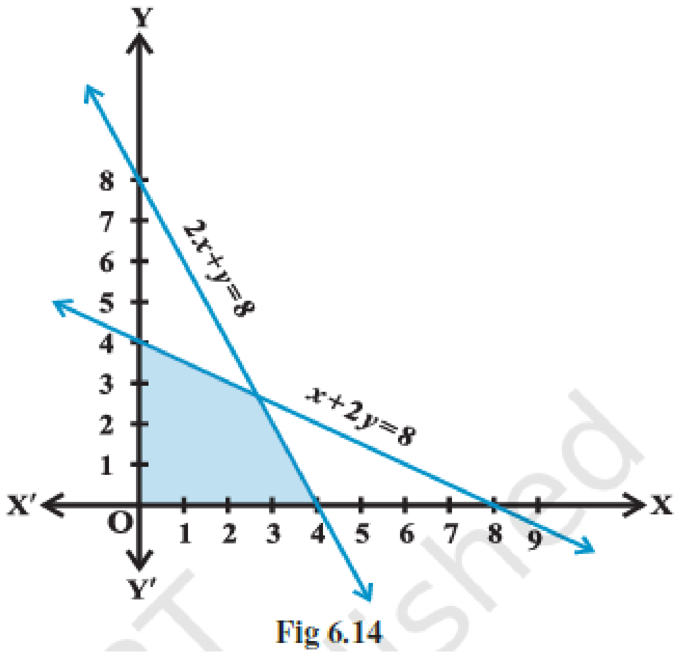


Fig 6.14

x+2y = 8

x	y
0	4
8	0

2x+y = 8

x	y
0	8
4	0

PYQ & EXPECTED QUESTIONS

1. Solve $3x+4y \leq 60$; $x+3y \leq 30$; $x,y \geq 0$ graphically. (IMP 2015)
2. Solve the following inequalities graphically: $x+y \geq 5$; $x-y \leq 3$ (IMP 2014)
3. Solve graphically the inequalities: $x,y \geq 0$; $5x+y \geq 5$; $x+3y \geq 5$ (IMP 2011)