

Session 10

In this session we discuss the summation of natural numbers.

We do the summation by pairing process and obtain a formula to calculate the sum

Look at the pattern given below

$$1 + 2 + 3 + 4 = (1 + 4) + (2 + 3) = 5 + 5 = 5 \times 2 = (1 + 4) \times \frac{4}{2} = \frac{4(4+1)}{2}$$

$$1 + 2 + 3 + 4 + \dots + 10 = 11 \times 5 = (10 + 1) \times \frac{10}{2} = \frac{10(10+1)}{2}$$

$$1 + 2 + 3 + \dots + 20 = \frac{20(20+1)}{2}$$

Sum of natural numbers from 1 to 25 is $\frac{25(25+1)}{2}$ Now we can write a formula to find the sum of first n natural numbers

$$\text{Sum} = \frac{n(n+1)}{2}$$

- 1) Find the sum using the formula
 - a) Find the sum of first 10 natural numbers
 - b) Calculate $1 + 2 + 3 + \dots + 20$
 - c) Calculate the sum of counting numbers from 1 to 25
 - d) Find the sum $3 + 6 + 9 + 12 + \dots + 30$
 - e) Calculate the sum of all natural numbers from 1 to 100

Answer

$$\text{a) Sum} = \frac{n(n+1)}{2} = \frac{10(10+1)}{2} = 5 \times 11 = 55$$

$$\text{b) } 1 + 2 + 3 + \dots + 20 = \frac{20(20+1)}{2} = 10 \times 21 = 210$$

$$\text{c) Sum} = \frac{n(n+1)}{2} = \frac{25(25+1)}{2} = 25 \times 13 = 325$$

$$\text{d) } 3 + 6 + 9 + \dots + 30 = 3(1 + 2 + 3 + \dots + 10) = 3 \times \frac{10(10+1)}{2} = 3 \times 55 = 165$$

$$\text{e) Sum} = \frac{100(100+1)}{2} = 5050$$

- 2) A boy put 1 rupee in the first day, 2 rupees in the second day, 3 rupees in the third day in a bag. He continues this process.
 - a) Write the amount in the bag in each day as a sequence
 - b) How much money will be in the bag after 10 days?
 - c) Calculate the total amount after 30 days

Answer

a) $1, 3, 6, 10 \dots$

b) Amount after 10 days $= \frac{10(10+1)}{2} = 5 \times 11 = 55$

c) Total amount after 30 days $= \frac{30(30+1)}{2} = 30 \times 31 = 930$

3) Consider the sequence $1, 3, 6, 10, 15 \dots$. All of its terms are the sum of counting numbers from 1 in the order.

a) $1 = 1, 3 = 1 + 2, 6 = 1 + 2 + 3$. Write three more terms as the sum

b) What is the tenth term of the sequence?

c) Is 60 a term of this sequence?

d) Write the algebraic form of this sequence

e) What is the 20th term of this sequence?

Answer

a) $x_4 = 1 + 2 + 3 + 4$

$$x_5 = 1 + 2 + 3 + 4 + 5$$

$$x_6 = 1 + 2 + 3 + 4 + 5 + 6$$

b) $x_{10} = 1 + 2 + 3 + \dots + 10 = \frac{10(10+1)}{2} = 5 \times 11 = 55$

c) Tenth term is 55. Its 11th term will be $55 + 11 = 66$. So we can say 60 is not a term of the sequence.

d) $x_n = \frac{n(n+1)}{2}$

e) $x_{20} = \frac{20(20+1)}{2} = 210$

4) Look at the pattern given below

$$\begin{array}{cccc} & & 1 & \\ & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 & 9 \\ \hline \end{array}$$

a) Write the number of numbers in each line as a sequence

b) Write the algebraic form of this sequence

c) How many numbers are there in 20th line of the pattern

d) Write the numbers at the right end of each line as another sequence

e) Which number comes at the right end of 30th line?

f) Which number comes in the left end of 30 th line?

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g) Calculate the sum of all numbers upto the end of 30 the line.

Answer

a) $1, 3, 5, 7 \dots$

b) $x_n = dn + (f - d) = 2n + (1 - 2) = 2n - 1$

c) $x_{20} = 2 \times 20 - 1 = 39$

d) $1, 4, 9, 16, 25 \dots$

e) $30^2 = 900$

f) $29^2 + 1 = 842$

g) $\text{Sum} = \frac{900(900+1)}{2} = 405450$

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