

TEXT BOOK QUESTIONS - ARITHMETIC SEQUENCES

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1

Write the algebraic expression for each of the sequences below:

- i) Sequence of odd numbers
- ii) Sequence of natural numbers which leave remainder 1 on division by 3.
- iii) The sequence of natural numbers ending in 1.
- iv) The sequence of natural numbers ending in 1 or 6.

Answer.

i) 1, 3, 5, 7, 9, 11, . . .

Term position	1	2	3	4	5	. . .	<i>n</i>
Term	1	3	5	7	9	. . .	
	$= 2 - 1$	$= 4 - 1$	$= 6 - 1$	$= 8 - 1$	$= 10 - 1$. . .	
	$= 2 \times 1 - 1$	$= 2 \times 2 - 1$	$= 2 \times 3 - 1$	$= 2 \times 4 - 1$	$= 2 \times 5 - 1$		$2 \times n - 1$

Algebraic expression of the sequence = $2n - 1$

ii) 1, 4, 7, 10, 13, . . .

Term position	1	2	3	4	5	. . .	<i>n</i>
Term	1	4	7	10	13	. . .	
	$= 3 - 2$	$= 6 - 2$	$= 9 - 2$	$= 12 - 2$	$= 15 - 2$. . .	
	$= 3 \times 1 - 2$	$= 3 \times 2 - 2$	$= 3 \times 3 - 2$	$= 3 \times 4 - 2$	$= 3 \times 5 - 2$		$3 \times n - 2$

Algebraic expression of the sequence = $3n - 2$

iii) 1, 11, 21, 31, 41, 51, . . .

Term position	1	2	3	4	5	...	n
Term	1 $= 10 - 9$ $= 10 \times 1 - 9$	11 $= 20 - 9$ $= 10 \times 2 - 9$	21 $= 30 - 9$ $= 10 \times 3 - 9$	31 $= 40 - 9$ $= 10 \times 4 - 9$	41 $= 50 - 9$ $= 10 \times 5 - 9$...	$10 \times n - 9$

Algebraic expression of the sequence = $10n - 9$

iv) $1, 6, 11, 16, 21, \dots$

Term position	1	2	3	4	5	...	n
Term	1 $= 5 - 4$ $= 5 \times 1 - 4$	6 $= 10 - 4$ $= 5 \times 2 - 4$	11 $= 15 - 4$ $= 5 \times 3 - 4$	16 $= 20 - 4$ $= 5 \times 4 - 4$	21 $= 25 - 4$ $= 5 \times 5 - 4$...	$5 \times n - 4$

Algebraic expression of the sequence = $5n - 4$

2

For the sequence of regular polygons starting with an equilateral triangle, write the algebraic expressions for the sequence of the sums of inner angles, the sums of the outer angles, the measures of an inner angle, and the measures of an outer angle.

Answer.

a) Sequence of the sums of inner angles = $180^\circ, 360^\circ, 540^\circ, 720^\circ, 900^\circ, \dots$

Term position	1	2	3	4	5	...	n
Term	180 $= 180 \times 1$	360 $= 180 \times 2$	540 $= 180 \times 3$	720 $= 180 \times 4$	900 $= 180 \times 5$...	$180 \times n$

Algebraic expression of the sequence = $180n$

b) Sequence of the sums of outer angles = $360^\circ, 360^\circ, 360^\circ, 360^\circ, 360^\circ, \dots$

Term position	1	2	3	4	5	...	n
Term	360	360	360	360	360	360	360

Algebraic expression of the sequence = 360

c) Sequence of the measures of an inner angle = $60^\circ, 90^\circ, 108^\circ, 120^\circ, \dots$

Term position	1	2	3	4	5	...	n
Term	60	90	108	120			
	$= \frac{180}{3}$	$= \frac{360}{4}$	$= \frac{540}{5}$	$= \frac{720}{6}$	$\frac{900}{7}$...	$\frac{180 \times n}{n+2}$

Algebraic expression of the sequence = $\frac{180 \times n}{n+2}$

d) Sequence of the measures of an outer angles = $120^\circ, 90^\circ, 72^\circ, 60^\circ, \dots$

Term position	1	2	3	4	5	...	n
Term	120	90	72	60			
	$= \frac{360}{3}$	$= \frac{360}{4}$	$= \frac{360}{5}$	$= \frac{360}{6}$	$\frac{360}{7}$...	$\frac{360}{n+2}$

Algebraic expression of the sequence = $\frac{360}{n+2}$

3

(3) Look at these pictures:



The first picture is got by removing the small triangle formed by joining the midpoints of an equilateral triangle. The second picture is got by removing such a middle triangle from each of the red triangles of the first picture. The third picture shows the same thing done on the second.

- i) How many red triangles are there in each picture?
- ii) Taking the area of the original uncut triangle as 1, compute the area of a small triangle in each picture.
- iii) What is the total area of all the red triangles in each picture?

iv) Write the algebraic expressions for these three sequences obtained by continuing this process.

Answer.

i) Number of red triangles in the first figure = 3

Number of red triangles in the second figure = 9

Number of red triangles in the third figure = 27

ii) The area of a small triangle in the first figure = $\frac{1}{4}$

The area of a small triangle in the second figure = $\frac{1}{16}$

The area of a small triangle in the third figure = $\frac{1}{64}$

iii) The total area of all the red triangles in the first figure = $\frac{3}{4}$

The total area of all the red triangles in the second figure = $\frac{9}{16}$

The total area of all the red triangles in the third figure = $\frac{27}{64}$

iv) a) Sequence of the number of red triangles in the each figure = 3, 9, 27, . . .

Term position	1	2	3	. . .	n
Term	3 = 3^1	9 = 3^2	27 = 3^3	. . .	3^n

Algebraic expression of the sequence = 3^n

b) Sequence of the area of a small triangle in the each figure

$$= \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \dots$$

Term position	1	2	3	...	<i>n</i>
Term	$\frac{1}{4}$ $= \left(\frac{1}{4}\right)^1$	$\frac{1}{16}$ $= \left(\frac{1}{4}\right)^2$	$\frac{1}{64}$ $= \left(\frac{1}{4}\right)^3$...	$\left(\frac{1}{4}\right)^n$

Algebraic expression of the sequence = $\left(\frac{1}{4}\right)^n$

c) Sequence of the total area of all the red triangles in the each figure

$$= \frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \dots$$

Term position	1	2	3	...	<i>n</i>
Term	$\frac{3}{4}$ $= \left(\frac{3}{4}\right)^1$	$\frac{9}{16}$ $= \left(\frac{3}{4}\right)^2$	$\frac{27}{64}$ $= \left(\frac{3}{4}\right)^3$...	$\left(\frac{3}{4}\right)^n$

Algebraic expression of the sequence = $\left(\frac{3}{4}\right)^n$