CHAPTER 1 : PHYSICAL WORLD

(Prepared By Ayyappan C, HSST ,GMRHSS ,Kasaragod)

SCOPE OF PHYSICS

Macroscopic domain

- The macroscopic domain includes phenomena at the laboratory, terrestrial and astronomical scales.
- **Classical Physics** deals mainly with macroscopic phenomena and includes subjects like **Mechanics**, **Electrodynamics, Optics** and **Thermodynamics**.
- **Mechanics -**founded on Newton's laws of motion
- **Electrodynamics -** deals with electric and magnetic phenomena associated with charged and magnetic bodies.
- **Optics –** deals with the phenomena involving light
- **Thermodynamics**. - it deals with systems in macroscopic equilibrium and is concerned with changes in internal energy, temperature, entropy, etc., of the system through external work and transfer of heat.

Microscopic domain

- The microscopic domain includes atomic, molecular and nuclear phenomena.
- Quantum Theory is currently accepted as the proper framework for explaining microscopic phenomena.

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CHAPTER 2

UNITS AND MEASUREMENT

(Prepared by AYYAPPAN C,HSST Physics, GMRHSS , Kasaragod)

Physical quantity

- Any quantity that can be measured
- A physical quantity can be classified in to two:-
  i) Fundamental quantity (Base quantity)
  ii) Derived quantity
- Quantities that cannot be expressed in terms of other quantities are known as **fundamental quantities.**
  Eg:- mass, length, time etc.
- Quantities which are derived from fundamental quantities are known as **derived quantities.**
  Eg:- force, velocity, area, volume ,etc

Unit

- Basic, internationally accepted **reference standard** used for measurement is called **unit.**
- The units for the fundamental or base quantities are called **fundamental or base units.**
- The units of all other physical quantities can be expressed as combinations of the base units.
- Units obtained for the derived quantities are called **derived units.**

Systems of Units

- A complete set of the base units and derived units, is known as the **system of units.**
- In CGS system the base units for length, mass and time were centimetre, gram and second respectively.
- In FPS system the base units for length, mass and time were foot, pound and secondrespectively.
- In MKS system the base units for length, mass and time were metre, kilogram and second respectively.

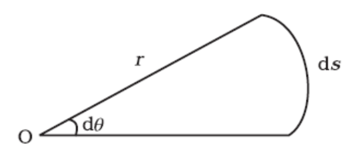
THE INTERNATIONAL SYSTEM OF UNITS

- SI system is the internationally accepted system of unit at present.
- In SI, there are seven base units and two supplementary units.

SI SYSTEM OF UNITS

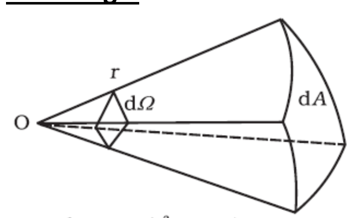
Sl No	Base quantity	SI Units	
		Name	**Symbol**
1	Length	metre	m
2	Mass	kilogram	kg
3	Time	second	s
4	Electric Current	ampere	A
5	Temperature	kelvin	K
6	Amount of substance	mole	mol
7	Luminous intensity	candela	cd
Sl No	**Supplementary quantity**	**SI Units**	
		Name	**symbol**
1	Plane angle	radian	rad
2	Solid angle	steradian	sr

Plane angle



The plane angle , $d\theta = \dfrac{ds}{r}$

Solid angle



The solid angle , $d\Omega = \dfrac{dA}{r^2}$

Prefixes used with SI units

Prefix	Symbol	Meaning
Tera -	**T**	10^{12}
Giga-	**G**	10^{9}
Mega-	**M**	10^{6}
Kilo-	**K**	10^{3}
Deci-	**d**	10^{-1}
Centi -	**c**	10^{-2}
Milli-	**m**	10^{-3}
Micro	**µ**	10^{-6}
Nano	**n**	10^{-9}
Pico	**p**	10^{-12}

SPECIAL UNITS FOR SHORT AND LARGE LENGTHS

Sl No	Unit Name	Symbol	Meaning
1	fermi	**f**	10^{-15} m
2	angstrom	**A^0**	10^{-10} m
3	Astronomical unit	**AU**	1.496 $x10^{11}$ m (Average distance of the sun from earth)
4	light year	**ly**	9.46 $x10^{15}$ m (distance that light travels with velocity of 3 x 10^8 m/s in one year)
5	parsec	**parsec**	3.08 x 10^{16} m (distance at which average radius of earth's orbit subtends an angle of 1 arc second)

DIMENSIONS OF PHYSICAL QUANTITIES

- All the physical quantities represented by derived units can be expressed in terms of some combination of sevenfundamental or base quantities.
- **The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity**.
- **Length has the dimension [L], mass [M], time [T], electric current [A], thermodynamic temperature [K], luminous intensity [cd], and amount of substance [mol].**
- In mechanics, all the physical quantities can be written in terms of the dimensions [L], [M] and [T].

Dimensional formulae

- The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the *dimensional formula of the given physical quantity.*
- For example, the dimensional formula of the volume is $[M^0L^3T^0]$.

Dimensional equation

- An equation obtained by equating a physical quantity with its dimensional formula is called the **dimensional equation of the physical** quantity.
- For example, the dimensional equations of volume [*V*], speed [*v*], force [*F*] and mass density [ρ] may be expressed as

$$[V]=[M^0L^3T^0]$$

$$[v]=[M^0LT^{-1}]$$

$$[F]=[MLT^{-2}]$$

$$[\rho]=[ML^3T^0]$$

APPLICATIONS OF DIMENSIONAL ANALYSIS

- Dimensional analysis can be used to:

a) To check the dimensional consistency of equations

b) To deduce relation among physical quantities.

To check the dimensional consistency of equations

- The principle of homogeneity is used to check the dimensional correctness of equations.

Principle of homogeneity

- The magnitudes of physical quantities may be added together or subtracted from one another only if they have the same dimensions.
- The equation $P=AB+CD$, **is dimensionally correct only if** $[P]=[AB]=[CD]$.

PROBLEM

- Check the dimensional consistency of the equation $v=u+at$.

Solution

- We have , [v] = $[M^0LT^{-1}]$
  [u] = $[M^0LT^{-1}]$
  [at] = $[M^0LT^{-2}]$ [T] =$[M^0LT^{-1}]$
- Thus [v] =[u] =[at] , the equation is dimensionally correct.

Limitations of dimensional analysis

- The dimensional consistency does not guarantee correct equations.
- The arguments of special functions, such as the trigonometric, logarithmic and exponential functions are dimensionless.
- A pure number, ratio of similar physical quantities, such as angle as the ratio (length/length), refractive index as the ratio (speed of light in vacuum/speed of light in medium) etc., has no dimensions.

- Dimensionless constants cannot be obtained by this method.
- **If an equation fails consistency test, it is proved wrong, but if it passes, it is not proved right.**
- It does not distinguish between the physical quantities having same dimensions.

To deduce relation among physical quantities

- For this we should know dependence of the physical quantity on other quantities and consider it as a product type of the dependence.

Derivation of the equation for time period of a pendulum using dimensions

- Let the period of oscillation of the simple pendulum depends on its length (*l), mass of the bob* (*m) and acceleration due to gravity (g).*
- The dependence of time period *T on* the quantities *l, g and m as a product may be* written as :

$$t = kl^x g^y m^z$$

- Where k is a constant
- Taking dimensions on both sides

$$[L^0 M^0 T^1] = [L]^x [LT^{-2}]^y [M]^z = [L^{x+y} T^{-2y} M^z]$$

- On equating the dimensions on both sides, we get
  x+y =0 , thus x= - y
  z=0 , -2y =1, thus y = -1/2
  therefore x= ½.
- So that $t = kl^{\frac{1}{2}} g^{-\frac{1}{2}}$
- **Or** $t = k\sqrt{\frac{l}{g}}$
- But k=2π , thus $t = 2\pi\sqrt{\frac{l}{g}}$

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CHAPTER THREE
MOTION IN A STRAIGHT LINE

(Prepared by AYYAPPAN C,HSST Physics, GMRHSS , Kasaragod)

MOTION

- Motion is change in position of an object with time.
- Branch of physics which deals with the motion of objects – **Mechanics**
- Mechanics is classified into
  i)Statics ii) Kinematics iii) Dynamics
- **Statics** deals with object at rest under the action of forces.
- In **Kinematics**, we study ways to describe motion without going into the causes of motion.
- **Dynamics** deals with objects in motion by considering the causes of motion.

POINT OBJECT

- If the size of the object is much smaller than the distance it moves, it is considered as point object.
- Examples

a) a railway carriage moving without jerks between two stations.
b) a monkey sitting on top of a man cycling smoothly on a circular track.

FRAME OF REFERENCE

- A place from which motion is observed and measured is called **frame of reference**.
- Example: Cartesian coordinate system with a clock – the reference point at the origin.

TYPES OF MOTION

- Based **on the number of coordinates required** to describe motion, motion can be classified as:
  a) One dimensional motion (Rectilinear motion)
  b) Two dimensional motion
  c) Three dimensional motion.

One dimensional motion

- Motion along a straight line is called one dimensional motion or rectilinear motion.
- Only one coordinate is required to describe this motion.
- In one-dimensional motion, there are *only two* directions (backward and forward, upward and downward) in which an object can move
- Example :
  i) a car moving on a straight road.
  ii) Freely falling body

Two dimensional motion

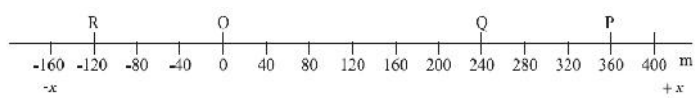
- Motion in a plane is called two dimensional motions.
- Two coordinates are required to represent this motion.
- Example :
  i) A car moving on a plane ground
  ii) A boat moving on a still lake

Three dimensional motion

- Motion in a space is called three dimensional motion.
- Three coordinates are required to represent this motion.
- Example :
  i) Movement of gas molecules
  ii) A flying bird

PATH LENGTH

- The length of the path covered by an object is called path length.
- It is the total distance travelled by the object.
- Path length is a scalar quantity — a quantity that has a magnitude only and no direction.



- For example, the path length of the car moving from O to P and then from P to Q is 360+120 = 480 m.

DISPLACEMENT

- It is the change of position in a definite direction.
- Displacement is a vector quantity –have both magnitude and direction.
- It can be positive, negative or zero.
- In one dimensional motion direction, the two directions can be represented using positive (+) and negative (-) signs.

- If x_1 and x_2 are the positions of an object in time t_1 and t_2, the displacement in time interval $\Delta t = t_2 - t_1$, is given by

$$\Delta x = x_2 - x_1$$

- If $x_2 > x_1$, displacement is positive
- if $x_2 < x_1$, displacement is negative.
- **The magnitude of displacement may or may not be equal to the path length traversed by an object.**
- If the motion of an object is along a straight line and in the **same direction**, the magnitude of displacement is equal to the total path length.

<u>Uniform motion</u>

- If an object moving along the straight line covers equal distances in equal intervals of time, it is said to be in **uniform motion** along a straight line.

<u>AVERAGE VELOCITY</u>

- Ratio of total displacement to the total time .
- $Average\ Velocity = \frac{Total\ Displacement}{Total\ time\ interval}$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

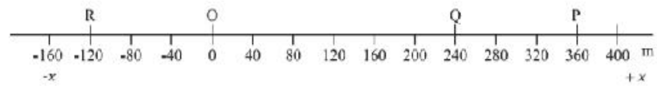
- The SI unit for velocity is **m/s** or **m s^{-1}**
- The unit **km h^{-1}** is used in many everyday applications
- $1km/h = \frac{5}{18} m/s$
- Average velocity is a vector quantity
- Average velocity can be positive or negative or zero..
- **Slope** of the Displacement-Time graph gives the average velocity.

<u>AVERAGE SPEED</u>

- Ratio of total path length travelled to the total time interval
- $Average\ Speed = \frac{Total\ Path\ length}{Total\ time\ interval}$
- Average speed over a finite interval of time is greater or equal to the magnitude of the average velocity
- If the motion of an object is along a straight line and in the **same direction**, the magnitude of average velocity is equal to average speed.
- SI unit of average speed is same as that of velocity.

<u>PROBLEM</u>

- A car is moving along a straight line, It moves from O to P in 18 s and returns from P to Q in 6.0 s. What are the average velocity and average speed of the car in going (a) from O to P ? and (b) from O to P and back to Q ?



<u>Solution</u>

a) Average velocity

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{+360m}{18s} = +20m/s$$

Average speed, $= \frac{360m}{18s} = 20m/s$

b) Average velocity

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{+240m}{(18+6)s} = +10m/s$$

Average speed $= \frac{360+120}{(18+6)s} = 20m/s$

<u>AVERAGE ACCELERATION</u>

- Ratio of change in velocity to time interval

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

- Where v_2 and v_1 are the instantaneous velocities or simply velocities at time t_2 and t_1 .
- SI unit is m/s^2.
- Slope of the velocity-time graph gives average acceleration.

<u>INSTANTANEOUS ACCELERATION (ACCELERATION)</u>

- It is the acceleration at an instant..
- It is the average acceleration as the as the time interval tends to zero

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- The instantaneous acceleration is the slope of the tangent to the *v–t* curve at that instant.
- Acceleration can be positive, negative or zero.
- It is a vector quantity.

GRAPHS RELATED TO MOTION

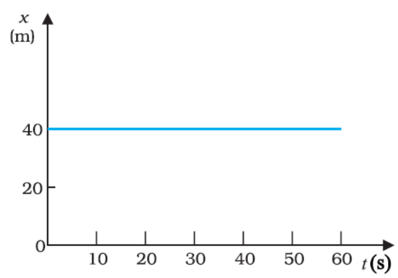
POSITION-TIME GRAPH (x –t Graph)

- It is the graph drawn taking time along x-axis and position along y-axis
- Slope of the x-t graph gives the average velocity.
- Slope of the tangent at a point in the x-t graph gives the velocity at that point.

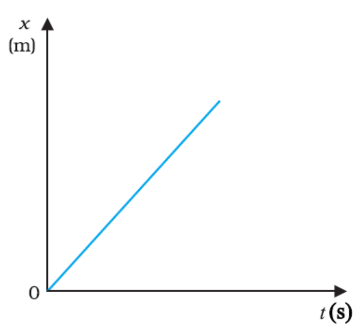
Uses of Position –Time Graph

- To find the position at any instant
- To find the velocity at any instant
- To obtain the nature of motion

Position- time graph of stationary object

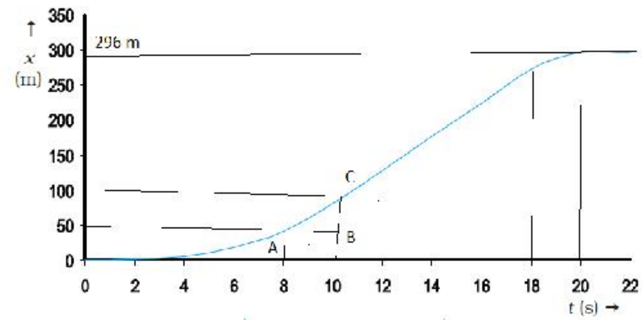


Position- time graph of an object in uniform motion

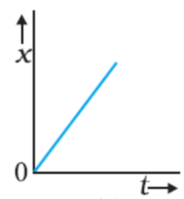


Position-time graph of a car

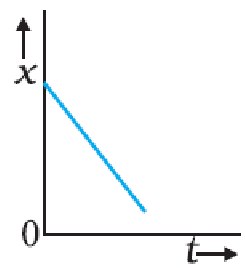
- The car starts from rest at time t = 0 s from the origin O and picks up speed till t = 10 s and thereafter moves with uniform speed till t = 18 s. Then the brakes are applied and the car stops at$t$ = 20 s and x = 296 m.



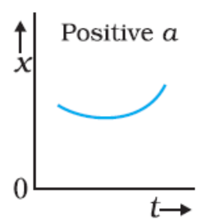
Position-time graph of an object moving with positive velocity



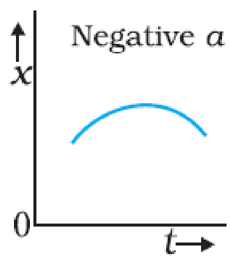
Position-time graph of an object moving with negative velocity



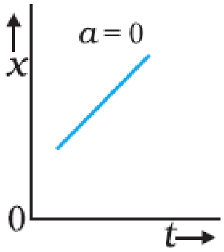
Position-time graph for motion with positive acceleration



Position-time graph for motion with negative acceleration

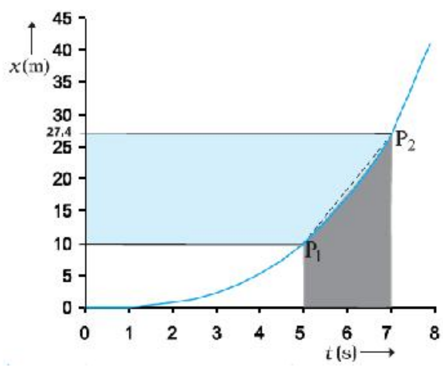


Position-time graph for motion with zero acceleration



PROBLEM

- Calculate the average velocity between 5s and 7s from the graph.



