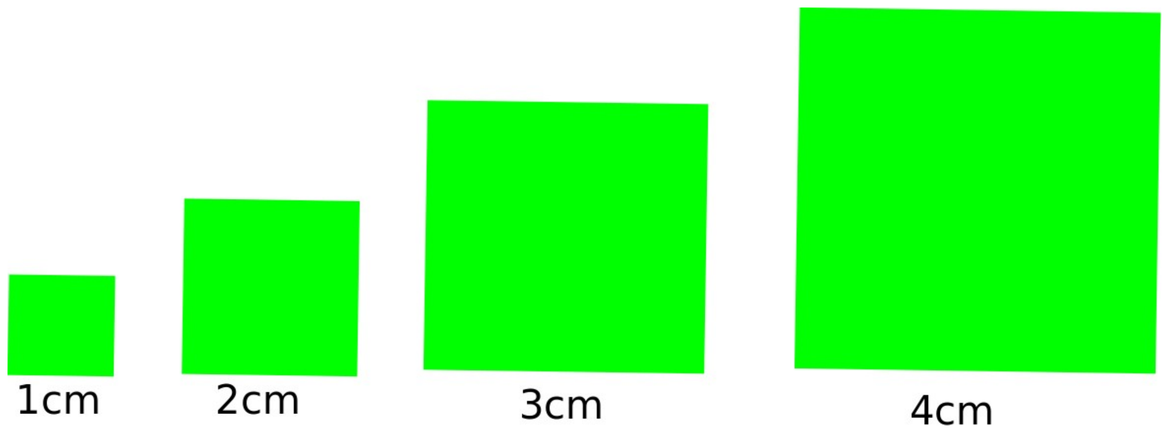
UNIT 1

Arithmetic Sequence

Activity 1



consider the squares given above. Length of sides of the squares are 1cm, 2cm , 3cm etc.
The numbers showing lenth of sides are
1, 2, 3, 4,.............
Then can you write the numbers showing the following as above?
1) Perimeter:
2) Area :
3)Length of diagonal:

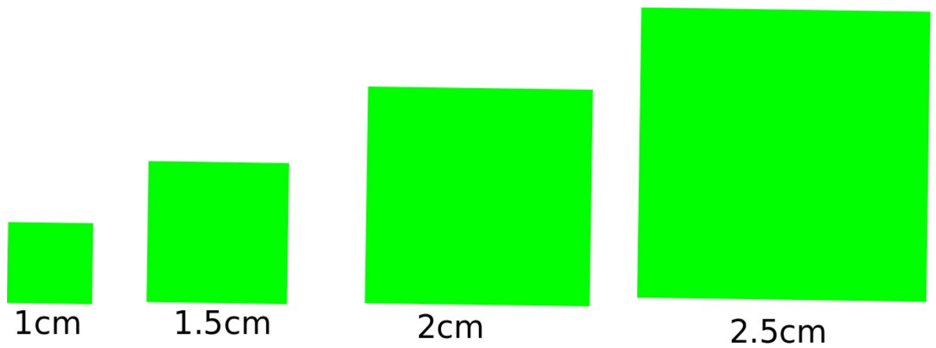
Sides : 1,2,3,4,---------------
Perimeter: (4 x one side) : 4,8,12,16, -----------
Area : (square of one side) : 1, 4 , 9, 16, -----------
Diagonal : (√2 times of one side) :√2, 2√2,3√2,4√2,--------

A set of numbers written like this, as the 1st , 2nd , 3rd and so on, according to a particular rule is called a number sequence.

