

Session 7

In this session we revise the basic concepts on arithmetic sequence that we studied earlier.

Examples

1) Revision of previous topics

- a) Write the arithmetic sequence with first term 7 and common difference 6. What is its algebraic form?

Answer

a) $7, 13, 19 \dots$

$$x_n = dn + (f - d) = 6n + (7 - 6) = 6n + 1$$

- b) Write an arithmetic sequence with first term $\frac{1}{2}$ and common difference $\frac{3}{2}$

Answer

b) $x_1 = \frac{1}{2}$

$$x_2 = \frac{1}{2} + \frac{3}{2} = \frac{1+3}{2} = \frac{4}{2} = 2$$

$$x_3 = 2 + \frac{3}{2} = \frac{2}{1} + \frac{3}{2} = \frac{4}{2} + \frac{3}{2} = \frac{7}{2}$$

$$x_4 = \frac{7}{2} + \frac{3}{2} = \frac{10}{2} = 5$$

Now we can write the sequence as $\frac{1}{2}, 2, \frac{7}{2}, 5 \dots$

- c) The difference between 10 th term and 20 th term of an arithmetic sequence is 60. What is the difference between 5 th term and 8 th term ?

Answer

c) $x_{20} - x_{10} = 60, 10d = 60, d = 6$

$$x_8 - x_5 = 3d = 3 \times 6 = 18$$

- d) The difference between 17 th term and 25 th term of an arithmetic sequence is 80. 17 th term is 174. What is the common difference and first term? Write the algebraic form of the sequence? What is the 100 th term of the sequence?

Answer

$$\begin{aligned} \text{d) } x_{25} - x_{17} &= 8d = 80, d = 10 \\ f &= x_{17} - 16d = 174 - 16 \times 10 = 174 - 160 = 14. \\ x_n &= dn + (f - d) = 10n + (14 - 10) = 10n + 4 \\ x_{100} &= 10 \times 100 + 4 = 1004 \end{aligned}$$

- e) Algebraic form of an arithmetic sequence is $3n + 5$. What is the common difference of the sequence? Write the sequence numerically. What is the remainder obtained on dividing the terms of the sequence by its common difference?

Answer

$$\begin{aligned} \text{e) } d &= 3 \\ \text{Sequence } &3 \times 1 + 5, 3 \times 2 + 5, 3 \times 3 + 5 \dots \\ &8, 11, 14 \dots \\ \text{remainder is } &2 \end{aligned}$$

2) Applications (uses) of algebraic form

- a) How many terms are there in the sequence $1, 3, 5, 7 \dots 99$

Answer

$$\begin{aligned} \text{a) } x_n &= dn + (f - d) = 2n + (1 - 2) = 2n - 1 \\ 2n - 1 &= 99, 2n = 100, n = 50. \\ \text{There are } &50 \text{ terms below } 100 \text{ in this sequence.} \end{aligned}$$

- b) What is the 30 th term of the arithmetic sequence $10, 17, 24, 31 \dots$

Answer

$$\begin{aligned} \text{b) } x_n &= dn + (f - d) = 7n + (10 - 7) = 7n + 3 \\ x_{30} &= 7 \times 30 + 3 = 213 \end{aligned}$$

- c) How many terms are there in the arithmetic sequence $3, 7, 11 \dots$ just below 100. What is the term just below 100?

Answer

$$\begin{aligned} \text{c) } x_n &= dn + (f - d) = 4n + (3 - 4) = 4n - 1 \\ 4n - 1 &< 100, 4n < 100 + 1, 4n < 101, n < \frac{101}{4} = 25.2 \\ \text{That is } &n = 25. \text{ There are } 25 \text{ terms below } 100 \end{aligned}$$

- d) How many numbers are there below 100 which gives the remainder 2 on dividing by 3

Answer

d) 2, 5, 8, 11...

$$3n - 1 < 100, 3n < 101, n < \frac{101}{3}$$

$n < 33.6$. That is $n = 33$. There are 33 terms below 100

e) Algebraic form of an arithmetic sequence is $\frac{3}{7}n + 1$

What is the first integer term of this sequence . Write the sequence of integer terms. How many integer terms are there below 100 in this sequence ?

Answer

e) $n = 7$ ആയാൽ $x_7 = \frac{3}{7} \times 7 + 1 = 3 + 1 = 4$

$n = 7, 14, 21, 28 \dots$ എന്നീ സംഖ്യകളായാൽ $x_7 = 4, x_{14} = 7, x_{21} = 10 \dots$

First integer term is 4.

Sequence 4, 7, 10...

The algebraic form of the sequence of integer terms is $3n + 1$.

$3n + 1 < 100, 3n < 99, n < 33$ That is 32 terms below 100