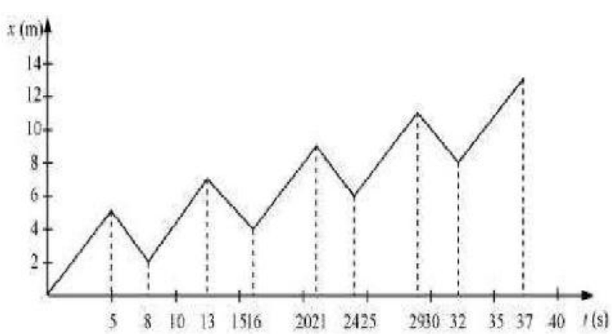
Solution

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(27.4 - 10.0)\,\mathrm{m}}{(7-5)\,\mathrm{s}} = 8.7\,\mathrm{m\,s^{-1}}$$

PROBLEM-2

- A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the *x-t* graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

Solution



Time taken to fall in pit =37s.

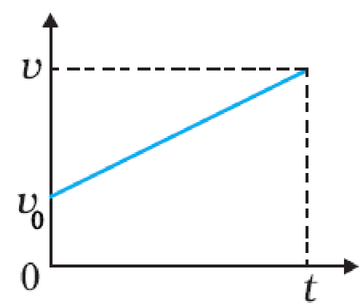
VELOCITY – TIME GRAPH (v-t GRAPH)

- A graph with velocity along Y –axis and time along X-axis.
- The **acceleration** at an instant is **the slope** of the tangent to the *v–t curve at that instant.*
- *Area under the v-t graph gives the* **displacement.**

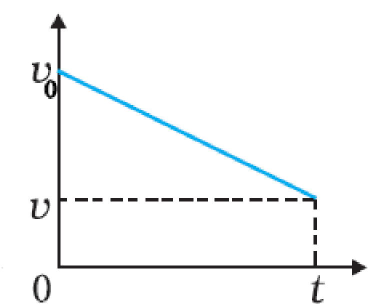
Uses of v-t graph

- To find the displacement
- To find the velocity at any time
- To find the acceleration at any time
- To know the nature of motion

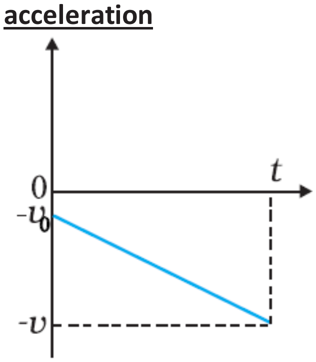
v-t graph of motion in positive direction with positive acceleration



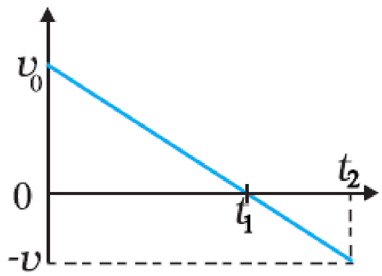
v-t graph of motion in positive direction with negative acceleration



v-t graph of motion in negative direction with negative acceleration

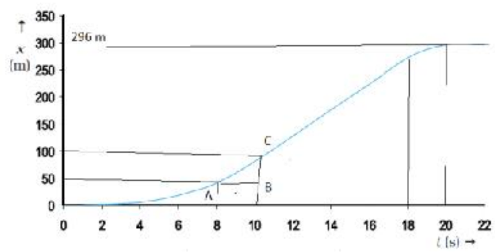


v-t graph of motion of an object with negative acceleration that changes direction at time t_1.

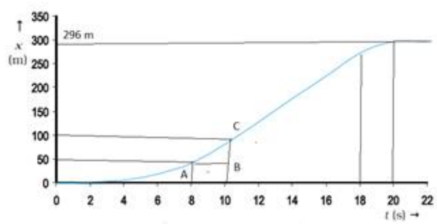


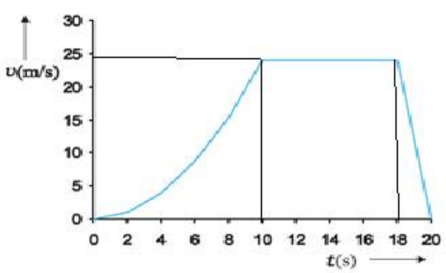
PROBLEM-1

- Draw v-t graph from the given x-t graph.



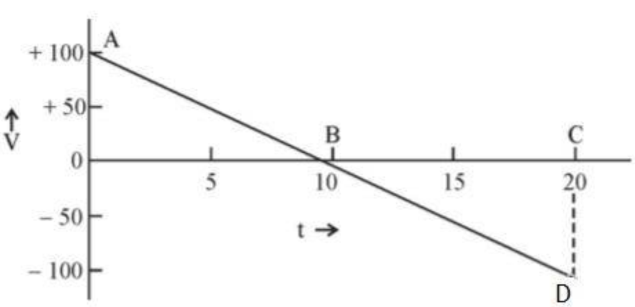
Solution





PROBLEM-2

- Velocity-time graph of a ball thrown vertically upwards with an initial velocity is shown in figure.



a) What is the magnitude of initial velocity of the ball?

b) Calculate the distance travelled by the ball during 20 s, from the graph.

c) Calculate the acceleration of the ball from the graph

Solution

a) 100 m/s

b) $\text{Distance} = \text{area of } \Delta OAB + \text{ area of } \Delta BCD$

$$= \left(\frac{1}{2}\times 10\times 100\right) + \left(\frac{1}{2}\times 10\times 100\right) = 1000m$$

c) Acceleration = slope of the graph

$$slope = \frac{0-100}{10} = -10$$

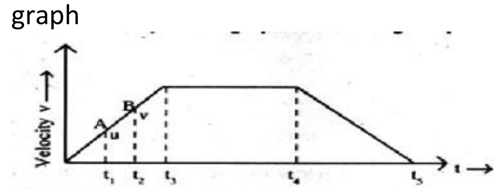
- Therefore acceleration = -10m/s^2

ACCELERATION –TIME GRAPH

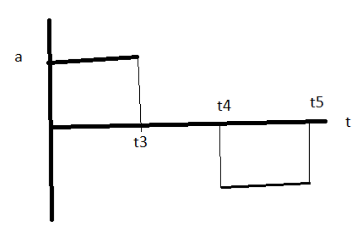
- A graph with acceleration along Y –axis and time along X-axis.
- Area under acceleration – time graph gives velocity.

PROBLEM

- The graph shows the velocity – time graph of a moving body in a one dimensional motion. Draw the corresponding acceleration – time graph

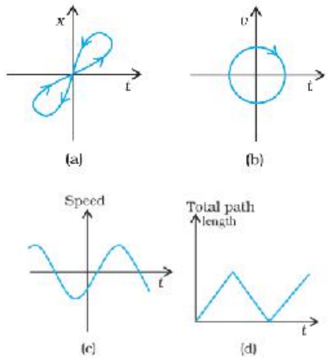


Solution



PROBLEM

- which of these *cannot* possibly represent one-dimensional motion of a particle.

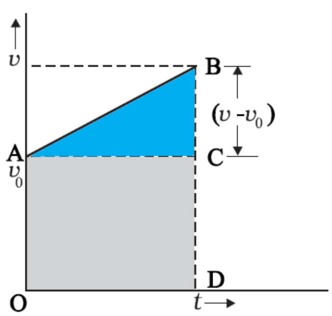


Solution

a) No – because a particle cannot have two positions at the same instant of time.
b) No – because particle can never have two values of velocities at the same instant of time.
c) No- speed cannot be negative
d) No – total path length cannot decrease with time.

KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

velocity –time graph of an object moving with uniform acceleration and with initial velocity v_0



Velocity – Time Relation

- We have

$$Acceleration = \frac{Change\ in\ velocity}{Time\ taken}$$

$$a = \frac{v - v_0}{t}$$

- Where v- final velocity, a – acceleration v_0 –initial velocity

$$at = v - v_0$$

Or

$$\therefore v = v_0 + at$$

Displacement-Time Relation

- We know , area under v-t graph = Displacement
- Thus the displacement at any time interval 0 and t, is given by

$$Displacement = Area\ of\ \Delta\ ABC + Area\ of\ \square\ OACD$$

- Thus $x = \frac{1}{2} \times (v - v_0) \times t + v_0 t$
- But $v - v_0 = at$
- Thus

$$x = \frac{1}{2} \times at \times t + v_0 t$$

$$x = \frac{1}{2} \times at^2 + v_0 t$$

- Therefore

$$x = v_0 t + \frac{1}{2} at^2$$

- If x_o is the initial displacement

$$x - x_0 = v_0 t + \frac{1}{2} at^2$$

Velocity –Displacement Relation

- We have

$$Average\ Velocity = \frac{Displacement}{Time}$$

- Thus
$$Dispalcement = Average\ Velocity\ \times Time$$

- Therefore

$$x = \frac{(v + v_0)}{2} \times t$$

- But

$$t = \frac{v - v_0}{a}$$

- Thus

$$x = \frac{(v + v_0)}{2} \times \frac{(v - v_0)}{a} = \frac{v^2 - v_0{}^2}{2a}$$

Therefore

$$v^2 = v_0{}^2 + 2ax$$

- If x_o is the initial displacement

$$v^2 = v_0{}^2 + 2a(x - x_0)$$

Thus the Equations of motion are

$$v = v_0 + at$$

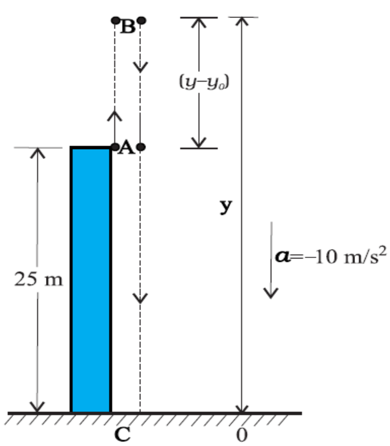
$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0{}^2 + 2a(x - x_0)$$

PROBLEM

- A ball is thrown vertically upwards with a velocity of 20 m s^{-1} from the top of a multistorey building. The height of the point from where the ball is thrown is 25.0 m from the ground.
  a) How high will the ball rise ?
  b) how long will it be before the ball hits the ground? Take g = 10 m s^{-2}

Solution



a) Given v_0 = +20 m/s , a = -g =10 m/s , v = 0

- Using the equation

$$v^2 = v_0{}^2 + 2a(y - y_0)$$

- We get

$$0 = 20^2 + 2 \times (-10)(y - y_0)$$

- Solving we get

$$(y - y_0) = 20m$$

b) We have y_0 = 25 m, y = 0 m , v_o = 20 m /s, a = –10m /s^2,

- Using the equation

$$y - y_0 = v_0 t + \frac{1}{2}at^2$$

$$0 - 25 = 20t - \frac{1}{2} \times 10t^2$$

$$5t^2 - 20 - 25 = 0$$

- Solving this quadratic equation we get, t=5s.

MOTION OF AN OBJECT UNDER FREE FALL

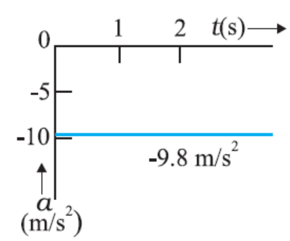
- A body falling under the influence of acceleration due to gravity alone is called **free fall** (air resistance neglected)
- If the height through which the object falls is small compared to the earth's radius, g can be taken to be constant, equal to 9.8 m s^{-2}.
- Free fall is an example of motion with uniform acceleration.
- Since the acceleration due to gravity is always downward, it is in the negative direction.
- Acceleration due to gravity = - g = - 9.8m/s^2.

Equations of motion of a freely falling body

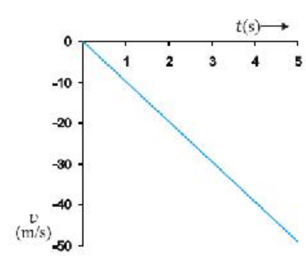
- For a freely falling body with v_0=0 and y_0 =0, the equations of motion are

$$v = 0 - g\,t \qquad = -9.8\,t \quad \text{m s}^{-1}$$
$$y = 0 - \tfrac{1}{2}\,g\,t^2 \qquad = -4.9\,t^2 \quad \text{m}$$
$$v^2 = 0 - 2\,g\,y \qquad = -19.6\,y \quad \text{m}^2\,\text{s}^{-2}$$

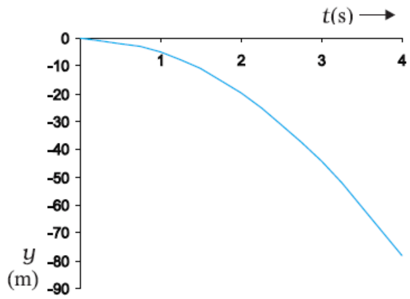
Acceleration –Time graph of a freely falling body



Velocity – Time graph of a freely falling body



Position –Time graph of a freely falling body



Galileo's law of odd numbers

- The distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity [*namely,* 1: 3: 5: 7……]

Proof

- Divide time interval of motion into equal intervals
- The distance travelled is found out using

$$y = -\frac{1}{2}gt^2$$

t	Displacement y	Y in terms of y_0 $= -\frac{1}{2}g\tau^2$	Distance travelled in successive intervals	Ratio of distances
0	0	0		
τ	$= -\frac{1}{2}g\tau^2$	y_0	y_0	1
2τ	$= -4\times\frac{1}{2}g\tau^2$	$4y_0$	$3y_0$	3
3τ	$= -9\times\frac{1}{2}g\tau^2$	$9y_0$	$5y_0$	5
4τ	$= -16\times\frac{1}{2}g\tau^2$	$16y_0$	$7y_0$	7

- Thus ratio of distances is found to be 1:3:5:7:…..

STOPPING DISTANCE OF VEHICLES

- When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance.
- Stopping distance is an important factor considered in setting speed limits, for example, in school zones
- Stopping distance depends on the initial velocity (v_0) *and the* braking capacity, or deceleration (–a) *that* is caused by the braking.

Equation for Stopping Distance

- Let the distance travelled by the vehicle before it stops be ,d.
- Substituting v=0 , x=d and acceleration = - a in the equation

$$v^2 = {v_0}^2 + 2ax$$

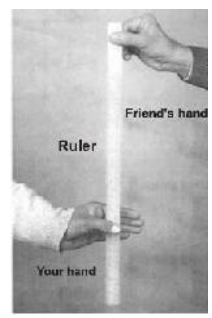
$$0 = {v_0}^2 - 2ad$$

$$ {v_0}^2 = 2ad$$

- Thus stopping distance , $d = \frac{{v_0}^2}{2a}$
- Thus stopping distance is proportional to square of initial velocity.

REACTION TIME

- Reaction time is the time a person takes to observe, think and act.



- Dropping a ruler the reaction time can be calculated using the formula

$$t_r = \sqrt{\frac{2d}{g}}$$

- Where d is the distance moved before reaction.
- train B moves south with a speed of 90 km h–1. What is the

a) Velocity of B with respect to A ?,
b) Velocity of ground with respect to B ?
c) velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 km h–1 with respect to the train A) as observed by a man standing on the ground ?

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CHAPTER 4
MOTION IN A PLANE

(Prepared By Ayyappan C, HSST , GMRHSS , Kasaragod)

SCALARS

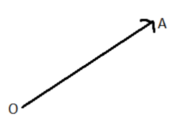
- A scalar quantity is a quantity with magnitude only.
- It is specified by a single number, along with the proper unit.
- Examples are : the distance , mass, temperature time etc.
- Scalars can be added, subtracted, multiplied and divided just as the ordinary numbers.
- Scalars can be added or subtracted with quantities with same units only. However, you can multiply and divide scalars of different units.

VECTORS

- A **vector quantity is a quantity that has both** a magnitude and a direction.
- A vector is specified by giving its magnitude by a number and its direction.
- Examples are displacement, velocity, acceleration and force.

Representation of Vectors

- Vectors are represented using a straight-line with an arrow head.

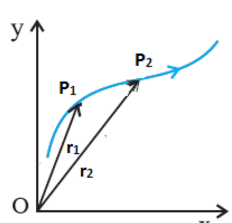


- The length of the line is equal to or proportional to the magnitude of the vector and the arrow head shows the direction.