So the above number patterns are number sequences

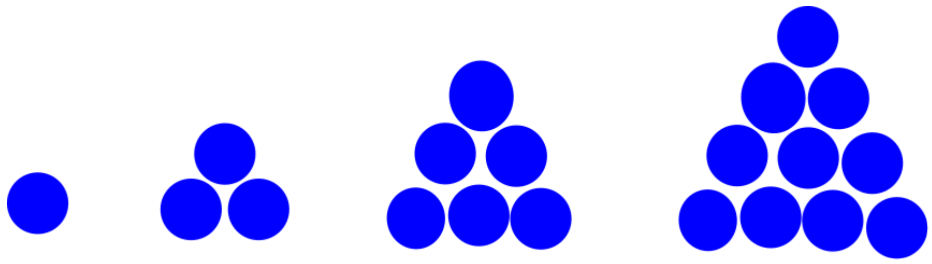
Activity 2
write the next two terms of the above sequences

Activity3
Consider the square patterns below and write the number sequences showing
a) Length of sides
b)Perimeter
3) Area
4)Length of diagonals



*Also write the next two terms of these sequences
Activity 4
Write the following sequences
*sequence of odd natural numbers
*sequence of even natural numbers
*sequence of prime numbers
*sequence of natural numbers which leaves remainder 1 on division by 5
*Sequence of natural numbers ending in 1 or 6
*sequence of natural numbers leaving remainder 2 on division by 5

Activity4



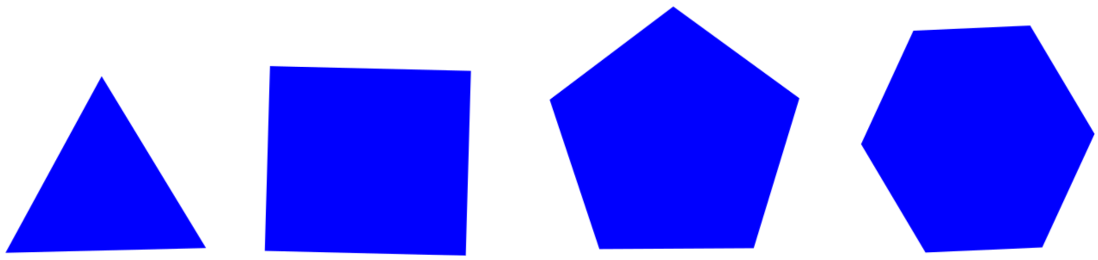
Look at the triangles made with balls

* Write the number of balls used in each triangle as a sequence

*Find out the number of balls needed to make the next three triangles.

Activity5

Make the following number sequences from the sequence of regular polygons



a)Number of sides:

b)Sum of inner angles:

c)Sum of outer angles:

d)One inner angle:

e)One outer angle:

Algebra of sequences

We have seen that the sequence of perimeters of squares of sides 1cm ,2cm, 3cm, ...is

4,8,12,......

Here 1^{st} term is 4 and 2^{nd} term is 8 and 3^{rd} term is 12 ....

Can you say what is its n^{th} term?

Position of terms:	1	2	3	4............
Terms :	4	8	12	16..........

So n^{th} term is 4n

The terms of the sequence are usually represented by $x_1, x_2, x_3, \ldots\ldots, x_n$

So we have $x_n = 4n$

What is the n^{th} term of the sequence 1,4,9,16,.......?

The n^{th} term of a sequence is also called its algebraic form

Now write down the n^{th} term (algebraic form) of the following sequences

sides :1, $1^1/_2$, 2, $2^1/_2$, ...

Perimeter :4, 6, 8, 10, ...

Area :1, $2^1/_4$, 4, $6^1/_4$, ...

Diagonal : $\sqrt{2}, 3\sqrt{2}/2, 2\sqrt{2}, 5\sqrt{2}/2$,...........

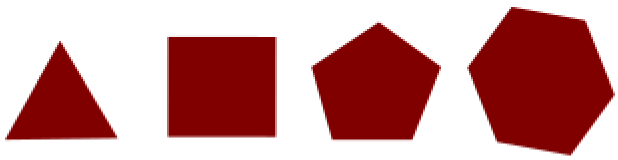
Write the algebraic expression for each of the following sequences

a)Sequence of odd numbers

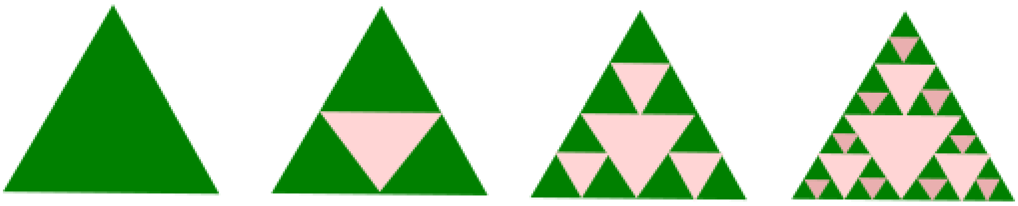
b) Sequence of natural numbers which leave remainder 2 on division by 3

c)Sequence of natural numbers ending in 2 or 7

d) Sequence of powers of 2



Consider the regular polygons, Write the algebraic expression for the sequence of sum of inner angles, sum of outer angles, measure of one inner angle, one outer angle



