

ONLINE MATHS CLASS - X - 05 (28 / 06 /2021)

1. ARITHMETIC SEQUENCE - CLASS 3

What did we study in the last class ?

- ★ Each term of a sequence is related to its position .
- ★ The n^{th} term of a sequence is its general form .
- ★ The n^{th} term of a sequence is also called its *algebraic form* .

Activity 1

Consider the following number sequences .

| | Number sequence |
|--|----------------------|
| Natural numbers | 1, 2, 3, 4, . . . |
| Even numbers | 2, 4, 6, 8, . . . |
| Multiples of 5 | 5, 10, 15, 20, . . . |
| Natural numbers which leave remainder 2 when division by 3 | 2, 5, 8, 11, . . . |
| Perimeter of the squares with length of the sides go 1, 2, 3, 4, . . . | 4, 8, 12, 16, . . . |

What are the special features of the above number sequences ?

| | |
|----------------------|---------------------------------------|
| 1, 2, 3, 4, . . . | 1, (1 + 1), (2 + 1), (3 + 1), . . . |
| 2, 4, 6, 8, . . . | 2, (2 + 2), (4 + 2), (6 + 2), . . . |
| 5, 10, 15, 20, . . . | 5, (5 + 5), (10 + 5), (15 + 5), . . . |
| 2, 5, 8, 11, . . . | 2, (2 + 3), (5 + 3), (8 + 3), . . . |
| 4, 8, 12, 16, . . . | 4, (4 + 4), (8 + 4), (12 + 4), . . . |

Finding

Here each sequence got by starting with a number and adding a fixed number repeatedly

Arithmetic sequences

A sequence got by starting with any number and adding a fixed number repeatedly is called an *arithmetic sequence* .

Activity 2

1. Consider the sequence of sums of the outer angles of polygons .

| Polygon | Triangle | Quadrilateral | Pentagon | Hexagon |
|---------------------|---------------|---------------|---------------|---------------|
| Sum of outer angles | 360° | 360° | 360° | 360° |

and continue like this

Sequence = 360° , 360° , 360° , 360° ,

Here the sequence start with 360 and adding 0 repeatedly . So this sequence is an arithmetic sequence .

2. Consider the sequence 1 , $1\frac{1}{2}$, 2 , $2\frac{1}{2}$, 3 , $3\frac{1}{2}$,

Here the sequence start with 1 and adding $\frac{1}{2}$ repeatedly . So this sequence is an arithmetic sequence .

3. Consider the sequence of squares with length of the sides go 1 , 2 , 3 , 4 ,

Length of the diagonal of a square = $\sqrt{2} \times \text{side}$

Sequence of the lengths of the diagonals = $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$,

Here the sequence start with $\sqrt{2}$ and adding $\sqrt{2}$ repeatedly . So this sequence is an arithmetic sequence .

3. An object moves along a straight line at 10 metres / second . Applying a constant force in the opposite direction , the speed is reduced by 2 metres / second .

The sequence of the speed is 10 , 8 , 6 , 4 , . . .

Here the terms are got by subtracting 2 repeatedly from 10 . This is also considered an arithmetic sequence . (we can interpret *subtract 2* as *adding - 2*)

Finding

In an arithmetic sequence , we add the same number to move from a term immediately after it . So if we subtract from any term , the term immediately before it , we get this number .

An **arithmetic sequence** is a sequence in which we get the same number on subtracting from any term , the term immediately preceding it .

Common difference of an arithmetic sequence

In an arithmetic sequence , we get the same number on subtracting from any term , the term immediately preceding it . This constant difference is called the **common difference** of an arithmetic sequence .

Note :

Very often , we find out whether a given sequence is an arithmetic sequence by checking whether the difference between the terms is constant .

Note :

Usually the terms in a sequence are written in algebra as

$x_1 , x_2 , x_3 , x_4 , x_5 , \dots$ or $y_1 , y_2 , y_3 , y_4 , y_5 , \dots$

| First term | Second term | Third term | Fourth term | ... |
|------------|-------------|------------|-------------|-----|
| x_1 | x_2 | x_3 | x_4 | ... |

| Sequence | Distance between two consecutive terms | | | |
|---|--|------------------------------------|------------------------------------|------------------------------------|
| | $x_2 - x_1$ | $x_3 - x_2$ | $x_4 - x_3$ | $x_5 - x_4$ |
| 1, 2, 3, 4, 5, . . . | $2 - 1 = 1$ | $3 - 2 = 1$ | $4 - 3 = 1$ | $5 - 4 = 1$ |
| 2, 4, 6, 8, 10, . . . | $4 - 2 = 2$ | $6 - 4 = 2$ | $8 - 6 = 2$ | $10 - 8 = 2$ |
| 5, 10, 15, 20, 25, . . . | $10 - 5 = 5$ | $15 - 10 = 5$ | $20 - 15 = 5$ | $25 - 20 = 5$ |
| 2, 5, 8, 11, 14, . . . | $5 - 2 = 3$ | $8 - 5 = 3$ | $11 - 8 = 3$ | $14 - 11 = 3$ |
| 4, 8, 12, 16, 20, . . . | $8 - 4 = 4$ | $12 - 8 = 4$ | $16 - 12 = 4$ | $20 - 16 = 4$ |
| 360, 360, 360, 360, 360, . . . | $360 - 360 = 0$ | $360 - 360 = 0$ | $360 - 360 = 0$ | $360 - 360 = 0$ |
| $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}, \dots$ | $2\sqrt{2} - \sqrt{2} = \sqrt{2}$ | $3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$ | $4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$ | $5\sqrt{2} - 4\sqrt{2} = \sqrt{2}$ |
| 10, 8, 6, 4, 2, . . . | $8 - 10 = -2$ | $6 - 8 = -2$ | $4 - 6 = -2$ | $2 - 4 = -2$ |

Activity 3 (Multiplying natural numbers and the adding / subtracting a fixed number)

| | Number sequence |
|---|--------------------------|
| Multiply natural numbers by 6 | 6, 12, 18, 24, 30, . . . |
| Multiply natural numbers by 6 and then add 1 | 7, 13, 19, 25, 31, . . . |
| Multiply natural numbers by 6 and then subtract 1 | 5, 11, 17, 23, 29, . . . |

Are these arithmetic sequences ?

| Sequence | Distance between two consecutive terms | | | |
|--------------------------|--|---------------|---------------|---------------|
| | $x_2 - x_1$ | $x_3 - x_2$ | $x_4 - x_3$ | $x_5 - x_4$ |
| 6, 12, 18, 24, 30, . . . | $12 - 6 = 6$ | $18 - 12 = 6$ | $24 - 18 = 6$ | $30 - 24 = 6$ |
| 7, 13, 19, 25, 31, . . . | $13 - 7 = 6$ | $19 - 13 = 6$ | $25 - 19 = 6$ | $31 - 25 = 6$ |
| 5, 11, 17, 23, 29, . . . | $11 - 5 = 6$ | $17 - 11 = 6$ | $23 - 17 = 6$ | $29 - 23 = 6$ |

Since the difference between any term and the term before it is a constant, the above sequences are arithmetic sequences.