TYPES OF VECTORS

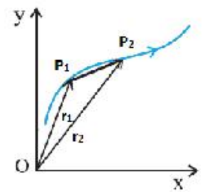
Position Vectors

- To describe the position of an object moving in a plane an arbitrary point is taken as origin.
- A vector drawn from the origin to the point is known as position vector.



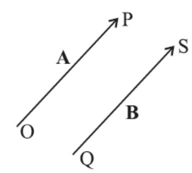
Displacement vectors

- A vector joining the initial and final positions of an moving object is known as displacement vector.
- The magnitude of the displacement vector is either less or equal to the path length of an object between two points



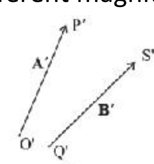
Equal vectors

- Two vectors A and B are said to be equal if, and only if ,they have the same magnitude and same direction.



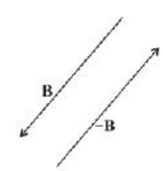
Unequal vectors

- Two vectors A and B are said to be unequal if, they have the different magnitude or direction.



Negative vector

- Negative of a vector has the same magnitude but opposite direction.



Null vector (Zero vector)

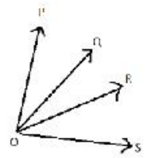
- A vector with zero magnitude and arbitrary direction
- Examples are :
  - Displacement of a stationary object
  - Velocity of a stationary object

Collinear vectors

- Vectors with same direction or opposite direction
- Their magnitudes may or may not be equal

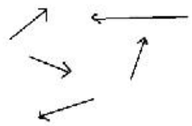
Co-initial vectors

- Vectors having same initial point



Coplanar vectors

- Vectors lying on the same plane



UNIT VECTORS

- A vector with unit magnitude
- It is used to denote a direction
- Any vector can be represented as the product of its magnitude and a unit vector

$$\vec{A} = |\vec{A}|\,\hat{A}$$

- Where $\hat{A}$ is unit vector
- Thus unit vector, $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

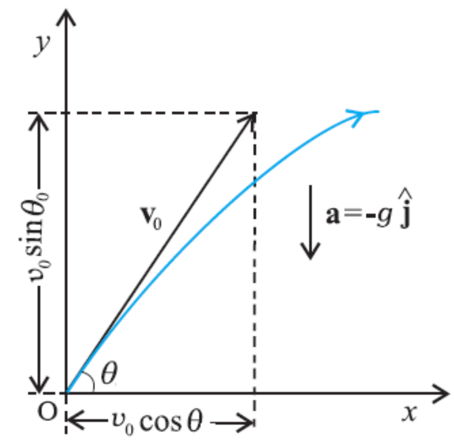
Orthogonal unit vectors

- Unit vectors along the x, y, z axes of a rectangular coordinate system is called orthogonal unit vectors.
- They are denoted as $\hat{i}$, $\hat{j}$ and $\hat{k}$

PROJECTILE MOTION

- An object that is in flight after being thrown or projected is called a projectile.
- The horizontal component of velocity remains unchanged.
- Due gravity vertical component of velocity changes with time.
- It is assumed that air resistance has negligible effect on motion of the projectile.
- The trajectory or path of a projectile is **parabola.**

Motion of an object projected with velocity v_0 at an angle θ



- After the object has been projected, the acceleration acting on it is that due to gravity which is directed vertically downward:
- That is $\vec{a} = -g\hat{j}$ or in component form

$$a_x = 0,\ a_y = -g$$

- The components of initial velocity $\mathbf{v_o}$ are :

$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$

- If we take the initial position to be the origin of the reference frame (x_0=0 , y_0=0), the equations of motion for the projectile is given by

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = (v_0 \cos\theta)t$$

$$y = v_{0y}t + \frac{1}{2}a_y t^2 = (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

- Also

$$v_x = v_{0x} + a_x t = v_0 \cos\theta$$

$$v_y = v_{0y} + a_y t = v_0 \sin\theta - gt$$

Equation of path of a projectile

- We have from the equation of motion

$$t = \frac{x}{(v_0 \cos\theta)}$$

- Substituting this in the equation

$$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

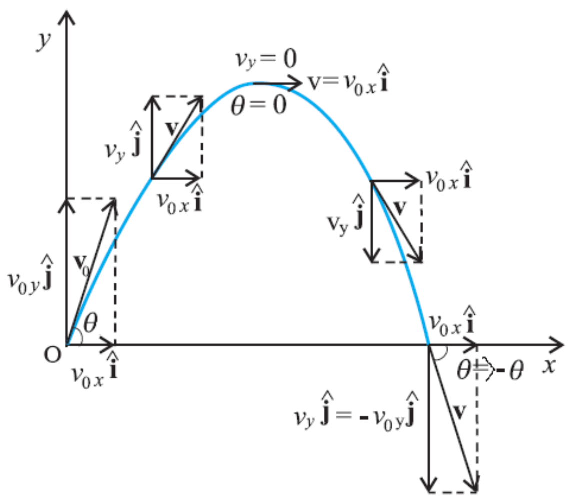
- We get

$$y = (v_0 \sin\theta)\frac{x}{(v_0\cos\theta)} - \frac{1}{2}g\left(\frac{x}{(v_0\cos\theta)}\right)^2$$

$$y = (\tan\theta)x - \frac{g}{2(v_0\cos\theta)^2}x^2$$

- This equation is of the form
  $$y = ax + bx^2$$
- This is the equation of a parabola.

The parabolic path of a projectile



- At the highest point , velocity is zero, but still there is acceleration due to gravity.

Time of maximum height (t_m)

- At maximum height $v_y = 0$,
- If t_m is the time of maximum height, then

$$v_y = v_0\sin\theta - gt_m = 0$$

$$t_m = \frac{v_0\sin\theta}{g}$$

Time of Flight of the projectile (T)

- The total time during which the projectile is in flight is called *time of flight* .

Equation of Time of Flight

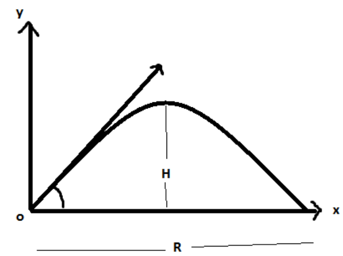
- During time of flight we have, the vertical displacement y=0, thus

$$y = (v_0\sin\theta)T - \frac{1}{2}gT^2 = 0$$

$$T = \frac{2v_0\sin\theta}{g}$$

- Thus time of flight $T = 2t_m$

Maximum Height of a Projectile (H)



- We have the vertical displacement,

$$y = (v_0\sin\theta)t - \frac{1}{2}gt^2$$

- At maximum height y =H and $t = t_m$, then

$$H = (v_0\sin\theta)t_m - \frac{1}{2}g{t_m}^2$$

$$= \frac{{v_0}^2\sin^2\theta}{g} - \frac{1}{2}\left(\frac{{v_0}^2\sin^2\theta}{g}\right)$$

$$H = \frac{{v_0}^2\sin^2\theta}{2g}$$

Horizontal Range of a Projectile (R)

- The horizontal distance travelled by the projectile during the time of flight is called ***horizontal range.***
- **R = Horizontal velocity x Time of flight**

$$R = v_0\cos\theta \times \frac{2v_0\sin\theta}{g}$$

$$R = \frac{{v_0}^2(2\sin\theta\cos\theta)}{g} = \frac{{v_0}^2(\sin 2\theta)}{g}$$

$$R = \frac{{v_0}^2(\sin 2\theta)}{g}$$

Maximum horizontal range

- Range is maximum when $2\theta = 90^0$ or $\theta = 45^0$.
- Thus

$$R_{\max} = \frac{{v_0}^2}{g}$$

PROBLEM -1

- A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 m s^{-1}. Neglecting air resistance, find
  a) the time taken by the stone to reach the ground.
  b) the speed with which it hits the ground. (Take g = 9.8 m s^{-2}).

Solution

- We choose the origin of the *x*-,and *y* axis at the edge of the cliff and *t* = 0 s at the instant the stone is thrown.

a) We have

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

- Here y_0 =0, v_{0y} =0, a_y = - g = 9.8 m/s^2 and y = -490 m, therefore

$$-490 = -\frac{1}{2} \times 9.8t^2$$

$$t = 10s$$

b) The components of velocity are given by

$$v_x = v_{0x} + a_x t = v_{0x} = 15m/s$$

$$v_y = v_{0y} + a_y t = 0 - 9.8 \times 10 = -98m/s$$

- Therefore the speed of the stone is

$$v = \sqrt{v_x^{\ 2} + v_y^{\ 2}} = \sqrt{15^2 + 98^2} = 99m/s$$

PROBLEM -2

- A cricket ball is thrown at a speed of 28 m s^{-1} in a direction 30° above the horizontal. Calculate
  (a) the maximum height
  (b) the time taken by the ball to return to the same level
  (c) the distance from the thrower to the point where the ball returns to the same level.

Solution

a) The maximum height is

$$H = \frac{v_0^{\ 2}\sin^2\theta}{2g} = \frac{(28 \times \sin 30)^2}{2 \times 9.8} = 10m$$

b) Time of flight is

$$T = \frac{2v_0 \sin\theta}{g} = \frac{2 \times 28 \times \sin 30}{9.8} = 2.9s$$

c) Horizontal range is

$$R = \frac{v_0^{\ 2}(\sin 2\theta)}{g} = \frac{28^2 \times (\sin 2 \times 30)}{9.8} = 69m$$

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CHAPTER 5

LAWS OF MOTION

(Prepared By Ayyappan C, GMRHSS, Kasaragod)

MOMENTUM (P)

- Momentum is the product of its mass and velocity

$$\vec{P} = m\vec{v}$$

- Momentum is a vector quantity

Some Situations relating momentum and applied force

Situation -1

i) A much greater force is needed to push the truck than the car to bring them to the same speed in same time.

ii) A greater opposing force is needed to stop a heavy body than a light body in the same time, if they are moving with the same speed.

iii) If two stones, one light and the other heavy, are dropped from the top of a building, a person on the ground will find it easier to catch the light stone than the heavy stone.

Reason

- In these cases change in momentum is greater for a heavy body.
- External force required is proportional to change in momentum for the given time.

Situation -2

i) A bullet fired by a gun can easily pierce human tissue before it stops, resulting in casualty.

ii) The same bullet fired with moderate speed will not cause much damage

Reason

- Velocity is high for a bullet from a gun - the change in momentum is high
- External force required to stop the bullet is proportional to change in momentum for a given time.

Situation -3

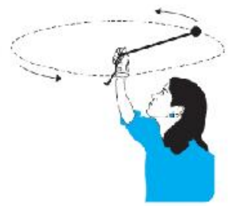
i) A seasoned cricketer catches a cricket ball coming in with great speed far more easily than a novice, who can hurt his hands in the act

Reason

- External force depends on the time in which the momentum change is brought about.
- The change in momentum brought about in a shorter time needs greater applied force and vice versa.

Situation -4

- Suppose a stone is rotated with uniform speed in a horizontal plane by means of a string, the magnitude of momentum is fixed, but its direction changes
- The force needed to change in momentum is provided by our hand through the string.
- Our hand needs to exert a greater force if the stone is rotated at greater speed or in a circle of smaller radius, or both



Reason

- External force is proportional to change in momentum.

NEWTON'S SECOND LAW OF MOTION

- **The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.**
- That is

$$\vec{F} \propto \frac{\Delta\vec{p}}{\Delta t} \quad \text{or} \quad \vec{F} = k\frac{\Delta\vec{p}}{\Delta t}$$

- Where Δp – change in momentum in the time interval Δt and k – constant of proportionality.
- Taking the limit $\Delta t \rightarrow 0$,

$$\vec{F} = k\frac{d\vec{p}}{dt}$$

- For a body of fixed mass m,

$$\frac{d\vec{p}}{dt} = m\frac{dv}{dt} = m\vec{a}$$

- Thus $\vec{F} = km\vec{a}$
- The S I unit of force (newton) is defined such that k=1.

- Therefore

$$\vec{F} = m\vec{a}$$

- This law is applicable to both single particle and a system of particles.

Definition of newton

- One newton is that force, which causes an acceleration of $1 m/s^2$, to a mass of 1kg.

$$1N = 1kgms^{-2}$$

Newton's second law in vector component form

- The second law of motion is a vector law. It is equivalent to three equations, one for each component of the vectors

$$F_x = \frac{dp_x}{dt} = ma_x$$

$$F_y = \frac{dp_y}{dt} = ma_y$$

$$F_z = \frac{dp_z}{dt} = ma_z$$

- Thus , if the force makes an angle with the velocity of a body , it changes only the component of velocity along the direction of force.
- The component of velocity normal to the force remains unchanged.

PROBLEM

- A bullet of mass 0.04 kg moving with a speed of $90 m s^{-1}$ enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet?

Solution

- Given m=0.04kg, v_0 = 90 m/s , x= 0.6m, v=0
- The acceleration of the bullet is given by

$$v^2 = v_0{}^2 + 2ax$$

$$\Rightarrow 0 = 90^2 + 2 \times a \times 0.6$$

$$a = -\frac{90^2}{2 \times 0.6} = -6750 m/s$$

- The resistive force is

$$F = ma = 0.04 \times (-6750) = -270N$$

Impulse

- The product of force and time.

$$\boldsymbol{Impulse = Force \times Time\ Duration}$$

= Change in Momentum

$$I = F \times \Delta t = \Delta p$$

- Unit of impulse is **newton-second (Ns).**

Impulsive force

- A large force acting for a short time to produce a finite change in momentum.
- Examples are force when a ball hits on a wall, force exerted by a bat on a ball, force on a nail by a hammer etc.

PROBLEM

- A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of $12 m s^{-1}$. If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball. (Assume linear motion of the ball)

Solution

$$\text{Impulse} = \text{Change in momentum}$$

$$= 0.15 \times 12 - (-0.15 \times 12) = 3.6Ns$$

THE LAW OF CONSERVATION OF MOMENTUM

- **The total momentum of an isolated system (a system with no external force) of interacting particles is conserved.**
- **From Newton's second law**

$$F = \frac{dp}{dt}$$

When F=0, we get

$$F = \frac{dp}{dt} = 0$$

$$dp = 0$$

$$\therefore p = \text{constant}$$

- **Therefore, when F=0, initial momentum = final momentum.**

Applications of conservation of momentum

Recoil of a gun

- Velocity of a bullet- muzzle velocity
- Movement of gun backward, when a bullet is fired- recoil of gun
- According to conservation of momentum momentum before firing = momentum after firing
- Thus

$$0 = mu + MV$$

Where m- mass of bullet, u- velocity of bullet, M- mass of gun, V- recoil velocity of gun

- Therefore

$$V = \frac{-mu}{M}$$

- The negative sign shows that velocity of gun is opposite to that of bullet
- Recoil velocity is very small (since M > m)

Rocket propulsion

- When a rocket is fired, fuel is burnt in the combustion chamber.
- The hot gas at very high pressure escapes through the nozzle with a very high velocity
- The escaping gas has a very high momentum
- In order to conserve momentum the rocket moves in the forward direction.

FRICTION

- ***Friction*** is the force which opposes the relative motion between two surfaces in contact.
- It acts ***tangential*** to the surface of contact.
- Arises due to

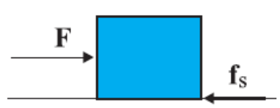
a) adhesive force between surfaces
b) irregularities of plane surface

- There are two types

I) Static friction
II) Kinetic friction

Static friction

- Friction between two surfaces in contact as long as the bodies is at rest.



- Its value increases from zero to a maximum value called limiting friction (f_s^{max}).
- Limiting friction is the static frictional force just before sliding.

Laws of Static Friction

- The *magnitude of limiting friction* is *independent of area* of the contact between the bodies.
- The limiting friction is proportional to the normal reaction N.

$$f_s^{\max} \propto N$$

$$f_s^{\max} = \mu_s N$$

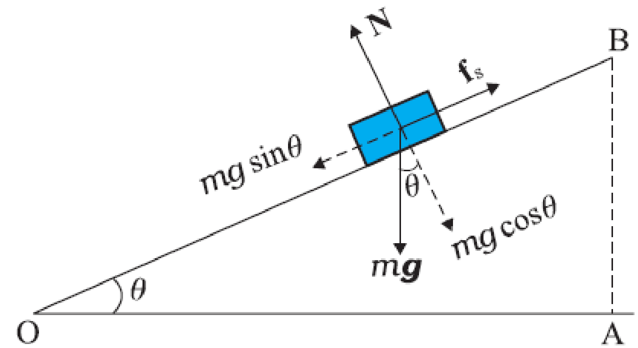
- μ_s - coefficient of static friction.

$$\mu_s = \frac{f_s^{\max}}{N}$$

- Thus , value of static friction may be written as

$$f_s \le \mu_s N$$

Angle of friction (θ)



- The angle at which the body begin to slide on an inclined plane is called **angle of limiting static friction or angle of repose**
- The weight of the body can be resolved in to two components.
- Just before sliding

$$mg\sin\theta = f_s^{\max}$$

$$mg\cos\theta = N$$

- Dividing the two equations

$$\frac{mg\sin\theta}{mg\cos\theta} = \frac{f_s^{\max}}{N}$$

$$\tan\theta = \frac{f_s^{\max}}{N} = \mu_s$$

- Thus coefficient of static friction is the tangent of the angle of limiting friction.