Look at the picture above,
From an equilateral triangle, small equilateral triangles are removed to get the next triangles
Write the sequence showing

a) **Number of green triangles in each figure.**
b) **Taking area of large triangle as 1, area of each green triangles in each picture**
c) **Total area of green triangles in each picture**
d) **Write the algebraic form of each sequence**

Arithmetic sequence

Let us examine the sequences that we have already constructed

a)1,2,3,4,..........
b)4,8,12,16,.....
c)1, $1^1/_2$, 2, $2^1/_2$, ...
d)4, 6, 8, 10, ...
e) $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$,........
f) 1,4,9, 16,...........
g) 1, 1/4,1/16, ...
h)60,90,108,120,......

What are the properties of 1st four sequences? Is there any thing common?

Starting with a number and adding a fixed number repeatedly
In other words difference between adjacent terms are equal

ie, $x_2-x_1=x_3-x_2=x_4-x_3=$...........
such sequences are called arithmetic sequences

A sequence got by starting with any number and adding a fixed number repeatedly is called an arithmetic sequence

consider the sequence,
5,8,11,14,.............
$x_2 - x_1 = 8-5 = 3$
$x_3 - x_2 = 11 - 8 = 3$
$x_4 - x_3 = 14 - 11 = 3$
Here the differences are same, so it is an arithmetic sequence.
Consider another sequence,
1 , 11, 21, ...
here $x_2 - x_1 = 11 - 1 = 10$
$x_3 - x_2 = 21 - 11 = 10$
Here the differences are common or same, so it is an arithmetic sequence.
Here the difference is called common difference
A sequence in which when one term is subtracted from the next term we get the same number is called an arithmetic sequence. And the difference is the common difference . Common difference is represented by the letter "d".
So, $d = x_2 - x_1 = x_3 - x_2 = \ldots\ldots\ldots\ldots$
Check whether the following sequences are arithmetic sequence
If arithmetic sequence find its common difference.
1)1, 4, 9, ...
Here 4-1=3
9-4 =5 ,Difference is not same
so not an arithmetic sequence
2) 360, 360, 360, ...
here 360 – 360 = 0
360 – 360 = 0
here difference equal , so it is an arithmetic sequence
d = 0
3)Sequence of odd numbers
1, 3, 5, ...
3 – 1 = 2
5 – 3 = 2
difference same, so it is an arithmetic sequence
d = 2
4)sequence of even numbers
5)Sequence of fractions got as half the odd numbers
ie, sequence of odd numbers is 1,3,5, ...

sequence of half of odd numbers is, 1/2,3/2,5/2,.......

3/2 - 1/2 = 2/2 =1

5/2 – 3/2 = 2/2 =1

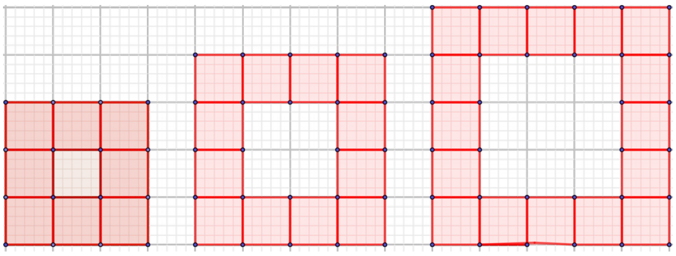
since difference same it is an arithmetic sequence.

d= 1

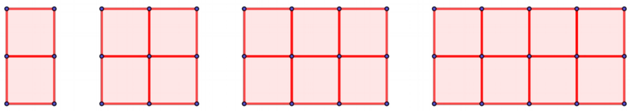
5) sequence of powers of 2

6) sequence of reciprocals of natural numbers

7)If the pattern is continued , do the numbers of coloured squares form an arithmetic sequence



8)See the pictures



a) How many small squares are there in each rectangle?

b)How many large squares?

