Session 8

Let us think about a property of arithmetic sequence)

■ Consider some arithmetic sequences.

$2, 5, 8, 11$

$3, 7, 11, 15, 19, 23$

$10, 15, 20, 25, 30, 35, 40, 45$

All these sequences have even number of terms

We can make pairs by taking terms equidistant from both ends.

$(2, 11), (5, 8)$.Pair sum is 13 in both pairs.

Can you see this property in other sequences also

If an arithmetic sequence has even number of terms , the sum of the terms equidistant from both ends are equal.

■If the arithmetic sequence contains odd number of terms there is a term which comes in the middle without pair. This middle term is half of the pair sum

Look at the sequences given below $2, 5, 8, 11, 14$

$3, 7, 11, 15, 19, 23, 27$

$10, 15, 20, 25, 30, 35, 40, 45, 50$

In the first squence, pair sum is $2 + 14 = 16$, middle term is half of 16 . It is 8.Note that the number of terms $n = 5$. So middle term is $\frac{5+1}{2}$ th term. That is 3rd term

In the second squence, pair sum is $3 + 27 = 30$, middle term is half of 30 . It is 15.Note that the number of terms $n = 7$. So middle term is $\frac{7+1}{2}$ th term. That is 4rd term

In the third squence, pair sum is $10 + 50 = 60$, middle term is half of 60 . It is 30.Note that the number of terms $n = 9$. So middle term is $\frac{9+1}{2}$ th term. That is 5rd term

Examples

1) 5 th term of an arithmetic sequence is 38

    a) What is the sum of first and nineth terms?

    b) What is the sum of second and eigthth terms ?

    c) If the first term is 10 then what is the common difference?

    d) What is the nineth term of the sequence?

a) $x_1 + x_9 = 2 \times x_5 = 2 \times 38 = 76$

If the arithmetic sequence contains odd number of terms there is a term which comes in the middle without pair. This middle term is half of the pair sum

b) $x_2 + x_8 = 2 \times x_5 = 2 \times 38 = 76$

If the arithmetic sequence contains odd number of terms there is a term which comes in the middle without pair. This middle term is half of the pair sum

c) $x_5 - x_1 = 4d, 38 - 10 = 4d$
$4d = 28, d = 7$

d) $x_9 + x_1 = 76, x_9 + 10 = 76, x_9 = 66$

2) The sum of the 1st and 15 th terms of an arithmetic sequence is 56.

a) What is the 8 th term of the sequence?

b) What is the sum of second and 14 th terms of the sequence?

c) What is the sum of 7 th and 9 th terms of the sequence?

d) If the first term is 7 then what is the common difference?

e) Write the algebraic form of the sequence

a) $x_8 = \frac{56}{2} = 28$

b) $x_2 + x_{14} = 56$

c) $x_7 + x_9 = 56$

d) $x_8 - x_1 = 7d, 28 - 7 = 7d$
$7d = 21, d = 3$

e) $x_n = dn + (f - d) = 3n + (7 - 3) = 3n + 4$

3) Algebraic form of an arithmetic sequence is $7n + 1$

a) What is the 10 th term of the sequence?

b) What is the sum of 1st and 19 th terms of this sequence?

c) What is the sum of 9 th and 11th terms of this sequence?

4) Choose the correct answer.

1) What is the algebraic form of the sequence $2, 4, 6, 8 \cdots$ $nn + 22n2n - 1$

2) If the difference between 5 th term and 10 th term of an arithmetic sequence is 20 then what is the difference between 10 th term and 20 th term 10203040

3) What is the first term of the arithmetic sequence whose algebraic form is $4n + 5$ 4591

4) What is the tenth term of the sequence whose algebraic form is $-10n + 100$ -1501

Answer

a) $2n$
   $x_n = dn + (f - d) = 2n + (2 - 2) = 2n + 0 = 2n$

b) 40
   $5d = 20$. So $10d = 40$

c) 9
   $f = 4 \times 1 + 5 = 9$

d) 0
   $x_{10} = -10 \times 10 + 100 = -100 + 100 = 0$

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