PROBLEM-1

- Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the co-efficient of static friction between the box and the train's floor is 0.15.

Solution

- Since the acceleration of the box is due to the static friction,

$$ma = f_s \le \mu_s N = \mu_s mg$$
$$a \le \mu_s g$$
$$a_{\max} = \mu_s g = 0.15 \times 10 m/s^2 = 1.5 m/s^2$$

PROBLEM-2

- A mass of 4 kg rests on a horizontal plane. The plane is gradually inclined until at an angle θ= 15° with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface?

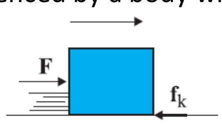
Solution

- We have

$$\tan\theta = \mu_s$$
$$\mu_s = \tan 15^0 = 0.27$$

Kinetic friction

- Friction experienced by a body when it moves



- Two types:
  - i) ***Sliding friction***
  - ii) ***Rolling friction***
- ***Rolling friction < sliding friction < static friction***

Laws of kinetic friction

- *Kinetic friction does not depend on the nature of the two surfaces in contact.*
- *Kinetic friction is proportional to the normal reaction.*

$$f_k \propto N$$
$$f_k = \mu_k N$$

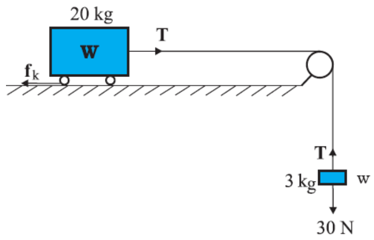
- μ_k is the coefficient of kinetic friction
- Coefficient of kinetic friction is less than that of static friction

Rolling friction

- Friction when a body rolls on a surface
- Very small compared to sliding friction-surface area of contact is small
- Advantage of Rolling friction is made use in ball-bearings

PROBLEM

- What is the acceleration of the block and trolley system shown in the figure, if the coefficient of kinetic friction between the trolley and the surface is 0.04? What is the tension in the string? (Take g = 10 m s^{-2}). Neglect the mass of the string.



Solution

- Net force on 2kg mass is
  $30 - T = 2a$, a –acceleration
- Net force on trolley is
  $$T - f_k = 20a$$
- Now $f_k = \mu_k N$
  μ_k=0.04,
  N= 20 x 10= 200 N
- Thus
  $$T - 0.04 \times 200 = 20a$$
  $$T - 8 = 20a$$
- Solving the equations , we get
  a=22/23 =0.96m/s^2 and T = 27.1 N

FRICTION AS A NECESSARY EVIL

- Friction is considered as a **necessary evil**, because it has both advantages and disadvantages.

Advantages of friction

- We are able to walk on the ground due to friction
- We can hold an object in hand due to friction
- Meteors burn in air due to friction.

Disadvantages of friction

- When a vehicle moves lot of energy is lost to overcome friction
- Excess heat produced in machines causes wear and tear to parts
- Atmospheric friction is disadvantageous to rockets and satellites

Ways to minimize friction

- Using lubricants like, grease, oil, wax etc.
- Using ball bearings or roll bearings
- Using anti-friction metals or alloys
- Separating the surfaces by an air cushion
- Streamlining the body of vehicles
- Polishing the surfaces.

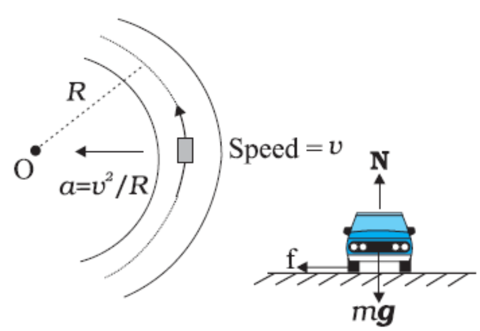
CIRCULAR MOTION

- Acceleration of a body moving in a circle of radius R with uniform speed v is v^2/R directed towards the centre.
- According to the second law, the force providing this acceleration is

$$f_C = \frac{mv^2}{R}$$

- This force directed forwards the centre is called the **centripetal force**.
- For a stone rotated in a circle by a string, the centripetal force is provided by the tension in the string.
- The centripetal force for motion of a planet around the sun is the gravitational force on the planet due to the sun.
- For a car taking a circular turn on a horizontal road, the centripetal force is the force of friction.

Motion of a car on a level road



- Three forces act on the car

i) The weight of the car, *mg*
ii) Normal reaction, *N*
iii) Frictional force, *f*

Maximum speed of the car on a level road

- As there is no acceleration in the vertical direction

$$N - mg = 0$$
$$N = mg$$

- The centripetal force required for the circular motion is provided by the frictional force between road and the car tyres.
- Thus

$$f \le \mu_s N = \frac{mv^2}{R}$$

$$\therefore v^2 \le \frac{\mu_s RN}{m} = \frac{\mu_s Rmg}{m}$$

$$v^2 \le \mu_s Rg$$

- Thus for a given value of μ_s and R, the maximum speed of circular motion of the car is given by

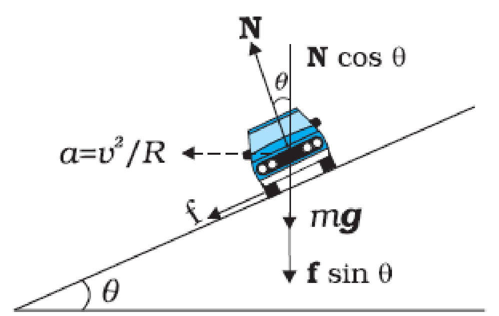
$$v_{\max} = \sqrt{\mu_s Rg}$$

Motion of a car on a banked road

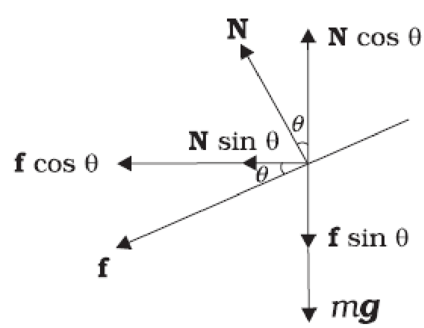
Banking of roads

- The phenomenon of raising outer edge of the curved road above the inner edge is called banking of roads.
- We can reduce the contribution of friction to the circular motion of the car if the road is banked

Forces on a car in a banked road



Maximum possible speed of a car on a banked road – with friction



- Since there is no acceleration along the vertical direction, the net force along this direction must be zero.
- Thus

$$N\cos\theta = mg + f\sin\theta$$

- The centripetal force is provided by the horizontal components of *N and f.*

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$$

- But for maximum speed , v_{max}, $f = \mu_s N$
- Thus

$$N\cos\theta = mg + \mu_s N\sin\theta$$

$$N(\cos\theta - \mu_s\sin\theta) = mg$$

$$N = \frac{mg}{(\cos\theta - \mu_s\sin\theta)}$$

- Also

$$N(\sin\theta + \mu_s\cos\theta) = \frac{mv_{\max}^2}{R}$$

- Substituting for N in this equation we get,

$$\frac{mg(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{mv_{\max}^2}{R}$$

$$v_{\max}^2 = \frac{Rg(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}$$

- Therefore

$$v_{\max} = \sqrt{\frac{Rg(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}}$$

- Dividing numerator and denominator by cosθ, we get

$$v_{\max} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{(1 - \mu_s\tan\theta)}}$$

- Thus maximum possible speed of a car on a banked road is greater than that on a flat road.

Speed of the car – without friction

- If there is no friction, μ_s=0,therefore the speed of the car is

$$v_0 = \sqrt{Rg\tan\theta}$$

- This is called the **optimum speed.**
- At this speed, frictional force is not needed to provide the necessary centripetal force.
- Driving at this speed on a banked road will cause little wear and tear of the tyres.

PROBLEM

- A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the (a) optimum speed of the racecar to avoid wear and tear on its tyres, and (b) maximum permissible speed to avoid slipping ?

Solution

- Given , $\theta=15^0$,μ_s =0.2, R=300m, g =9.8m/s^2

a) Optimum speed is

$$v_0 = \sqrt{Rg\tan\theta} = \sqrt{300\times 9.8\times\tan 15^0} = 28.1m/s$$

b) Maximum speed is

$$v_{\max} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{(1 - \mu_s\tan\theta)}} = 38.1m/s$$

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Chapter 6

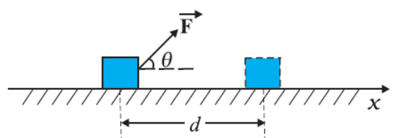
WORK, ENERGY AND POWER

(Prepared By Ayyappan C,HSST, GMRHSS, Kasaragod)

WORK

- The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement.
- The work and energy have the same dimensions $[ML^{-2}T^{-2}]$
- The SI unit is joule (J).

Work done by a constant force



- **The work done by the constant force F , is**
  $$W = Fd\cos\theta$$
- Or $W = \vec{F} \bullet \vec{d}$
- Work done can be zero, positive or negative

Special cases

- If ϴ=0, then maximum work is done given by $W = Fd$.
- If ϴ=90^0, then work done =0
- If ϴ is between 0^0 and 90^0, the work done is positive.
- If ϴ is between90^0 and 180^0, the work done is negative.

Situations in which Work done = 0

- **the displacement is zero (d=0)**:
  - A weightlifter holding a 150 kg mass steadily on his shoulder for 30 s does no work on the load during this time.
- **the force is zero (F=0)**:
  - A block moving on a smooth horizontal table is not acted upon by a horizontal force (since there is no friction), but may undergo a large displacement
- **the force and displacement are mutually perpendicular (**ϴ = 90^0)
  - For the block moving on a smooth horizontal table, the gravitational force *mg does no work since it acts at right* angles to the displacement.

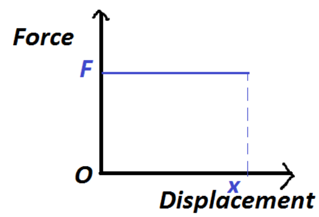
Situations in which work done is negative

- A ball is thrown in the upward direction – work done by the gravitational force is negative.
- The work done by the frictional force, when we push the book to a distance is negative
- The work done by the gravitational force, when we are lifting a bucket of water from the well is negative

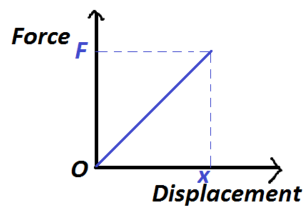
Force –Displacement graph (F-d Graph)

- A graph drawn with displacement along X –axis and force along Y- axis.
- **Area** under F-d graph gives **the work done**.

F-d graph of work done by a constant force



F-d graph of work done by a uniformly varying force



ENERGY

- Energy is the capacity for doing work.
- It can be measured by the work that the body can do.
- Joule is the SI unit of energy.

Alternative units of Work /Energy

erg	**10^{-7}J**
electron volt (eV)	**1.6 x 10^{-19} J**
calorie (cal)	**4.186 J**
kilowatt hour (kWh)	**3.6 x 10^6 J**

MEHANICAL ENERGY

- The energy of an object due to its motion or position.
- Total mechanical energy is the sum of kinetic and potential energy

KINETIC ENERGY

- The kinetic energy of an object is a measure of the work an object can do by the virtue of its motion

- Kinetic energy an object of mass m *moving with* velocity **v**, is

$$K = \frac{1}{2}m(\vec{v}.\vec{v}) = \frac{1}{2}mv^2$$

- Kinetic energy is a scalar quantity.
- In terms of momentum , p

$$K = \frac{p^2}{2m}$$

- The dimensions are $[ML^2T^{-2}]$
- The SI unit is joule (J).

POTENTIAL ENERGY

- Potential energy is the 'stored energy' by virtue of the position or configuration of a body.
- Eg: energy in a stretched string
- The potential energy is released in the form of kinetic energy.
- It is a scalar quantity.
- The dimensions of potential energy are $[ML^2T^{-2}]$.
- The SI unit is joule (J).

Gravitational Potential Energy (V)

- Gravitational potential energy of an object at a height h, is the negative of work done by the gravitational force in raising the object to that height.

$$V(h) = mgh$$

- The gravitational force can be written as

$$F = -\frac{dV(h)}{dh} = -mg$$

- Thus the gravitational force F equals the negative of the derivative of $V(h)$ with respect to h.
- The negative sign indicates that the gravitational force is downward.

Conservative Force

- A force *is conservative if*
    1) *it can be derived* from a scalar quantity *V(x).*
    2) the work done by the force depends only on initial and final positions.
- Examples are, gravitational force, electric force , spring force etc
- The work done by a conservative force in a closed path is zero.
- The change in potential energy of a conservative force is equal to the negative of the work done by the force.

$$\Delta V = -F(x)\Delta x$$

Non conservative forces

- The forces in which the work done depends on the factors like velocity or path taken.
- Example: frictional force, viscous force etc.

PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

- **The total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.**
- **If forces are conservative**

$$K + V = \text{constant}$$

Proof

- If a body undergoes displacement Δx , under the action of conservative forces, F(x), from work – energy theorem,

$$\Delta K = \mathrm{F(x)\Delta x}$$

- The change in potential energy is given by

$$\Delta V = -\mathrm{F(x)\Delta x}$$

- Adding the two equations

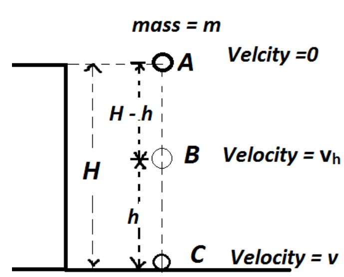
$$\Delta K + \Delta V = \mathrm{F(x)\Delta x - F(x)\Delta x} = 0$$

$$\Delta(\mathrm{K+V}) = 0$$

$$\mathrm{K+V} = \text{constant}$$

Conservation of Mechanical Energy in a Freely Falling Body

- Consider a ball of mass m *being dropped from a cliff of* height h.



Total Energy at the point A

- Kinetic energy at A is zero (K=0), since v=0
- Potential energy at A is , V = mgH

- Thus total energy at A, is

$$E = K + V = 0 + \mathrm{mgH} = \mathrm{mgH}$$

Total Energy at the point B

- Kinetic energy at B is

$$K = \frac{1}{2} m {v_h}^2$$

- But we have

$${v_h}^2 - 0^2 = 2g(H-h)$$

$${v_h}^2 = 2g(H-h)$$

- Thus

$$K = \frac{1}{2} m {v_h}^2 = mg(H-h)$$

- Potential energy at B is , V = mgh
- The total energy at B is

$$E = K + V = \mathrm{mg(H - h)} + \mathrm{mgh} = \mathrm{mgH}$$

Total Energy at the point C

- The kinetic energy at C is

$$K = \frac{1}{2} m v^2$$

- But we have

$$v^2 - 0^2 = 2gH$$

$$v^2 = 2gH$$

- Thus

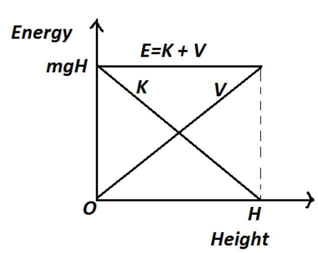
$$K = \frac{1}{2} m v^2 = mgH$$

- The potential energy at C is, V = 0
- The total energy at C

$$E = K + V = \mathrm{mgH} + 0 = \mathrm{mgH}$$

- **Therefore total energy at A = total energy at B = Total energy at C = mgH = a constant**

Graph of the variation of kinetic energy and potential energy of a freely falling body



POWER

- **Power** is defined as the time rate at which work is done or energy is transferred.
- **Average power** is given by

$$P_{av} = \frac{W}{t}$$

- Where W – total work done , t – total time

- **The instantaneous power** is given by

$$P = \frac{dW}{dt}$$

- Power is a **scalar** quantity

- SI unit – **watt (W)**

- Dimensions are **[ML^2T^{-3}]**

- Another unit of power is **horse – power (hp)**

$$1hp = 746W$$

- Horse –power is used to describe the output of automobiles, motorbikes, etc.

Relation connecting power, force and velocity

- We have the work done

$$dW = \vec{F} \bullet d\vec{r}$$

- Where F – force , dr – displacement.