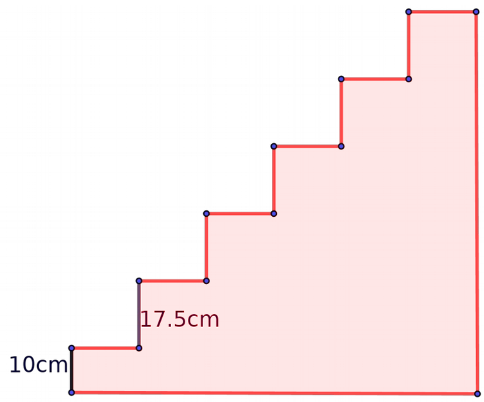
c) How many square in all?

Are all these patterns arithmetic sequence?

9)In the stair case shown here the height of the first step is 10cm and the height of each step after it is 17.5cm

a) Write the height of each step from the ground as a sequence

b) Is it an arithmetic sequence? If so find its common difference



Position and terms of the sequences

consider the arithmetic sequence

1, 6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61, 66, 71, ...

x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, ...

The positions of these terms are marked below
What is the common difference? 6 – 1 = 5
Let us consider 5th term 21,when one 5 is added we get 6th term
When two times 5 is added ,we get 5+2 = 7th term
When 3 times common difference is added we get 8th term
Now complete the table below

1, 6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61, 66, 71, ……………..
x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, ……..

Term	Position of term	Number of times common difference added	New term	New position of term	Number of terms move forward
21	x_5	2	31	x_7	2
36	x_8	4	56	x_{12}	4
11	x_2	11	61	x_{13}	
1				x_{14}	
16					
	x_7			x_{13}	
		5	66		
6					7

Now complete the table below

A S	d	x_m	x_n	$(x_m – x_n)$	m-n	$\frac{(x_m – x_n)}{m - n}$
2,5,8,11,14,17 …	3	X_6 =17	x_1=2	15	5	3
1,7,13,19,25,.. ..		x_4	x_3			
0,4,8,12,16,.... .		x_7	x_5			
10,17,24,31,...		x_8	x_4			

Compare the table and write your conclusions

In an arithmetic sequence , term difference is proportional to position difference and the constant of proportionality is the common difference.

Ie, $$d = \frac{(x_m - x_n)}{m - n}$$

1)Write down the missing terms in the following arithmetic sequences

a) 24, 42, - , - , ...

b) - , 24 , 42, - , ...

c) - , - , 24 , 42, ...

d) 24, - 42 , - , ...

e) - , 24 , - , 42, ...

f) 24 , - , - , 42, ...

2)The 5th term of an arithmetic sequence is 38 and 9th term is 66. what is its common difference? 25th term?

Here $x_5 = 38$,

$x_9 = 66$

so $d = (x_9 - x_5)/(9 - 5) = (66 - 38)/4 = 28/4 = 7$

$x_{25} = x_5 + 20d = 38 + 20 \times 7 = 38 + 140 = 178$

3)Find common difference of the following arithmetic sequences

a) 3rd term =34, 6th term= 67

b) 3rd term= 43, 6th term = 76

c)3rd term = 2, 5th term = 3

d)4th term =2 , 7th term =3

e)2nd term =5 , 5th term = 2

4)Is 101 a term of the arithmetic sequence 13, 24, 35, ...? What about 1001?

We have already discussed that difference of terms of an arithmetic sequence is a multiple of common difference

ie, difference is divisible by common difference

If 101 is a term , then difference between 101 and 13 (or any other term of this sequence) will be divisible by common difference

here d= 24 – 13 = 11

(101 – 13)/11 = 88/11 = 8

since difference is divisible by common difference 101 is a term of this sequence.

If we consider 1001,

$(1001 - 13)/11 = 988/11 = 89^{9}/_{11}$, Here difference is not divisible by common difference, so 1001 is not a term of this sequence.

