Online Class – X – 04 24 / 06 / 2021

1. Arithmetic Sequence – Class 2

To view class

Algebra of sequences

- Consider sequence of counting numbers

1, 2, 3, 4, 5...............

The numbers forming a sequence are called its ***terms***

Here

1st term = 1, 2nd term= 2 , 3rd term= 3, 4th term = 4 ,.............

10th term = 10, ... nth term= n

The **terms** in a sequence are written in algebra as

$x_1, x_2, x_3, x_4,$ ....

$\therefore x_1 = 1$

$x_2 = 2$

$x_3 = 3$

$x_4 = 4$, ...$x_{10} = 10$ ...., $x_n = n$

Algebraic form

nth term of a sequence is the general term and hence it is called the ***algebraic form*** of the sequence

Algebraic form of sequence of counting numbers is **n**

- **Consider sequence of even numbers**

  2, 4, 6, 8, 10, 12, ...

  Here

		This can be written as
1st term	$(x_1) = 2$	$2 = 2 \times 1$
2nd term	$(x_2) = 4$	$4 = 2 \times 2$
3rd term	$(x_3) = 6$	$6 = 2 \times 3$
4th term	$(x_4) = 8$	$8 = 2 \times 4$
5th term	$(x_5) = 10$	$10 = 2 \times 5$
⋮		
10th term	$(x_{10}) = 20$	$20 = 2 \times 10$
⋮		
20th term (x_{20})		$x_{20} = 2 \times 20 = 40$
⋮		
50th term (x_{50})		$x_{50} = 2 \times 50 = 100$
⋮		
nth term (x_n)		$x_n = 2 \times n = 2n$

Algebraic form of sequence of even numbers is 2n

- **Consider sequence of perfect squares**

  1, 4, 9, 16, 25, 36, ...

Here

$x_1 = 1 = 1^2$

$x_2 = 4 = 2^2$

$x_3 = 9 = 3^2$

$x_4 = 16 = 4^2$

⋮

$x_{20} = 20^2 = 400$

⋮

$x_{50} = 50^2 = 2500$

⋮

$x_n = n^2$

Algebraic form of sequence of perfect squares is n^2

- **Consider sequence of reciprocals of counting numbers**

  Counting numbers : 1, 2, 3, 4, 5 ....

Reciprocals : $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$,...............

$x_1 = \frac{1}{1} = 1$

$x_2 = \frac{1}{2}$

$x_3 = \frac{1}{3}$

$x_4 = \frac{1}{4}$

⋮

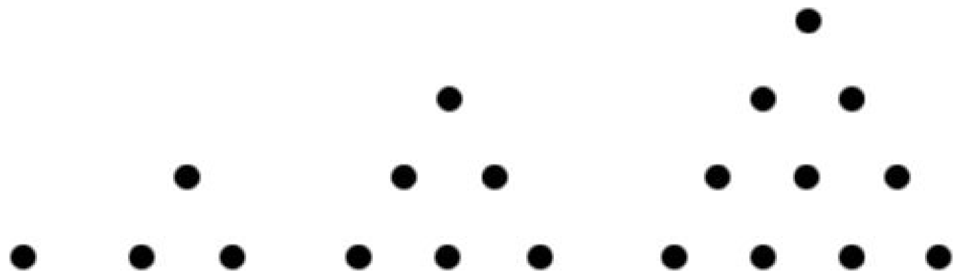
$x_{12} = \frac{1}{12}$

⋮

$x_n = \frac{1}{n}$

Algebraic form of sequence of reciprocal of counting numbers is $\frac{1}{n}$

- **Consider sequence of triangles made with dots**



Number of dots

1	3	6	10
$x_1 = 1$	$x_2 = 1+2$	$x_3 = 1+2+3$	$x_4 = 1+2+3+4$

x_5 = Number of dots in 5^{th} triangle = $1 + 2 + 3 + 4 + 5$

⋮

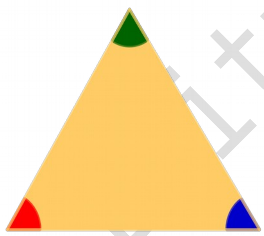
x_{10} = Number of dots in 10^{th} triangle = $1 + 2 + 3 + 4 + \ldots\ldots + 10$

⋮

x_n = Number of dots in n^{th} triangle = $1 + 2 + 3 + 4 + \ldots\ldots + n$

- **Consider sequence of sum of interior angles of regula polygons**

1. **Triangle**

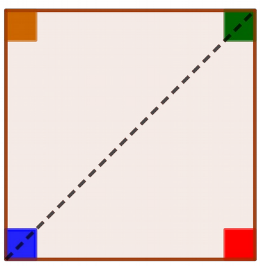


1 Triangle

Sum of interior angles = $180^0 \times 1$

$x_1 = 180^0 \times 1$

2. **Square**

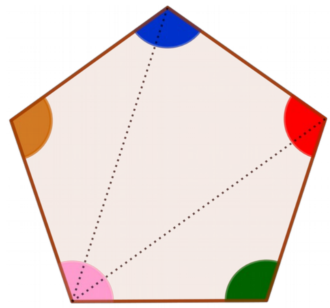


2 Triangles

Sum of interior angles = $180^0 \times 2$

$x_2 = 180^0 \times 2$

3. Pentagon

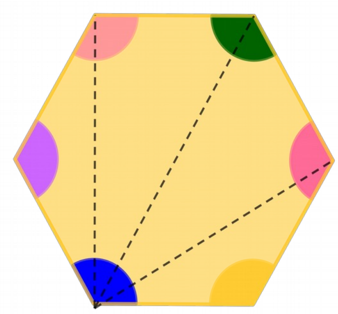


3 Triangles

Sum of interior angles = $180^0 \times 3$

$x_3 = 180^0 \times 3$

4. Hexagon



4 Triangles

Sum of interior angles = $180^0 \times 4$

$x_4 = 180^0 \times 4$

Algebraic form of sum of interior angles of polygon is

$180^0 \times n$

Discussion: Is it possible to write algebraic form of every sequence ?

Consider sequence of prime numbers

2, 3, 5, 7, 11, 13, 17, 19, 23, ...

Here we cannot write a general term.

There is no algebraic form exists for the sequence of prime numbers

So some sequences have no algebraic form.

Prime numbers are the numbers that have only two factors, 1 and the number itself.

Assignment

T.B Page 18

(1) Write the algebraic expression for each of the sequences below:

a) Sequence of odd numbers

b) Sequence of natural numbers which leave remainder 1 on division by 3.

c) The sequence of natural numbers ending in 1.

d) The sequence of natural numbers ending in 1 or 6.

(2) For the sequence of regular polygons starting with an equilateral triangle, write the algebraic expressions for

a) the sequence of the sums of inner angles

b) the sums of the outer angles

c) the measures of an inner angle

d) the measures of an outer angle.

(3) Look at these pictures:

The first picture is got by removing the small triangle formed by joining the midpoints of an equilateral triangle.
The second picture is got by removing such a middle triangle from each of the red triangles of the first picture.
The third picture shows the same thing done on the second.

i) How many red triangles are there in each picture?

ii) Taking the area of the original uncut triangle as 1, compute the area of a small triangle in each picture.

iii) What is the total area of all the red triangles in each picture?

iv) Write the algebraic expressions for these three sequences obtained by continuing this process.