

## Session 9

Consider an arithmetic sequence having 10 terms.

2, 5, 8, 11, 14, 17, 20, 23, 26, 29

We can make 5 pairs taking the terms equidistant from both ends.

The pairs are (2, 29), (5, 26), (8, 23), (11, 20), (14, 17)

Pair sum is 31.

There are 5 pairs with this sum. So we can find the sum of the terms of the arithmetic sequence as  $31 \times 5 = 155$

There is a short cut method to find the sum of the terms.

In the first pair we can see first term 2 and tenth term 29. Its sum is 31, which is the pair sum.

$$\text{Sum} = 31 \times 5 = (2 + 29) \times \frac{10}{2}$$

If  $x_1$  is the first term and  $x_n$  is the  $n$ th term of an arithmetic sequence then

$$\text{Sum of the terms} = (x_1 + x_n) \times \frac{n}{2}$$

---

Let us consider an arithmetic sequence having odd number of terms

2, 5, 8, 11, 14, 17, 20

Pairing can be done as follows

(2, 20), (5, 17), (8, 14). The middle term 11 left without a pair. The pair sum is 22.

The sum of the terms  $22 \times 3 + 11 = 77$

Using the short cut method,  $\text{Sum} = (2 + 20) \times \frac{7}{2} = 22 \times \frac{7}{2} = 77$

---

Again consider the above arithmetic sequence having odd number of terms.

2, 5, 8, 11, 14, 17, 20

We calculate the sum of the terms as 77.

On dividing the sum of the terms by the number of terms we get the middle term.

That is  $77 \div 7 = 11$ .

We can conclude today's new concepts as follows

If the first term is  $x_1$  and  $n$ th term is  $x_n$  then the sum of the terms is  $(x_1 + x_n) \times \frac{n}{2}$

This is the general formula for calculating the sum of the terms of an arithmetic sequence.

If the number of terms is odd, sum of the terms divided by the number of terms gives the middle term.

## Examples

- 1) First term of an arithmetic sequence is 12 and common difference 3.

- a) Write the algebraic form of this sequence.
- b) What is the 20 th term of the sequence ?
- c) Calculate the sum of first 20 terms of the sequence.

2

**Answer**

a)  $x_n = dn + (f - d) = 3n + (12 - 3) = 3n + 9$

b)  $x_{20} = 3 \times 20 + 9 = 69$

c) Sum of 20 terms  $= (x_1 + x_{20}) \times \frac{20}{2} = (12 + 69) \times 10 = 81 \times 10 = 810$

2) The sum of the first 9 terms of an arithmetic sequence is 72

- a) What is the fifth term ?
- b) If the common difference is 3 then what is its first term?
- c) Write the algebraic form of this sequence.
- d) Find 30 th term of the sequence .
- e) Calculate the sum of first 30 terms of this sequence.

**Answer**

a) While considering 9 terms, fifth term will be the middle term. So,  $x_5 = \frac{72}{9} = 8$

b)  $d = 3$ . Therefore  $x_1 = x_5 - 4d = 8 - 4 \times 3 = 8 - 12 = -4$

c)  $x_n = dn + (f - d) = 3n + (-4 - 3) = 3n + (-7) = 3n - 7$

d)  $x_{30} = 3 \times 30 - 7 = 90 - 7 = 83$

e) Sum of 30 terms  $= (x_1 + x_{30}) \times \frac{30}{2} = (-4 + 83) \times 15 = 79 \times 15 = 1185$

3) 10 th term of an arithmetic sequence is 27 and its 30 th term is 67

- a) What is the common difference of this sequence?
- b) Find its first term
- c) Write the algebraic form of this sequence
- d) Calculate the sum of first 20 terms of this sequence .

### Answer

a)  $x_{30} - x_{10} = 20d$

$$20d = 67 - 27 = 40, d = 2$$

b)  $x_1 = x_{10} - 9d = 27 - 9 \times 2 = 27 - 18 = 9$

c)  $x_n = dn + (f - d) = 2n + (9 - 2) = 2n + 7$

d) Sum of 20 terms =  $(x_1 + x_{20}) \times \frac{20}{2}$

For this we find  $x_{20}$  first.

$$x_{20} = 2 \times 20 + 7 = 47$$

$$\text{Sum} = (9 + 47) \times \frac{20}{2} = 56 \times 10 = 560$$

- 4) Write an arithmetic sequence having 5 terms. Third term is 20 and common difference 4. What is the sum of the terms of this sequence.

### Answer

★ Third term  $x_3$  is 20. Keep 20 in the middle and arrange two terms in the right and two terms in the left

★ 12, 16, 20, 24, 28

★ sum of the terms  $5 \times x_3 = 5 \times 20 = 100$

or

$$\text{Sum} = (x_1 + x_5) \times \frac{5}{2} = (12 + 28) \times \frac{5}{2} = 40 \times \frac{5}{2} = 20 \times 5 = 100$$