Now do the following activities

a) Is 123 a term of 11,22,33,.....? What about 132?

b)Is 100 a term of 12 , 23, 34 ,.....? What about 1000?

c)Is 100 a term of 21 , 32 , 43,.....? What about 1000?

d)Is 3 a term of 1/4 , 1/2 , 3/4 ,.....? What about 4?

e)Is 3 a term of 3/4, $1^{1}/_{2}$, $2^{1}/_{4}$,.....? What about 4?

Consider an arithmetic sequence with 10 terms,

How many times common difference is added to get 10th term

What about a sequence with 20 terms? 19 common difference is added to 1st .

What about ,if 30 terms?

29 times common difference is added

..

If n terms, no of times common difference added is n-1

ie,$x_1 + (n-1)d = x_n$

so $(n-1)d = x_n - x_1$

$n-1 = (x_n - x_1)/d$

$n = (x_n - x_1)/d + 1$

so Number of terms in the sequence 10,14,18 ,.........,90

is n= (90-10)/4 + 1= 80/4 + 1 =20+1= 21

Find out number of terms of the following sequences

a) 20, 23, 26,........,110

b)-6.0,6, ...78.

c)95,102,109, ..., 165.

d)80,74,68, ...8.

How many three digit numbers are divisible by 7?

Sequence of three digit numbers divisible by 7 are 105,112,119,.....994.

so n = (994- 105)/7 +1 =889/7 +1= 127 +1=128

Now find the following

*Find the number of three digit numbers leaves remainder 1 on division by 7

*Find the number of natural numbers divisible by 3 between 160 and 400

*Find number of natural numbers leave remainder 1 on division by 3 between 150 and 600

Algebra of arithmetic sequence

Usually terms of an arithmetic sequence are represented by $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, \ldots\ldots$

Let the first term is f and the common difference is d, then

$x_1 = f$

$x_2 = f+d$

$x_3 = f+d+d = f+2d$

$x_4 = f+ 2d+d= f+3d$

$x_5 = f+4d$

$x_6 = ?$

....

$x_{10} = ?$

....

$x_{20} = ?$

....

$x_{100} = ?$

....

$x_m = ?$

$x_p = ?$

$x_n = ?$

So $x_n = f+(n-1)d$

From this we get, $x_n = f+nd - d$

$x_n = dn + f - d$

So n^{th} term or algebraic form of an arithmetic sequence ,

$x_n = dn + (f-d)$

Find the n^{th} term of

a) 2, 7, 12, ...

Here $f= 2$, $d = 7 - 2 = 5$

$x_n = dn+f-d = 5n + 2 - 5 = 5n - 3$

b)5, 11,16,............

c)-6, 0 , 6, ...

d) 100,95, 90, ...

e) 3, 0, -3, ...

f)1/2, 1, $1^1/_2$, ...

g) 2/7, 5/7, 8/7, ...

h) 11/8, 14/8, 17/8, ...

Now complete the following table below

sequence	f	d	x_n	Coefficient of n	Sum of coefficients
2,7,12,.........	2	5	3n – 1	3	3+ -1 =2
5,11,16,............					
-6, 0 ,6, ...					
100,95,90, ...	100	-5	-5n+105		
3, 0, -3, ...					
1/2, 1, $1^1/_2$, ...	1/2	1/2	$^1/_2$n+0	1/2	1/2
2/7,5/7,8/7, ...					
11/8,14/817/8, ...					

What is the general form of x_n ?

It is of the form "an+b"

What is the relation between coefficient of n and d?

Common difference d = coefficient of n

ie, d=a

What is the relation between first term and sum of coefficients?

X_1 = a+b

a)x_n=2n+3

f=x_1 =2 x1+3 =2+3 =5,

d= 2

sequence is 5,7,9, ...