- Thus the instantaneous power is given by

$$P = \frac{dW}{dt} = \vec{F} \bullet \frac{d\vec{r}}{dt}$$

- That is

$$P = \vec{F} \bullet \vec{v}$$

Unit of electrical energy

- Electrical energy is often expressed in **kilowatt hour (kWh)**

$$1kWh = 3.6 \times 10^6 J$$

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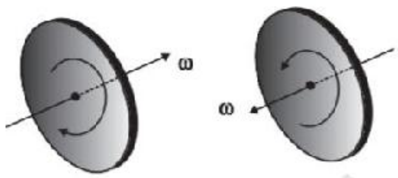
XI PHYSICS - CHAPTER 7
SYSTEMS OF PARTICLES AND ROTATIONAL MOTION

(Prepared By Ayyappan C, HSST , GMRHSS , Kasaragod, Mob: 9961985448)

ANGULAR VELOCITY AND ITS RELATION WITH LINEAR VELOCITY

- The average angular velocity of the particle over the interval Δt *is* **$\Delta\theta/\Delta t$.**
- The instantaneous angular velocity
  $\omega = d\theta/dt$.



- The general relation connecting angular velocity and linear velocity is given by

$$\vec{v} = \vec{\omega} \times \vec{r}$$

- The angular velocity is a vector quantity**.**

Angular acceleration

- Angular acceleration α is the time rate of change of angular velocity.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

- If the axis of rotation is fixed, the direction of ω and hence, that of α is fixed.

Moment of force (Torque)

- The **rotational analogue of force** is moment of force or torque.
- Torque is given by

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- The moment of force (or torque) is a vector quantity.
- The symbol τ stands for the Greek letter tau.
- The magnitude of τ is

$$\tau = rF\sin\theta$$

- Moment of force has dimensions same as those of work or energy **[ML^2T^{-2}]**
- Moment of a force is a vector, while work is a scalar.
- The SI unit of moment of force is **Newton-metre** (Nm).

Angular momentum of a particle

- Angular momentum is the **rotational analogue of linear momentum**.
- The angular momentum is given by

$$\vec{l} = \vec{r} \times \vec{p}$$

- The magnitude of the angular momentum vector is

$$l = rp\sin\theta$$

Relation Between Angular Momentum and Torque

- **We have**

$$\vec{l} = \vec{r} \times \vec{p}$$

- Differentiating with respect to time,

$$\frac{d\vec{l}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

- But ,

$$\frac{d}{dt}(\vec{r} \times \vec{p}) = (\vec{r} \times \frac{d\vec{p}}{dt}) + (\frac{d\vec{r}}{dt} \times \vec{p})$$

$$\frac{d}{dt}(\vec{r} \times \vec{p}) = (\vec{r} \times \vec{F}) + (\vec{v} \times m\vec{v})$$

- Here F= (dp/dt) and p= mv
- Since (**v x v) = 0**

$$\frac{d}{dt}(\vec{r} \times \vec{p}) = (\vec{r} \times \vec{F}) + 0$$

- **Thus**

$$\frac{d\vec{l}}{dt} = \vec{r} \times \vec{F}$$

- Therefore

$$\frac{d\vec{l}}{dt} = \vec{\tau}$$

- Thus, the time rate of change of the angular momentum of a particle is equal to the torque acting on it.

Torque and angular momentum for a system of particles

- For a system of *n particles, the total angular momentum is*

$$\vec{L} = l_1 + l_2 + \ldots\ldots + l_n$$

$$\vec{L} = \sum_{i=1}^{n} \vec{r}_i \times \vec{p}_i$$

- Thus

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{ext}$$

Conservation of Angular Momentum

- If the external torque acting on a system is zero, then

$$\frac{d\vec{L}}{dt} = 0$$
$$\vec{L} = \text{constant}$$

- Thus **if total external torque on a system is zero the angular momentum is conserved**.

MOMENT OF INERTIA

- Moment of inertia is the rotational analogue of mass of a body.
- The moment of inertia given by

$$I = \sum_{i=1}^{n} m_i r_i^{\,2}$$

- It is independent of the magnitude of the angular velocity.
- It is regarded as a measure of **rotational inertia of the body**
- Unit is kgm^2.

The moment of inertia of a rigid body depends on :

- the mass of the body,
- its shape and size
- distribution of mass about the axis of rotation,
- The position and orientation of the axis of rotation.

Rotational kinetic energy

- The kinetic energy in terms of moment of inertia is
- We have kinetic energy of a particle

$$k_i = \frac{1}{2} m_i v_i^{\,2}$$

- The velocity is given by

$$v_i = r_i \omega$$

- Thus for a system of particles

$$K = \frac{1}{2} \sum_{i=1}^{n} m_i r_i^{\,2} \omega^2$$

- Therefore

$$K = \frac{1}{2} I \omega^2$$

- where ω - angular velocity, I – moment of inertia
- or

$$K = \frac{L^2}{2I}$$

- where L – angular momentum

Moment of Inertia of a thin Ring

- Consider a thin ring of radius *R and mass M,* rotating in its own plane around its centre with angular velocity ω.
- Each mass element of the ring is at a distance R from the axis, and moves with a speed *Rω.*
- The kinetic energy is therefore,

$$K = \frac{1}{2} M v^2 = \frac{1}{2} M R^2 \omega^2$$

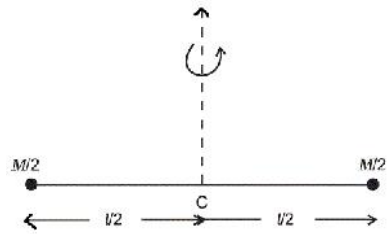
- Therefore comparing the equation with

$$K = \frac{1}{2} I \omega^2$$

- We get $I = MR^2$

Moment of Inertia of a rigid Rod

- Consider a rigid massless rod of length with a pair of small masses, rotating about an axis through the centre of mass perpendicular to the rod.



- Each mass *M/2 is at a distance l/2 from the axis.*
- The moment of inertia of the masses is therefore given by

$$I = \frac{M}{2}\left(\frac{l}{2}\right)^2 + \frac{M}{2}\left(\frac{l}{2}\right)^2$$

- Thus

$$I = \frac{Ml^2}{4}$$

Radius of Gyration

- In general moment of inertia can be written as

$$I = Mk^2$$

- Here the length *k is a* geometric property of the body and axis of rotation. It is called the **radius of gyration.**
- **The radius of gyration**

$$k = \sqrt{\frac{I}{M}}$$

- **The radius of gyration of a body about an axis** may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.

Moment of inertia of different bodies

No	Body	Axis	I
1	Thin circular ring radius R	Perpendicular to plane ,at centre	MR^2

2	Thin circular ring radius R	Diameter	$\frac{MR^2}{2}$
3	Thin rod ,length L	Perpendicular to rod ,at mid point	$\frac{ML^2}{12}$
4	Circular disc radius R	Perpendicular disc at centre	$\frac{MR^2}{2}$
5	Circular disc radius R	diameter	$\frac{MR^2}{4}$
6	Hollow cylinder radius R	Axis of cylinder	MR^2
7	Solid cylinder radius R	Axis of cylinder	$\frac{MR^2}{2}$
8	Solid sphere radius R	Diameter	$\frac{2}{5}MR^2$

Practical uses of moment of inertia

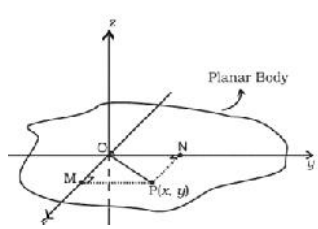
- The machines, such as steam engine and the automobile engine, etc., that produce rotational motion have a disc with a large moment of inertia, called a **flywheel.**
- Because of its large moment of inertia, the flywheel resists the sudden increase or decrease of the speed of the vehicle.
- It allows a gradual change in the speed and prevents jerky motions, thereby ensuring a smooth ride for the passengers on the vehicle.

Theorem of Perpendicular Axes

- **It states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.**
- **Thus**

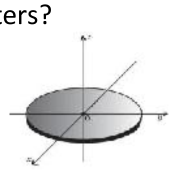
$$I_z = I_x + I_y$$

- This theorem is applicable to bodies which are planar.



PROBLEM

- What is the moment of inertia of a disc about one of its diameters?



Solution

- The moment of inertia of the disc about an axis perpendicular to it and through its centre is

$$I_z = \frac{MR^2}{2}$$

- Where M –mass, R – radius
- By symmetry of the disc, the moment of inertia about any diameter is same.

$$I_x = I_y$$

- Using perpendicular axis theorem

$$I_z = I_x + I_y = 2I_x$$

$$2I_x = \frac{MR^2}{2}$$

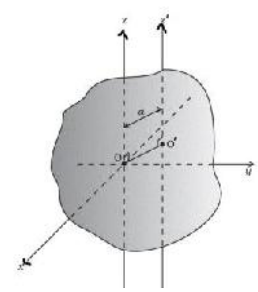
$$I_x = \frac{MR^2}{4}$$

Theorem of parallel axes

- **The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.**

$$I_{z'} = I_z + Ma^2$$

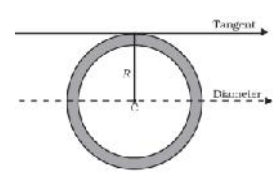
- Where a –distance between two parallel axes.
- This theorem is applicable to a body of any shape.



POBLEM

- What is the moment of inertia of a ring about a tangent to the circle of the ring?

Solution



$$I_{\text{tangent}} = I_{dia} + MR^2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

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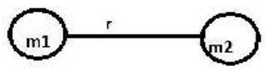
Chapter 8
GRAVITATION

(Prepared By Ayyappan C, HSST , GMRHSS, Kasaragod,)

UNIVERSAL LAW OF GRAVITATION

- ***Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.***



- Mathematically,

$$\boxed{|\mathbf{F}| = G\ \frac{m_1\ \ m_2}{r^2}}$$

- where **G** is the ***universal gravitational constant***
- The value of the gravitational constant G is experimentally determined by English scientist Henry Cavendish in 1798**.**

G = 6.67x10^{-11} N m^2/kg^2

ACCELERATION DUE TO GRAVITY OF THE EARTH
Acceleration due to gravity on the surface

- The gravitational force acting on a body on the surface of earth is given by

$$F = \frac{GMm}{R^2}$$

Where G- gravitational constant, M- mass of earth, m- mass of the body, R- radius of the earth.

- The weight experience by the body is
F = m*g, where g – acceleration due to gravity*
- *Thus ,*

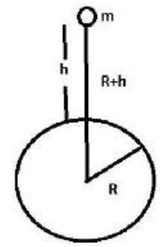
$$mg = \frac{GMm}{R^2}$$

- Therefore

$$g = \frac{GM}{R^2}$$

- The mass of the earth can be calculated using the values of acceleration due to gravity, G and radius of earth.
- This is the reason for the statement **"Cavendish weighed the earth".**

Variation of acceleration due to gravity with height



- The gravitational force on the mass *m at a height h above the* surface of the earth is

$$F = \frac{GMm}{(R+h)^2}$$

- Th**e** weight of the body at the height h is **mg$_h$** , where g$_h$ is the acceleration due to gravity at height**.**
- Thus

$$\mathbf{mg_h} = \frac{GMm}{(R+h)^2}$$

Therefore ,

$$\mathbf{g_h} = \frac{GM}{(R+h)^2}$$

- **If R>>h**

$$\mathbf{g_h} = \frac{GM}{R^2(1+\frac{h}{R})^2}$$

- ie , $\mathbf{g_h} = g(1+\frac{h}{R})^{-2}$,
- since $g = \frac{GM}{R^2}$
- Or

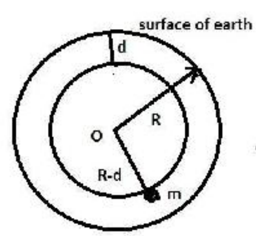
$$\mathbf{g_h} = g(1+\frac{h}{R})^{-2}$$

- Using binomial expression and neglecting the higher order terms we get

$$\mathbf{g_h} = g(1-\frac{2h}{R})$$

- Thus for small heights h above the value of g decreases

Variation of g with depth



- .If ‘ρ’ is the mean density of earth, then mass of earth is
- Mass = Volume x Density, ie

$$M = \frac{4}{3}\pi R^3 \rho$$

- Similarly mass of the small sphere of radius R-d is

$$M_S = \frac{4}{3}\pi (R-d)^3 \rho$$

- **Thus**

$$\frac{M_S}{M} = \frac{\frac{4}{3}\pi (R-d)^3 \rho}{\frac{4}{3}\pi R^3 \rho}$$

$$\frac{M_S}{M} = \frac{(R-d)^3}{R^3}$$

- **The acce**leration due to gravity on the surface of earth is

$$g = \frac{GM}{R^2}$$

- Thus the acceleration due to gravity on body at a depth d is

$$g_d = \frac{GM_S}{(R-d)^2}$$

- Thus dividing the two equations and substituting for M_S/M. we get

$$\frac{g_d}{g} = \frac{(R-d)}{R}$$

- Simplifying

$$g_d = g(1 - \frac{d}{R})$$

- Thus, as we go down below earth’s surface, the acceleration due gravity decreases.
- At the centre of the earth acceleration due to gravity is zero.

PROBLEMS

1. **Find the value of acceleration due to gravity at a height 100 km above the surface. ($g=9.8 m/s^2$, $R = 6.37 \times 10^3$ km)**

Solution

$$\mathbf{g_h} = g(1 - \frac{2h}{R}) \qquad g_h = 9.5\ m/s^2.$$

2. At what height above the surface of earth the value of g is reduced to 1/4th of the value of g on earth’s surface.

Solution

$$g(1 - \frac{2h}{R}) = \frac{1}{4}g$$

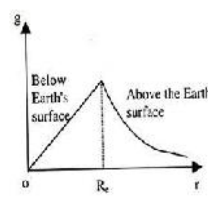
$$h = \frac{3R}{8}$$

$$\frac{g}{(1+\frac{h}{R})^2} = (g/4)$$

Or $(1+\frac{h}{R})^2 = 4$

$$(1+\frac{h}{R}) = 2, h = R$$

3. At what height the value of g will be half that on the surface of earth?
   Solution : h= 0.414R
4. Draw graph showing variation of g with distance from the centre.



$$\boxed{g = \frac{4}{3}\pi G\rho r; \quad = \frac{GM}{r^2};}$$

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CHAPTER NINE

MECHANICAL PROPERTIES OF SOLIDS

(Prepared By Ayyappan C, HSST Physics, GMRHSS , Kasaragod, *Mob: 9961985448*)

STRESS:

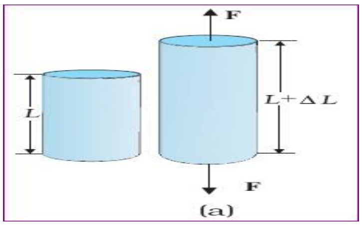
- The ***restoring force per unit area*** is known as stress.
- If *F is the force applied* and *A is the area of cross section of the* body, magnitude of the **stress = *F/A***
- The SI unit of stress is N m^{-2} or pascal (Pa)
- Its dimensional formula is $\boxed{[\,ML^{-1}T^{-2}\,]}$

Types of stress

- **Longitudinal stress or linear stress**
- **Normal stress or hydraulic stress**
- **Shearing stress or tangential stress**

Longitudinal stress or linear stress

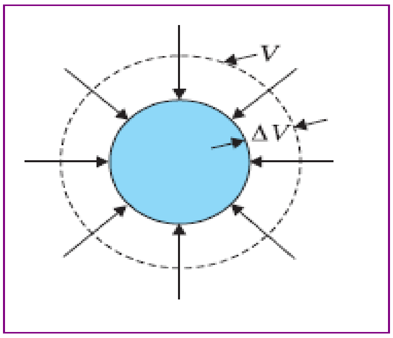
- This stress produces a change in length.
- The change in length may be elongation(tensile stress) or compression (compressive stress)



(a)

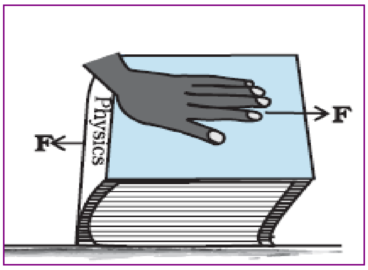
Normal stress or hydraulic stress or volume stress

- This stress produces a change in volume



Shearing stress or tangential stress

- This stress produces a change in shape



STRAIN:

- It is the ratio of change in dimension to the original dimension.
- It has no unit and dimensions.