***The algebraic form of some arithmetic sequence are given below**

Find its 1^{st} and common difference. Also form the sequence

a)x_n=5n – 1

b)x_n=-4n+1

c)x_n=6n- 5

d)x_n=$^1/_2$ n+ $^3/_2$

e)1 – n

f)95 – 2n

***What is the algebraic form of the following arithmetic sequence?**

a) 6th term 56 and 10th term 76

d= $(x_{10} – x_6)/(10 – 6)$ =(76 – 56) *4* =*20*/4 = 5

so $x_1 = x_6 – 5d$ = 56 – 5x4= 56 – 20 =36, so f=36.

x_n = dn +f-d =5n+36-5= 5n+31

b) 14th term 106 and 24th term 186

c)2nd term 12 and 32nd term 132

d)5th term14 and 14th term 5

e) 8th term 12 and 12th term 8

*** Now consider arithmetic sequence with 1st term 1/3 and common difference 1/6**

Is 1 a member of this sequence?

1/3 is same as 2/6

So , if we write the sequence we get,

2/6, 3/6, 4/6, 5/6 6/6,..........

yes ,6/6=1 is a member of this sequence

Are all the natural numbers in this sequence?

How can we find out it, it is not possible to write all the terms

Let us find out its nth term

x_n =dn+f-d =$^1/_6$n+1/3 – 1/6 = $^1/_6$n+2/6 – 1/6= $^1/_6$n+ 1/6 =(n+1)/6

As n=5, we get (5+1)/6=6/6 = 1

As n=11, we get (11+1)/6=12/6 = 2 and n=17,we get (17+1)/6=18/6=3 and so on

ie, as n=5,11,17,23,......... we get natural numbers 1,2,3,4,............

So all the natural numbers in this sequence

Now can you prove the arithmetic sequence with 1st term 1/3 and common difference 2/3 contains all odd numbers but no even number

Sums and Terms

consider the arithmetic sequence

2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35 , 38, 41 ,44,...........

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, \ldots\ldots\ldots\ldots$

Now complete the table below

Three consecutive terms	sum	Average=$\frac{sum}{3}$	Middle term
x_1, x_2, x_3	2+5+8=15	15/3=5	$x_2 = 5$
x_3, x_4, x_5			
x_7, x_8, x_9			
x_{11}, x_{12}, x_{13}			

What is the conclusion you get?
Is it true for every arithmetic sequence?
How can we prove it?

Let x-d, x, x+d be three consecutive terms of an arithmetic sequence
then average = $\frac{sum}{3} = \frac{(x-d)+x+(x+d)}{3} = \frac{3x}{3} = x$ = *middle term*

so the average of any 3 consecutive terms of any arithmetic sequence is the middle term

In other words

The sum of any three consecutive terms of an arithmetic sequence is three times its middle term

If a, b, c are 3 consecutive terms of an arithmetic sequence, then 2b=a+c

***Find the following**

a)If x,23, y are in arithmetic sequence, then find x+y
b) 34, x, 46 are in arithmetic sequence, find x
c)If x, y,z are in arithmetic sequence and x+y+z=45, find y and x+z.
d)If 3x+1, - 8 ,4x- 3 are in arithmetic sequence, find x.

When number of terms are odd

consider the arithmetic sequence

6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61 , 66, 71 ,76,..........
x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, ...

Now complete the table below

Terms	sum	Middle term	(Middle term) X (number of terms)
6+ 11+ 16+ 21+ 26	80	16	16x5 = 80
16+ 21+ 26+31 + 36			
6+ 11+ 16+ 21+26+31 +36			
16+21+26+31+36+41+46+51+56		36	

If the number of terms is odd , sum of terms is the product of middle term and number of terms

***In an arithmetic sequence the sum of first 7 terms is 84 find its 4^{th} term**

***In an arithmetic sequence, the sum of 1^{st} 5 terms 45 and sum 0f 1^{st} 15 terms is 285. Find its**
a)3rd term
b) 8^{th} term
c) 1^{st} term and common differences
d) write its algebraic form

***The sum of the 1^{st} five terms of an arithmetic sequence is 150 and the sum of 1^{st} ten terms is 550 , then find its**
a)3rd term

b) 8^{th} term
c)Write the sequence

***Write the 1^{st} three terms of the arithmetic sequence described below**

a)1st term 30 and sum of 1^{st} three terms is 300
b)1st term 30 and sum of 1^{st} four terms is 300
c)1st term 30 and sum of 1^{st} five terms is 300
d)1st term 30 and sum of 1^{st} six terms is 300

SUM OF POSITIONS AND SUM OF TERMS

consider the arithmetic sequence
2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35 , 38, 41 ,44,...........
x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, ...

Now complete the table below

Sum of terms	Sum of positions	Sum of terms	Sum of positions
$x_8+x_5=$	8+5=13	$x_1+x_{10}=31$	1+10=11
$x_{10}+x_3=$		$x_2+x_9=31$	2+9=11
$x_{12}+x_1=$		$x_3+x_8=31$	3+8=11
$x_5+x_9=$		$x_4+x_7=31$	4+7=11
$x_6+x_8=$		$x_5+x_6=31$	5+6=11
$x_{10}+x_4=$	10+4=14	$x_4+x_{13}=$	
$x_{13}+x_1=$		$x_7+x_{10}=$	
$x_{12}+x_2$		$x_8+x_9=$	

Compare the table and write your conclusions

In an arithmetic sequence , if the sums of positions of pairs of terms are equal, then the sum of pairs of term are also equal

***If the sum of 1^{st} and 20^{th} term of an arithmetic sequence is 78, find**
a)sum of 3^{rd} and 18^{th} term
b)sum of 7^{th} and 14^{th} term
c) If 8^{th} term is 30 ,what is its 13^{th} term
d)If 10^{th} term is 38 what is its 11^{th} term and 1^{st} term

Sum of consecutive natural numbers

Can you find out the following sums

*1+2+3+4+5+6 =?

*1+2+3+4+5+6+7+8+9+10 = ?

*1+2+3+.............+20=?

Is there any easy method to find out the sums?

In the first case, how much is 1+6?

1+6=7, 2+5=7, 3+4=7

How many 7? .....

so sum = 7x3 =21

ie, 6/2(1+6), Isn't it?

$$1+2+3+4+5+6$$

7

7

7

If we consider the numbers pairwise , we get 6/2=3 pairs

sum of each pair is 7

So sum = number of pairs x 7

Consider the next example,

$$1+2+3+.........+8+9+10$$

3+8=11

2+9=11

1+10=11

How many pairs? 10/2=5 pairs

Sum of each pair? 1+10=11

So sum =5x11=55

What about 3rd example?

1+2+3+4+...........+17+18+19+20
How many pairs?
Sum of each pair?
Then sum = ?
How much is 1+2+3+.......+48+49+50 ?
Find 1+2+3+.........+99+100
1+2+3+.........+25
1+2+3+.........+35
1+2+3+..........+x
1+2+3+...........+m
1+2+3+..........+n

So 1+2+3+4+............+n = $^{n}/_{2}$(n+1)
ie,1+2+3+..........+n = n(n+1)/2

So the sum of 1st n natural numbers is half of the product of last number n and next number n+1

Find the following sums
a)1+2+3+..........+99
b)1+2+3+ ....+200
c)1+2+3+...........+500
d)2+4+6+...........+100
e)7+14+21+..........+84
f)11+22+33+..........+121
g)6+7+8+.............+50

Sum of n terms of an Arithmetic Sequence

Find the following sums

2+5+8+11=

7+11+15+19+23+27=

6+11+16+21+26+31+36+41=?

If the number of terms are less we can easily find the sums

But if the number of terms large , it will be difficult

Is there any easy way?

Let us consider the 1^{st} one

2+5+8+11

let us take the terms in pairwise

2+11=13, 5+8=13, here there are 4/2 =2 pairs

sum of 1^{st} and last is same as sum of 2^{nd} and 3^{rd}

so sum = 2x13=26

consider the 2^{nd} one

7+11+15+19+23+27

7+27=?   34

11+23=?