Longitudinal (Linear) strain:

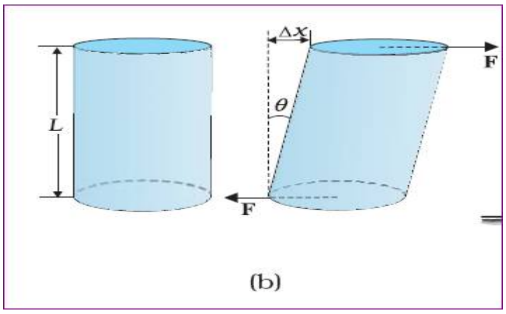
- It is the ratio of change in the length (ΔL) to the original length(L) of the body .
- **Longitudinal strain = $\Delta L/L$**

Volume strain:

- It is the ratio of change in volume (ΔV) to the original volume (V)
- **Volume strain = $\Delta V/V$**

Shearing strain :

- It is the angle turned by a straight line assumed on the body which was originally perpendicular to the tangential force**.**



(b)

$$\text{Shearing strain} \quad \frac{x}{L} = \tan\theta$$

- Usually θ is very small, tan θ is nearly equal to angle θ.
- **Thus, shearing strain = tan θ ≈ θ**

HOOKE'S LAW:

- For small deformations the stress is directly proportional to strain.

Thus,

$$\text{stress} \propto \text{strain}$$
$$\text{stress} = k \times \text{strain}$$

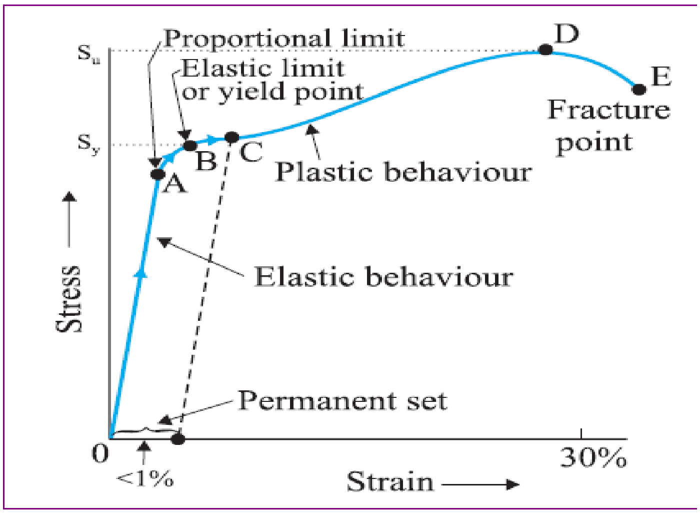
- Where *k is the proportionality constant and is* known as **modulus of elasticity**.

$$K = \frac{\text{Stress}}{\text{Strain}}$$

- Modulus of elasticity **depends on**, nature of the material of the body and temperature.
- It **is independent** of the dimensions of the body.
- S.I unit of 'k' is Nm^{-2} or Pascal [Pa]

Stress – Strain Curve:

- A graph drawn with strain along x-axis and strain along y-axis.



- The point B in the curve is known as ***yield point*** **(also known as elastic limit)** and the corresponding stress is known as ***yield strength*** (S_y) of the material.

- The point D on the graph is the ***ultimate tensile strength*** (S_u) *of the material.*

- If the ultimate strength and fracture points D and E are close, the material is said to be ***brittle.***

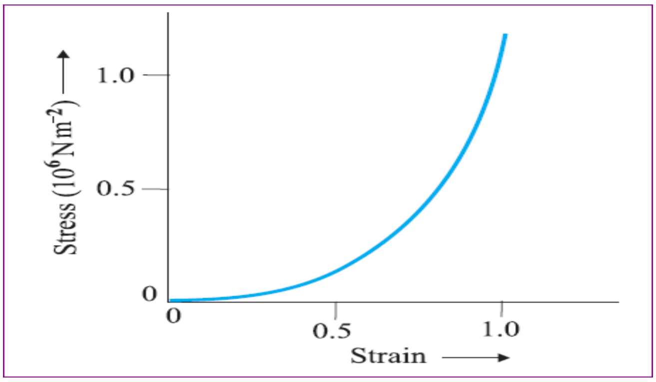
- If they are far apart, the material is said to be ***ductile*****.**

Elastomers:

- Substances which can be stretched to cause large strains are called elastomers.

- Eg: tissue of aorta, rubber etc

Stress-strain curve for the elastic tissue of Aorta



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CHAPTER 10

MECHANICAL PROPERTIES OF FLUIDS

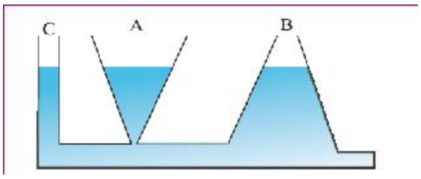
(Prepared By Ayyappan C, HSST, GMRHSS, Kasaragod)

Pascal's Law

- **The pressure in a fluid at rest is the same at all points if they are at the same height.**

Hydrostatic paradox

- The liquid level is independent of the shape of the container.
- Hydrostatic paradox is a consequence of Pascal's law.



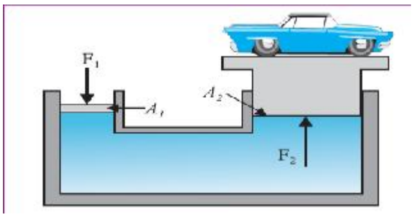
Hydraulic Machines

Pascal's law for transmission of fluid pressure

- Whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions.
- Hydraulic lift and hydraulic brakes are based on the Pascal's law.

Hydraulic Lift

- In a hydraulic lift two pistons are separated by the space filled with a liquid.



- A piston of small cross section A_1 *is used to exert a force* F_1 directly on the liquid.
- The pressure on the first piston is

$$P = F_1/A_1$$

- According to Pascal's law this pressure is transmitted throughout the liquid.
- Then the upward force on the second piston is ,

$$F_2 = PA_2 = \frac{F_1 A_2}{A_1}$$

- Therefore, the piston is capable of supporting a large force (large weight of, say a car, or a truck, placed on the platform)
- Thus, the applied force has been increased by a factor of A_2/A_1 , *this factor* is the **mechanical advantage** of the device.
- By changing the force at A_1, *the platform can be* moved up or down.

Hydraulic brakes

- When we apply a little force on the pedal with our foot the master piston moves inside the master cylinder, and the pressure caused is transmitted through the brake oil to act on a piston of larger area.
- A large force acts on the piston and is pushed down expanding the brake shoes against brake lining.
- Thus a small force on the pedal produces a large retarding force on the wheel.
- The pressure set up by pressing pedal is transmitted equally to all cylinders attached to the four wheels so that the braking effort is equal on all wheels.

BERNOULLI'S PRINCIPLE

- The total energy of an incompressible non viscous fluid in a steady flow from one point to another is a constant.
- It is a statement of conservation of energy.

Applicability of Bernoulli's theorem

- The fluids must be non viscous.
- Fluids must be incompressible
- This law does not hold for non steady or turbulent flow- since the velocity and pressure constantly changes with time.

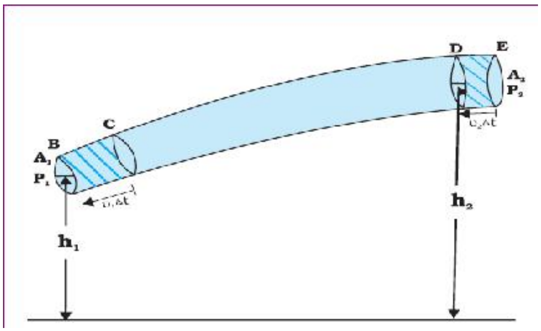
BERNOULLI'S EQUATION

- Bernoulli's equation is a general expression that relates the pressure difference between two points in a pipe to both velocity changes (kinetic energy change) and elevation (height) changes (potential energy change).
- Swiss Physicist Daniel Bernoulli developed this relationship.

$$P + \left(\frac{1}{2}\right)\rho v^2 + \rho gh = \text{constant}$$

- Where ρ- density of fluid, v- speed, P- pressure.
- The ***Bernoulli's relation*** may be stated as follows:
- As we move along a streamline the sum of the pressure (P), the kinetic energy per unit volume and the potential energy per unit volume (ρgh) remains a constant.

Derivation



- The figure shows, a fluid moving in a pipe of variable area of cross section and different heights.
- *v_1 is the speed at B and v_2 at D, then* fluid initially at B has moved a distance ***$v_1\Delta t$*** *to* C
- At the same interval Δ*t* the fluid initially at D moves to E, a distance equal to ***$v_2\Delta t$.***

Total work done on the fluid

- The ***work done on the fluid*** at left end (BC) is
$$\boxed{W_1 = P_1A_1(v_1\Delta t) = P_1\Delta V.}$$
- The ***work done by the fluid*** at the other end (DE) is
$$\boxed{W_2 = P_2A_2(v_2\Delta t) = P_2\Delta V}$$
- Thus the ***work done on the fluid*** at DE is
$$W_2 = -P_2\Delta V$$
- ***The total work** done **on the*** fluid is
$$\boxed{W_1 - W_2 = (P_1 - P_2)\ \Delta V}$$
- Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy.

Change in potential energy

- If the density of the fluid is ρ, the mass passing through the pipe in time **Δt is**
$$\boxed{\Delta m = \rho A_1 v_1 \Delta t = \rho \Delta V}$$
- Potential energy at height h_1 is
$$U_1 = \rho g h_1 \Delta V$$
- Potential energy at height h_2 is
$$U_2 = \rho g h_2 \Delta V$$
- Thus change in gravitational potential energy is
$$\boxed{\Delta U = \rho g \Delta V\ (h_2 - h_1)}$$

Change in kinetic energy

- The kinetic energy at the first end is
$$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}\rho v_1^2 \Delta V$$
- The kinetic energy at the last end is
$$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\rho v_2^2 \Delta V$$
- Thus the change in kinetic energy is
$$\boxed{\Delta K = \left(\frac{1}{2}\right)\rho\ \Delta V\,(v_2^2 - v_1^2)}$$

Work-energy theorem

- Applying Work-Energy Theorem we get
$$\boxed{(P_1 - P_2)\ \Delta V = \left(\frac{1}{2}\right)\rho\ \Delta V\,(v_2^2 - v_1^2) + \rho g \Delta V\ (h_2 - h_1)}$$
- Dividing each term by Δ*V*
$$\boxed{(P_1 - P_2) = \left(\frac{1}{2}\right)\rho\ (v_2^2 - v_1^2) + \rho g\ (h_2 - h_1)}$$
- Rearranging the above terms
$$\boxed{P_1 + \left(\frac{1}{2}\right)\rho v_1^2 + \rho g h_1 = P_2 + \left(\frac{1}{2}\right)\rho v_2^2 + \rho g h_2}$$
- This is **Bernoulli's equation**
- In general
$$\boxed{P + \left(\frac{1}{2}\right)\rho v^2 + \rho g h = \text{constant}}$$

Flow through a horizontal pipe

- If the pipe is horizontal **h_1=h_2 ,**then ΔU =0
- Bernoulli's Theorem becomes ,
$$\boxed{P + \frac{1}{2}\rho V^2 = \text{Constant}}$$

Bernoulli's equation for a stationary fluid

- When fluid is at rest the velocity is zero
- Thus the equation becomes
$$\boxed{P_1 + \rho g h_1 = P_2 + \rho g h_2}$$
- Or
$$\boxed{(P_1 - P_2) = \rho g\ (h_2 - h_1)}$$
- This is ***Pascal's law***

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CHAPTER ELEVEN
THERMAL PROPERTIES OF MATTER
(*Prepared By Ayyappan C, HSST Physics, GMRHSS Kasaragod, Mob: 9961985448*)

THERMAL EXPANSION

- The increase in the dimensions of a body due to the increase in its temperature is called thermal expansion.
- The expansion in length is called **linear expansion.**
- **The expansion in area is called area expansion.**
- **The expansion in volume is called volume expansion.**
- ***The fractional change in dimension [ratio of change in dimension to original dimension] is proportional to change in temperature***.
- The corresponding proportionally constant is called **co-efficient of thermal expansion or thermal expansivity.**

Type of thermal expansion	Linear expansion	Area expansion	Volume expansion
The dimension that changes	length	Area	Volume
Coefficient of	$\alpha_\ell = \frac{\Delta\ell}{\ell\Delta T}$	$\alpha_a = \frac{\Delta A}{A\Delta T}$	$\alpha_v = \frac{\Delta V}{V\Delta T}$
thermal expansion (α)	linear expansivity or co-efficient of linear expansion	Area expansivity or co-efficient of area expansion	Volume expansivity or coefficient of volume expansion
Relation		$\alpha_a = 2\alpha_\ell$	$\alpha_v = 3\alpha_\ell$

- ***Show that the coefficient of volume expansion for ideal gas is reciprocal of temperature***
Proof : Ideal Gas Equation is

$$PV = PV = \mu RT \text{ .... } (1)$$

- At constant pressure

$$P\Delta V = \mu R\Delta T \text{ .... } (2)$$

- Dividing the equations we get

$$\frac{\Delta V}{V} = \frac{\Delta T}{T}$$

$$\frac{\Delta V}{V\Delta T} = \frac{1}{T} = \alpha_V$$

- **Obtain the following relations**

$$\text{(i) } \alpha_a = 2\alpha_\ell \qquad \text{(ii) } \alpha_v = 3\alpha_\ell$$

- Consider a cube of length ' l '. Due to the increase in temperature ' ΔT', length of cube increases by Δl in all directions. The Coefficient of linear expansion is

$$\alpha_\ell = \frac{\Delta\ell}{\ell\Delta T}$$

(i)Increase in area of cube ΔA – Final area - initial area $= (\ell+\Delta\ell)^2 - \ell^2 = 2\times\ell\times\Delta\ell$ [Neglecting $\Delta\ell^2$]
Area expansivity

$$\alpha_a = \frac{\Delta A}{A\Delta T} = \frac{2\ell\times\Delta\ell}{\ell^2\Delta T} = \frac{2\cdot\Delta\ell}{\ell\cdot\Delta T} = 2\cdot\alpha_\ell$$

Therefore, $\alpha_a = 2\cdot\alpha_\ell$

(ii) Due to ' ΔT ' the increase in volume of cube, $\Delta V = (\ell+\Delta\ell)^3 - \ell^3$

$$= 3\ell^2\Delta\ell$$

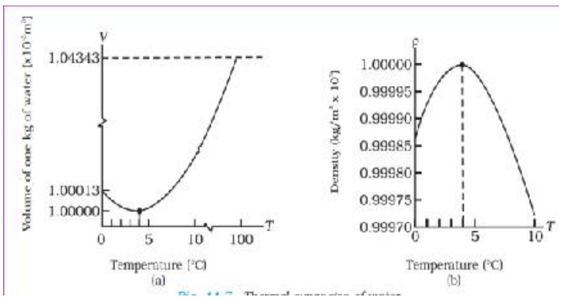
[Neglecting $\Delta\ell^2$ & $\Delta\ell^3$]

$$\alpha_v = \frac{\Delta V}{V\cdot\Delta T} = \frac{3\ell^2\cdot\Delta\ell}{\ell^3\times\Delta T} = 3\times\alpha_\ell$$

Therefore, $\alpha_v = 3\alpha_\ell$

ANOMALOUS BEHAVIOUR OF WATER

- **Water exhibits an anomalous behaviour; it contracts on heating between 0 °C and 4 °C.**
- **The volume of a given amount of water decreases as it is cooled from room temperature, until its temperature reaches 4 °C.**
- **Thus water has a maximum density at 4 °C.**



Environmental effect of Anomalous Behaviour of water

- **Bodies of water, such as lakes and ponds, freeze at the top first.**
- **As a lake cools toward 4 °C, water near the surface becomes denser and sinks; the water near the bottom rises.**

- **once the colder water on top reaches temperature below 4 °C, it becomes less dense and remains at the surface and freezes**
- **If water did not have this property, lakes and ponds would freeze from the bottom up, which would destroy much of their animal and plant life**

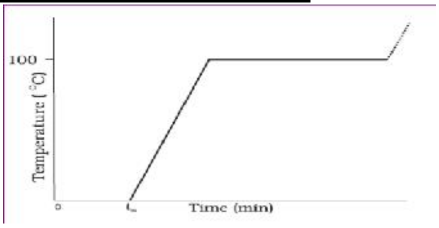
CHANGE OF STATE

- A transition from one state (solid, liquid or gas) to another state is called change of state.

Change of state	Name of transition
Solid → Liquid	Melting
Liquid → gas	Vapourisation
Liquid → solid	Fusion
Solid → gas (without forming liquid)	Sublimation

- During change of state, the two different state ***coexist in thermal equilibrium***.