15+19=?

so, sum = 3x34= 102

consider the sum

3+7+11+15+19+23+27+31+35+39

$$3+7+11+15+19+23+27+31+35+39$$

42
42
42
42
42

Taking pairwise we get 10/2= 5pairs
Sum of each pair= 3+39 =42
sum of terms =5x42 =210
ie,10/2(3+39)

Now find the following sums

a)5+8+11+14+17+20=? **{hint:6/2(5+20)}**
b)10+16+22+28+34 = ? **{hint:5/2(10+34)}**
c)1+3+5+7+9+.......+25=?
d)3+10+17+..........(15 terms)
e)2+7+12+...........(11terms)
f)6+10+14+...........(x terms)
g)4+11+18+..........(m terms)
h)5+9+13+............(n terms)

so, $x_1 + x_2 + x_3 + x_4 + \ldots\ldots\ldots\ldots + x_n = {}^{n}/_{2}(x_1+x_n)$

Sum of 1st n terms of an arithmetic sequence = $\dfrac{n(x_1+x_n)}{2}$

1)Find the sum of the 1st n terms of the following

a)2,8,14,…………

{ ans: f =2

d= 8 – 2 =6

x_n = dn+f-d =6n+2- 6 =6n – 4

Sum =$n(x_1+x_n)/2$ =n(2+6n-4)=n(6n -2)= $6n^2$- 2n}

b)5,9,13, ……….

c)76,70,64,……..

d) 1/2,5/2,9/2,…..

e) -6, -3, 0, ………

f)-12, -15, -18, ….

Now complete the table

A S	f	d	Sum of n terms	Sum of coefficients	Coefficient of n^2	d/2
2,8,14,……						
5,9,13,…..						
76,70,64,…						
1/2,5/2,9/2,….						
- 6, -3 , 0, ….						
- 12, - 15 ,- 18,….						

Compare the table and write the relation between d/2 and coefficient of n^2 in the sum of n terms.

Also write the relation between sum of coefficients and first term of the sequence

Also write the common form of sum of n terms

The sum of n terms of an arithmetic sequence is $2n^2+3n$

a) what is its first term and common difference?

b)what is its algebraic form(n^{th} term)?

Ans: a)first term = 2+3=5

d= 2x2 = 4

b) algebraic form = dn+f – d = 4n+5 – 4 = 4n+1

1)Algebraic form of sum of n terms of some arithmetic sequence are given below. Find its 1^{st} term common difference and algebraic form

a)$3n^2$- 2n

b) $5n^2 – 6n$

c) $1/2n^2 +3/2n$

d)$6n – 5n^2$

e)$4n – n^2$

2)Find the sum of all natural numbers divisible by 7 in between 100 and 300

3)Find the sum of all natural numbers which leaves remainder 1 on division by 3 below 120.

Ans: sequence of natural numbers which leaves remainder 1 on division by 3 below 120 is,

1,4,7,............118

number of terms , $n=\frac{x_n - x_1}{d} + 1 = \frac{(118-1)}{3} + 1$

$=117/3 + 1 = 39+1= 40$

Sum = $\frac{n}{2}(x_1 + x_n)$ $=\frac{40}{2}(1+118) =20\times119= 2380.$

4)Find the sum of all the natural numbers which leaves remainder 2 on division by 5 between 150 and 600.

5)Find the sum of all the three digit numbers which is a multiple of 11

6)Calculate the difference of sum of 1^{st} 20 terms and sum of next 20 terms of the sequence 3 , 10 , 17, ...

7)calculate the difference of the sums of 1^{st} 20 terms of the sequences 5,11,17,.... and 4,10,16,..........

{ans: 5, 11, 17,..................

<u>4, 10, 16,...........</u>

difference: 1, 1, 1, ...(twenty 1)

Difference of sums of 20 terms = 20x1=20}

8)Find the following sums

a)31+32+33+........+50

b)1.5+2.5+3.5+.........+21.5

c)1/2+3/2+5/2+..........+25/2

{ c) ans: 1/2+3/2+5/2+.....+25/2=(1+3+5+....+25)/2

total number of terms in 1,3,5,...,25 is 13

so sum =13/2(1+25) or 13^2

so required sum = 13^2/2= 169/2.}

9) 1

3 5

7 9 11

13 15 17 19

....

....

a)write the next two lines

b)write the sequence of number of numbers in each line

c)How many numbers in the 20^{th} line?

d)Find total number of numbers in 20 lines

e)What is the position of last term of 20^{th} line in the sequence 1,3,5,7,...........?

f)Find the 1^{st} and last term of the 20^{th} line

g)Find the sum of all the numbers in 20 lines