Temperature – time graph of ice



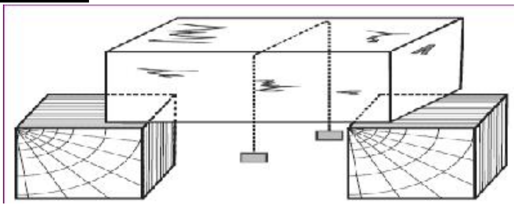
Melting point

- The temperature at which solid and liquid coexist in thermal equilibrium with each other is called melting point.
- The ***melting point decreases with increase in pressure***

Boiling point

- The temperature at which liquid and vapor state of substance coexist in thermal equilibrium with each other is called boiling point.
- The ***boiling point increases with increase in pressure*** and it decreases with decrease in pressure

Regelation



- When a metal wire loaded at both ends is kept over an ice block, it passes through the ice block to the other side without splitting it.
- The melting point of ice just below the wire decreases due to increase in pressure.
- As ice melts wire passes and refreeze (due to decrease in pressure). This process is called regelation.

Cooking is difficult at high altitude. Why *?*

- At high altitude, pressure is low. Boiling point decreasess with decrease in pressure.

For cooking rice pressure cooker is preferred. Why ?

- In pressure cooker, boiling point of water is increased by increasing pressure. Thus rice can be cooked at high temperature.

You might have observed the bubbles of steam coming from bottom of vessel when water is heated.These bubbles disappear as it reaches top of liquid just before boiling and they reach the surface at the time of boiling. Explain the reason ?

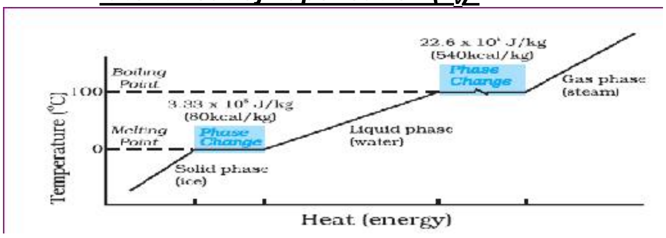
- Just before boiling, the bottom of liquid will be warm and at the top, liquid will be cool. So the bubbles of steam formed at bottom rises to cooler water and condense, hence they disappear. At the time of boiling, temperature of entire mass of water will be 100^{O}C. Now the bubbles reaches top and then escape.

LATENT HEAT

- *The amount of heat per unit mass transferred during change of state of substance is called latent heat of substance for the process.*
- *If mass m of a substance undergoes a change from one state to the other, then the quantity of heat required is given by*

$$\boxed{\begin{array}{ll} & Q = mL \\ \text{or} & L = Q/m \end{array}}$$

- Latent heat is characteristic of substance and it depends on pressure.
- Its unit is JKg^{-1}.
- The latent heat for a solid - liquid state change is called the ***latent heat of fusion (L_f), and that for a liquid-gas state change*** is called the ***latent heat of vaporisation (L_v).***



Burns from steam are usually more serious than boiling water. Why ?

- Latent heat of vapourisation for water is 22.6 x 10^5J Kg^{-1} (. ie; 22.6 x 10^5J heat is required to convert 1 kg of water into steam at 100^{0}C). So at 100^{0}C, steam carries 22.6 x 10^5 J. (more heat than water).

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CHAPTER TWELVE
THERMODYNAMICS

(Prepared By Ayyappan C, HSST Physics, GMRHSS, Kasaragod, *Mob: 9961985448*)

FIRST LAW OF THERMODYNAMICS

- **The amount of heat given to a system is equal to the sum of the increase in the internal energy of the system and the external work done.**

$$\Delta Q = \Delta U + \Delta W$$

- ***ΔQ = Heat supplied to the system by the* surroundings**
- ***ΔW = Work done by the system on the* surroundings**
- ***ΔU = Change in internal energy of the system***
- **At constant pressure**

$$\Delta W = P\Delta V$$

- **Thus**

$$\Delta Q = \Delta U + P\Delta V$$

THERMODYNAMIC PROCESSES

- It is any process in which there is some change in pressure, volume or temperature of a system.

Type of processes	Feature
Isothermal	Temperature constant
Isobaric	Pressure constant
Isochoric	Volume constant
Adiabatic	No heat flow between the system and the surroundings (ΔQ = 0)

Quasistatic process

- It is a process in which a thermodynamic system proceeds extremely slowly such that at every instant of time, the temperature and pressure are the same in all parts of the system.

Isothermal process

- A process in which the temperature of the system is kept fixed throughout is called an isothermal process.
- In such a process, if heat is developed in the system, it is given out to the surroundings or if heat is lost, it is taken from the surroundings.
- Eg: Melting , boiling, the expansion of a gas in a metallic cylinder placed in a large reservoir of fixed temperature etc.

Equation of state of isothermal process

$$PV = \text{constant}$$

Work done by an ideal gas during an isothermal process

- Suppose one mole of an ideal gas goes isothermally (at temperature *T) from its initial state (P_1, V_1) to* the final state (P_2, V_2).
- Let the volume of a gas having pressure P change by dV.
- Then work done, dW = P dV.
- Thus the total work done

$$W = \int_{V_1}^{V_2} P\,dV$$ But $$PV = RT. \text{ Or } P = \frac{RT}{V}$$

$$\therefore W = \int_{V_1}^{V_2} \frac{RT}{V}\,dV. \quad = RT\int_{V_1}^{V_2} \frac{dV}{V}$$

$$W = RT \ln\frac{V_2}{V_1}$$

- For an ideal gas, internal energy depends only on temperature.
- Thus, there is no change in the internal energy of an ideal gas in an isothermal process.
- ***For V2 > V1, W > 0; and for V2 < V1, W < 0.***
- **That is, in <u>an isothermal expansion</u>, the <u>gas absorbs heat</u> and does work while in <u>an isothermal compression,</u> work is done on the gas by the environment and <u>heat is released.</u>**

Adiabatic process

- In an adiabatic process, the system is insulated from the surroundings and heat absorbed or released is zero.

Equation of state for adiabatic process

- For an adiabatic process of an ideal gas

$$PV^{\gamma} = \text{const}$$

- where γ is the ratio of specific heats at constant pressure and at constant volume.

$$\gamma = \frac{C_p}{C_v}$$

- Also

$$TV^{\gamma-1} = \text{Constant}$$ or

$$P^{1-\gamma}T^{\gamma} = \text{Constant}$$

Work done in an adiabatic process

- Let an ideal gas undergoes adiabatic charge from (P1, V1, T1) to (P2, V2, T2).

$$W = \int_{V_1}^{V_2} P\,dV$$

$$W = k\int_{V_1}^{V_2} \frac{dV}{V^{\gamma}} \qquad \left(\text{since } P = \frac{k}{V^{\gamma}}\right)$$

- Here k is a constant

$$W = k\left|\frac{V^{-\gamma+1}}{-\gamma+1}\right|_{V_1}^{V_2} = \frac{k}{1-\gamma}\left[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}}\right] = \frac{1}{1-\gamma}\left[\frac{k}{V_2^{\gamma-1}} - \frac{k}{V_1^{\gamma-1}}\right] = \frac{1}{1-\gamma}\left[\frac{P_2V_2^{\gamma}}{V_2^{\gamma-1}} - \frac{P_1V_1^{\gamma}}{V_1^{\gamma-1}}\right]$$

$$W = \frac{1}{1-\gamma}\left[P_2V_2 - P_1V_1\right]$$

$$W = \frac{1}{1-\gamma}\left[P_2V_2 - P_1V_1\right] = \frac{R}{1-\gamma}(T_2 - T_1)$$

- That is

$$W = \frac{1}{\gamma-1}\left[P_1V_1 - P_2V_2\right] \quad \text{or} \quad W = \frac{R}{\gamma-1}\left[T_1 - T_2\right]$$

- If work is done by the gas in an adiabatic process (*$W > 0$) then $T2 < T1$.*
- If work is done on the gas (*$W < 0$), we get $T2 > T1$ i.e., the* temperature of the gas rises.

Isochoric process

- In an isochoric process, *V is constant.*
- *Thus work* done on or by the gas is zero.
- The heat absorbed by the gas goes entirely to change its internal energy and its temperature.

Isobaric process

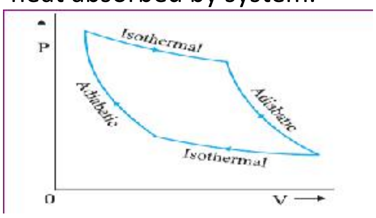
- In an isobaric process, *P is fixed. Work done by* the gas is

$$W = P(V_2 - V_1) = \mu R(T_2 - T_1)$$

- The heat absorbed goes partly to increase internal energy and partly to do work.
- The change in temperature for a given amount of heat is determined by the specific heat of the gas at constant pressure.

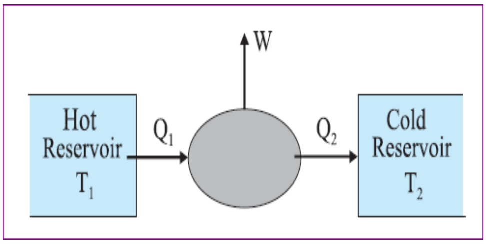
Cyclic process

- In cyclic process, the system returns to its initial state such that charge internal energy is zero.
- That is *$\Delta U = 0$ for a cyclic process*
- *Thus* the total heat absorbed equals the work done by the system
- The P - V diagram for cyclic process will be closed loop and area of this loop gives work done or heat absorbed by system.



HEAT ENGINES

- Heat engines converts' heat energy into mechanical energy.
- Heat engine is a device by which a system is made to undergo a cyclic process that results in conversion of heat to work
- ***Heat engines consists of*** **:**
- **Working substance** (the system which undergoes cyclic process) eg: mixture of fuel vapour and air in diesel engine, steam in steam engine.
- **An external reservoir at a high temperature** (T1) - it is the source of heat.
- ***An external reservoir at low temperature*** (T2) or sink



Working

- The working substances absorbs an energy Q1 from source reservoir at a temperature T1.
- It undergoes cyclic process and releases heat Q2 to cold reservoir.
- The change in heat (Q1 - Q2) is converted in to work (mechanical energy)

Efficiency of heat engine(η)

- The efficiency (η) of a heat engine is defined by

$$\eta = \frac{W}{Q_1}$$

- where *Q1 is the heat input i.e., the heat* absorbed by the system in one complete cycle and *W is the work done on the environment in* a cycle.
- Thus

$$W = Q_1 - Q_2$$

i.e.,

$$\eta = 1 - \frac{Q_2}{Q_1}$$

- In an external combustion engine, say a steam engine the system is heated by an external furnace.
- In an internal combustion engine, it is heated internally by an exothermic chemical reaction.

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CHAPTER THIRTEEN - KINETIC THEORY

(Prepared By Ayyappan C, HSST Physics, GMRHSS, Kasaragod, *Mob: 9961985448*)

KINETIC THEORY OF AN IDEAL GAS

- Kinetic theory of gases is a theory, which is based on the concept of molecular motion as is able to explain the behavior of gases.

Postulates of Kinetic Theory **:**

- **The molecules of a gas** are supposed to be **point masses**, the **size** of a molecule being **negligibl**e compared to the distance between them.
- There is **no force of attraction or repulsion between molecules**.
- The **molecules are in a state of random motion**, moving with all possible velocities in all possible directions.
- During their motion, they **collide with one another and also with the walls** of the container. These collisions **are elastic**.
- Between successive collisions, the molecules move in straight lines with uniform velocity. The distance travelled between two successive collisions is called **free path**. Average distance between the successive collisions is called **mean free path**
- **Time for a collision is negligibly small** compared to the time taken to traverse mean free path.
- The **mean KE of the molecule** is a constant at a given temperature and is **proportional to absolute temperature**.

Concept of Pressure.

- The pressure exerted by a gas may be defined as **the total momentum imparted to unit area** of the walls of the container **per second** due to molecular impacts (collisions).

Root mean square (rms) velocity of gas molecules.

- ***rms velocity*** of gas molecules is the square root of the mean of the squares of individual velocities of the molecules.
- If c_1, c_2, …….c_n are the velocities of a gas molecules, then mean square velocity,

$$\overline{c^2} = \frac{c_1^2 + c_2^2 + c_3^2 + \ldots\ldots + c_n^2}{n}$$

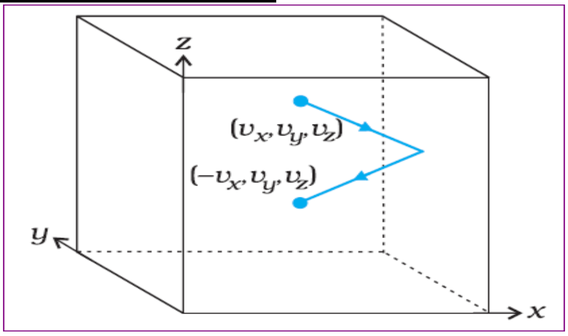
- Hence root mean square velocity

$$c_{rms} = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 + \ldots\ldots + c_n^2}{n}}$$

- At a temperature *T = 300 K, the root mean square speed* of a molecule in nitrogen gas is :

$$v_{rms} = 516 \text{ m s}^{-1}$$

Pressure of an Ideal Gas



- Consider a gas enclosed in a cube of side l.
- A molecule with velocity **(v_x, v_y, v_z)** *hits the planar wall parallel to yz plane* of area *A (= l^2).*
- The velocity after collision is **($-v_x$, v_y, v_z)** *.*
- The change in momentum of the molecule is **: *$-mv_x - (mv_x) = -2mv_x$* .**
- *By the* principle of conservation of momentum, the **momentum imparted to the wall in the collision = $2mv_x$.**
- In a small time interval Δ*t, a molecule with x-component of velocity* v_x will hit the wall if it is within the distance $v_x \Delta t$ from the wall.
- That is, all molecules within the volume *$Av_x \Delta t$ only can hit the wall in time Δt.*
- But, on the average, half of these are moving towards the wall and the other half away from the wall.
- Thus the number of molecules with velocity (v_x, v_y, v_z) *hitting the wall in time Δt is ½A v_x Δt n, where n is the number of molecules* per unit volume.
- The total momentum transferred to the wall by these molecules in time Δ*t is :*

$$Q = (2mv_x)\,(½\, n\, A\, v_x\, \Delta t\,)$$

- The force on the wall is the rate of momentum transfer *Q/Δt and pressure is force per unit* area :

$$P = Q\,/(A\,\Delta t) = n\,m\,v_x^{\,2}$$

- The above equation therefore, stands for pressure due to the group of molecules with speed v_x *in the x-direction and n* *stands for the* number density of that group of molecules.

- The total pressure is obtained by summing over the contribution due to all groups:

$$P = n\, m\, \overline{v_x^2}$$

- Now the gas is isotropic, i.e. there is no preferred direction of velocity of the molecules in the vessel.
- Therefore, by symmetry,

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$
$$= (1/3)\,[\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}] = (1/3)\,\overline{v^2}$$

- Thus

$$P = (1/3)\, n\, m\, \overline{v^2}$$

Kinetic Interpretation of Temperature

- We have

$$P = (1/3)\, n\, m\, \overline{v^2}$$

- We may write

$$PV = (1/3)\, nV\, m\, \overline{v^2}$$

$$PV = (2/3)\, N\, x\, ½\, m\, \overline{v^2}$$

- where *N (= nV) is the number of molecules in* the sample.
- The quantity in the bracket is the average translational kinetic energy of the molecules in the gas.
- Since the internal energy *E of an ideal* gas is purely kinetic,

$$E = N \times (1/2)\, m\, \overline{v^2}$$

- Thus

$$PV = (2/3)\, E$$

- But we have

$$PV = k_B NT \quad \text{or} \quad P = k_B nT$$

- Thus

$$E = (3/2)\, k_B NT$$

$$E/N = ½\, m\, v^2 = (3/2)\, k_B T$$

- Thus the ***average kinetic energy of a molecule is proportional to the absolute temperature of the gas***; it is independent of pressure , volume or the nature of the ideal gas.

Pressure of a Mixture Of Non- reactive Gases

- For a mixture of non-reactive ideal gases, the total pressure gets contribution from each gas in the mixture.
- Thus

$$P = (1/3)\,[n_1 m_1 \overline{v_1^2} + n_2 m_2 \overline{v_2^2} + \ldots]$$

- In equilibrium, the average kinetic energy of the molecules of different gases will be equal.
- That is,

$$½\, m_1\, \overline{v_1^2} = ½\, m_2\, \overline{v_2^2} = (3/2)\, k_B T$$

so that

$$P = (n_1 + n_2 + \ldots)\, k_B T$$

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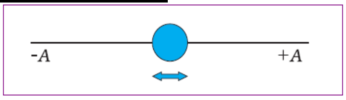
Chapter : 14
OSCILLATIONS
(Prepared By Ayyappan C, HSST, GMRHSS , Kasaragod)

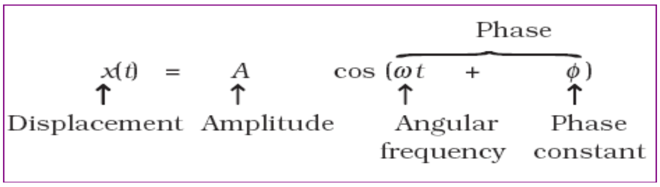
SIMPLE HARMONIC MOTION (SHM)

- In SHM the restoring force ***on the oscillating body is directly proportional to its displacement from the mean position, and is directed opposite to the displacement.***
- Eg: small oscillations of simple pendulum, swing, loaded spring, etc.

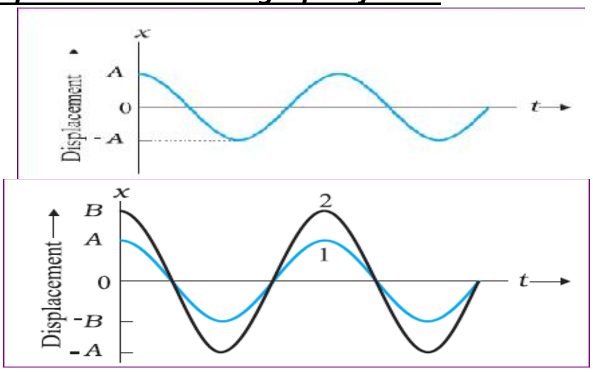
DISPLACEMENT OF SHM



- The displacement is given by
$$x(t) = A\cos(\omega t + \phi)$$



Displacement – Time graph of SHM



Amplitude(A):

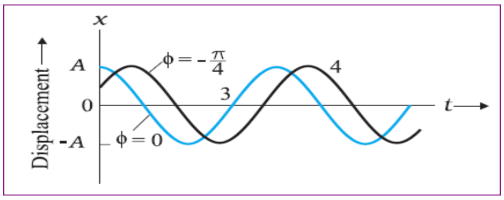
- It is the magnitude of the maximum displacement of the oscillating particle

Phase:

- The time varying quantity, ($\omega t + \varphi$), is called the **phase of the motion.**
- Phase describes the **state of motion** at a given time.

Phase constant (or phase angle):

- The constant ϕ is called the **phase constant.**
- The value of ϕ depends on the displacement and velocity of the particle at $t = 0$.

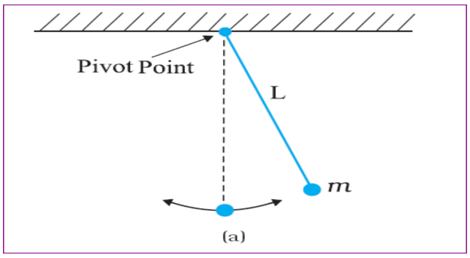


Angular frequency (ω):

- The angular frequency is, $\omega = \dfrac{2\pi}{T}$
- The SI unit of angular frequency is ***radians per second***.

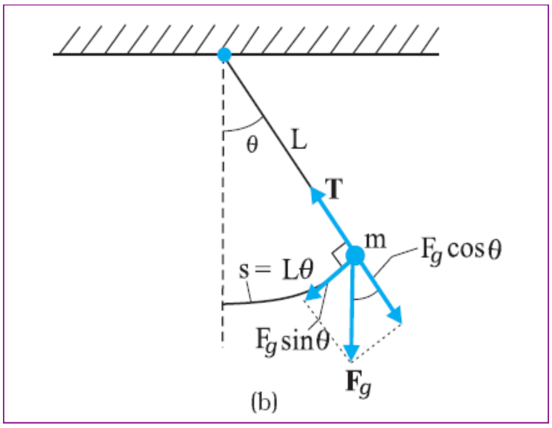
The Simple Pendulum

- A **simple pendulum,** consists of a particle of mass *m (called* the **bob** of the pendulum) suspended from one end of an unstretchable, massless string of length *L fixed at the other end.*



- **The forces acting on the bob are**:
  - Tension in the string
  - Gravitational force.

Expression for time period



- The string makes an angle θ with the vertical.
- We resolve the force F_g ***into a radial component*** F_g **cos θ** and a tangential component Fg sin θ.
- *The radial* component of force F_g **cos θ,** is **cancelled by the tension**.
- The tangential component, F_g **sin θ produces a restoring torque .**
- The restoring torque τ is $\tau = -LF_g \sin\theta$
- Where the negative sign indicates that the torque acts to reduce θ.
- For rotational motion we have $\tau = I\alpha$

- where **<u>I is the pendulum's moment of inertia</u>** about the pivot point and **<u>α is its angular acceleration</u>** about that point.
- Thus , $-LF_g \sin\theta = I\alpha$
- But $F_g = mg$,
- Thus $-Lmg\sin\theta = I\alpha$
- Or

$$\alpha = -\frac{mgL}{I}\sin\theta$$

- If θ is small $\sin\theta \approx \theta$, therefore

$$\alpha = -\frac{mgL}{I}\theta$$

- That is, **<u>the angular acceleration of the pendulum is proportional to the angular displacement θ but opposite in sign</u>**.
- Thus the motion of a **simple pendulum swinging through small angles is approximately SHM.**
- Comparing equations $a(t) = -\omega^2 x(t)$

  and $\alpha = -\dfrac{mgL}{I}\theta$, we get
- The angular frequency

$$\boxed{\omega = \sqrt{\frac{mgL}{I}}}$$

- And Period

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

- We have $I = mL^2$
- Thus

$$T = 2\pi\sqrt{\frac{L}{g}}$$

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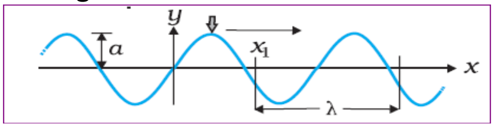
CHAPTER FIFTEEN

WAVES

Travelling or progressive wave

- A wave which **travels from one point of the medium to another** is called a travelling wave.



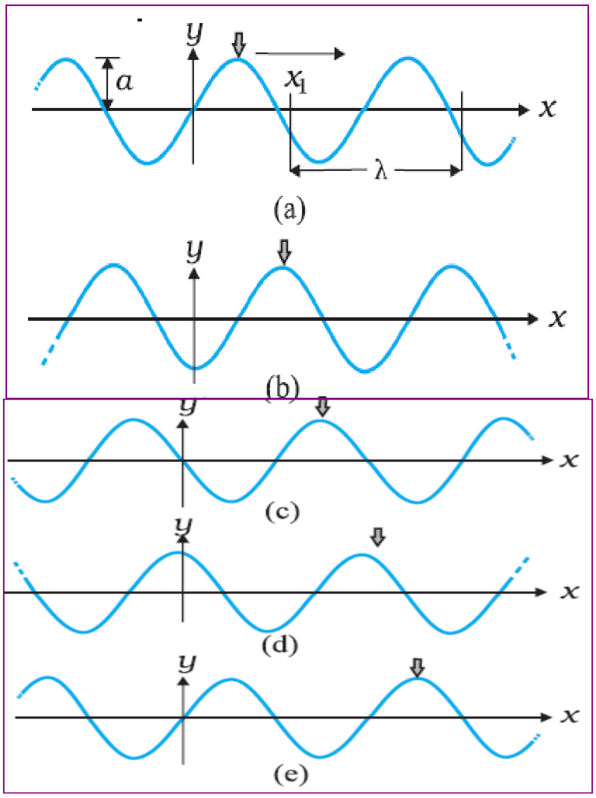
DISPLACEMENT RELATION IN A PROGRESSIVE WAVE

- *At any time t, the* displacement of a wave travelling in positive x-axis is given by

$$\boxed{y(x, t) = a \sin(kx - \omega t + \phi)}$$

- Where , **a- amplitude , k- angular wave number or propagation constant , ω- angular frequency , ϕ- initial phase angle and (kx- ωt+ ϕ) - phase**

Plots for a wave travelling in the positive direction of an x-axis at different values of time t.



- A **wave travelling in the negative direction of *x-axis* can be represented by**

$$\boxed{y(x, t) = a \sin(kx + \omega t + \phi)}$$

Amplitude

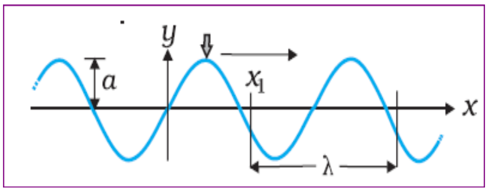
- The **amplitude *a of a wave*** is the **magnitude of the maximum displacement** of the elements from their equilibrium positions as the wave passes through them.
- It is a positive quantity, even if the displacement is negative.

Phase

- It describes the state of motion as the wave sweeps through a string element at a particular position *x*
- The constant ϕ is called the **initial phase angle.**
- **The** value of ϕ is determined by the initial (*t = 0)* displacement and velocity of the element (say, at *x = 0).*

Wavelength (λ)

- It is the minimum distance between two consecutive troughs or crests or two consecutive points in the same phase of wave motion.



Propagation constant *or the* angular wave number (k)

- For *t = 0* and ϕ = 0

$$\boxed{y(x, 0) = a \sin kx}$$

- By definition, the displacement *y is same at* both ends of this wavelength, that is at $x = x_1$ and at $x = x_1 + \lambda$.
- *Thus*

$$\boxed{\begin{aligned} a \sin k x_1 &= a \sin k(x_1 + \lambda) \\ &= a \sin(k x_1 + k\lambda) \end{aligned}}$$

- This condition can be satisfied only when,

$$\boxed{k\lambda = 2\pi n}$$

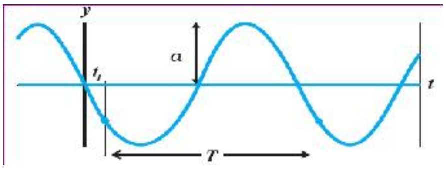
- where *n = 1, 2, 3... Since λ is defined as the least* distance between points with the same phase, *n =1 and therefore*

$$\boxed{k = \frac{2\pi}{\lambda}}$$

- *k is called the* ***propagation constant or the* angular wave number ; its SI unit is radian per** metre or rad m^{-1}

Period

- The **period of oscillation *T of a wave is*** the time any string element takes to move through one complete oscillation.



Angular Frequency

- The **angular frequency of the wave is given by**

$$\omega = 2\pi/T$$

- Its SI unit is rad $s^{-1.}$

Frequency

- It is the number of oscillations per unit time made by a string element as the wave passes through it
- The **frequency *v of a wave is defined as 1/T*** and is related to the angular frequency ω by

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

- It is usually measured in **hertz**

Displacement relation of a longitudinal wave

- In a longitudinal wave, the displacement of an element of the medium is parallel to the direction of propagation of the wave.
- The displacement function for a longitudinal wave is written as,

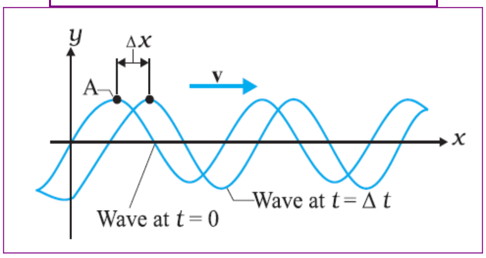
$$s(x, t) = a\sin(kx - \omega t + \phi)$$

- where *s(x, t) is the displacement of an element* of the medium in the direction of propagation of the wave at position *x and time t.*

THE SPEED OF A TRAVELLING WAVE

- The speed of a wave is related to its wavelength and frequency by the relation

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda\nu$$



- The speed **is determined by the properties of the medium.**

Speed of a Transverse Wave on Stretched String

- The speed of transverse waves on a string is determined by two factors,
  (i) the linear mass density or mass per unit length, μ, and
  (ii) (ii) the tension *T.*
- The linear mass density, μ, of a string is the mass *m of the string divided by its length l.* therefore its dimension is $[ML^{-1}]$.
- The tension *T* has the dimension of force $[M\,L\,T^{-2}]$.
- Let the speed $v = C\,\mu^{a}\,T^{b}$, *where c is a dimensionless constant.*
- *Taking dimensions on both sides*
  $\mathbf{[M^0L^1T^{-1}] = [M^1L^{-1}]^a[M\,L\,T^{-2}]^b}$
  $\mathbf{=[M^{a+b}L^{-a+b}T^{-2b}]}$
- Equating the dimensions on both sides we get
  a+b = 0 , therefore a=-b, -a+b = 1, therefore 2b=1 or b= ½ and a= - ½
- Thus $\mathbf{v = C\,\mu^{-½}\,T^{½}}$ **, or**

$$v = C\sqrt{\frac{T}{\mu}}$$

- It can be shown that C=1, therefore the speed of transverse waves on a stretched string is

$$v = \sqrt{\frac{T}{\mu}}$$

- The **speed** of a wave along a stretched ideal string **depends** only on the **tension and the linear mass** density of the string and does not depend on the frequency of the wave.

Speed of a Longitudinal Wave - Speed of Sound

- In a longitudinal wave the constituents of the medium oscillate forward and backward in the direction of propagation of the wave.
- The sound waves travel in the form of compressions and rarefactions of small volume elements of air.
- The speed of sound waves depends on
  i) Bulk modulus , B and
  ii) Density of the medium, ρ
- Using dimensional analysis we may write
  $\mathbf{v = C\,B^a\,\rho^b}$
- Taking dimensions $\mathbf{[M^0L^1T^{-1}] = [ML^{-1}T^{-2}]^a[M\,L^{-3}]^b = [M^{a+b}L^{-a-3b}T^{-2a}]}$
- Equating the dimensions on both sides we get
  a+b = 0 , therefore a=-b, -2a=-1, a=1/2 , therefore b=-1/2
- Therefore

$$v = C\sqrt{\frac{B}{\rho}}$$

- where *C is a dimensionless constant and can* be shown to be unity.
- Thus the speed of longitudinal waves in a medium is given by,

$$v = \sqrt{\frac{B}{\rho}}$$

- The **speed of propagation of a longitudinal wave** in a fluid therefore **depends only on the bulk modulus and the density of the medium.**
- The bulk modulus is given by

$$B = -\frac{\Delta P}{\Delta V/V}$$

- Here Δ*V/V is the fractional change in volume* produced by a change in pressure Δ*P.*

Speed of sound wave in a material of a bar

- The speed of a longitudinal wave in the bar is given by,

$$v = \sqrt{\frac{Y}{\rho}}$$

- where ***Y*** *is the* ***Young's modulus of the material*** of the bar.

Speed of sound in different media

Medium	Speed (m s⁻¹)
Gases	
Air (0 °C)	331
Air (20 °C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0 °C)	1402
Water (20 °C)	1482
Seawater	1522
Solids	
Aluminium	6420
Copper	3560
Steel	5941
Granite	6000
Vulcanised Rubber	54

Newton's Formula

- In the case of an ideal gas, the relation between pressure *P and volume V is given by*

$$PV = Nk_BT$$

- Therefore, **for an isothermal change** it follows that

$$V\Delta P + P\Delta V = 0$$

$$-\frac{\Delta P}{\Delta V/V} = P$$

- Thus **B=P**
- Therefore, the speed of a longitudinal wave in an ideal gas is given by,

$$v = \sqrt{\frac{P}{\rho}}$$

- This relation was first given by Newton and is known as **Newton's formula.**

Laplace correction

- According to Newton's formula for the speed of sound in a medium, we get for the speed of sound in air at STP,

$$v = \left[\frac{1.01\times10^5\ \mathrm{N\ m^{-2}}}{1.29\ \mathrm{kg\ m^{-3}}}\right]^{1/2} = 280\ \mathrm{m\ s^{-1}}$$

- This is about 15% smaller as compared to the experimental value of **331 m s**$^{-1}$
- Laplace pointed out that the **pressure variations in the propagation of sound waves are adiabatic** and not isothermal.
- For adiabatic processes the ideal gas satisfies the relation,

$$PV^{\gamma} = \text{constant} \quad \text{i.e.} \quad \Delta(PV^{\gamma}) = 0$$

$$P\gamma V^{\gamma-1}\Delta V + V^{\gamma}\Delta P = 0$$

- Thus for an ideal gas the adiabatic bulk modulus is given by,

$$B_{ad} = -\frac{\Delta P}{\Delta V/V} = \gamma P$$

- where γ is the ratio of two specific heats, Cp/Cv.
- The speed of sound is, therefore, given by,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

- This modification of Newton's formula is referred to as the **Laplace correction.**
- For **air** γ = 7/5,therefore the speed of sound in air at STP, we get a value ***331.3 m s***$^{-1}$, which agrees with the measured speed.

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