TEXT BOOK QUESTIONS - ARITHMETIC SEQUENCES

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1 Check whether each of the sequences given below is an arithmetic sequence. Give reasons. For the arithmetic sequences, write the common difference also.

i) Sequence of odd numbers

ii) Sequence of even numbers

iii) Sequence of fractions got as half the odd numbers

iv) Sequence of powers of 2

v) Sequence of reciprocals of natural numbers

Answer .

(i) Sequence = 1 , 3 , 5 , 7 , 9 , . . .

Here the sequence start with 1 and adding 2 repeatedly . So it is an arithmetic sequence .

Common difference = 2

(ii) Sequence = 2 , 4 , 6 , 8 , 10 , . . .

Here the sequence start with 2 and adding 2 repeatedly . So it is an arithmetic sequence .

Common difference = 2

(iii) Sequence = $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \ldots$

Here the sequence start with $\frac{1}{2}$ **and adding 1 repeatedly . So it is an arithmetic sequence .**

Common difference = 1

($\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, ... = $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, . . .)

(iv) Sequence = $2^1, 2^2, 2^3, 2^4, 2^5, \ldots$ = 2 , 4 , 8 , 16 , 32 , . . .

$x_2 - x_1 = 4 - 2 = 2$

$x_3 - x_2 = 8 - 4 = 4$

Here the difference of two consecutive terms is not a constant , So it is not an arithmetic sequence .

(v) Sequence = $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, . . .

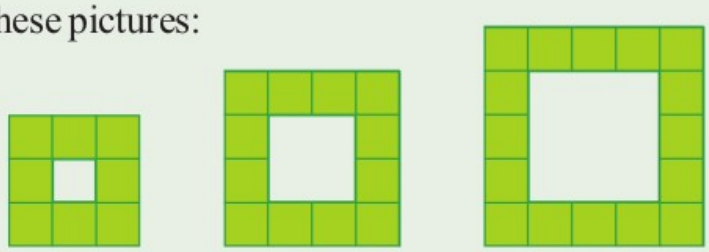
$x_2 - x_1$ = $\frac{1}{2} - \frac{1}{1} = \frac{1-2}{2\times1} = \frac{-1}{2}$

$x_3 - x_2$ = $\frac{1}{3} - \frac{1}{2} = \frac{2-3}{3\times2} = \frac{-1}{6}$

Here the difference of two consecutive terms is not a constant , it is not an arithmetic sequence .

2

Look at these pictures:



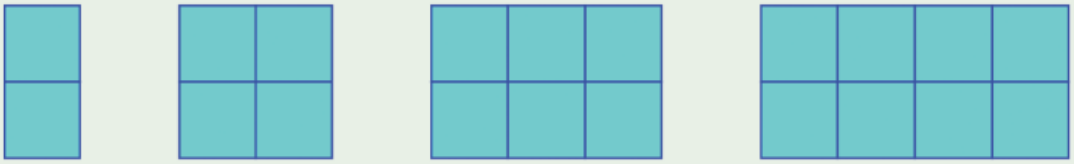
If the pattern is continued, do the numbers of coloured squares form an arithmetic sequence? Give reasons.

Answer .

Sequence = 8 , 12 , 16 , . . .

Here the sequence start with 8 and adding 4 repeatedly . So it is an arithmetic sequence .

3) See the pictures below:



i) How many small squares are there in each rectangle?

ii) How many large squares?

iii) How many squares in all?

Continuing this pattern, is each such sequence of numbers, an arithmetic sequence?

Answer .

(i) Number of small squares in the first rectangle = 2

Number of small squares in the second rectangle = 4

Number of small squares in the third rectangle = 6

Number of small squares in the fourth rectangle = 8

(ii) Number of large squares in the first rectangle = 0

Number of large squares in the second rectangle = 1

Number of large squares in the third rectangle = 2

Number of large squares in the fourth rectangle = 3

(iii) Number of squares in the first rectangle = 2

Number of squares in the second rectangle = 5

Number of squares in the three rectangle = 8

Number of squares in the fourth rectangle = 11

First sequence = 2 , 4 , 6 , 8 , . . .

Here the sequence start with 2 and adding 2 repeatedly . So it is an arithmetic sequence .

Second sequence = 0 , 1 , 2 , 3 , . . .

Here the sequence start with 0 and adding 1 repeatedly . So it is an arithmetic sequence .

Third sequence = 2 , 5 , 8 , 11 , . . .

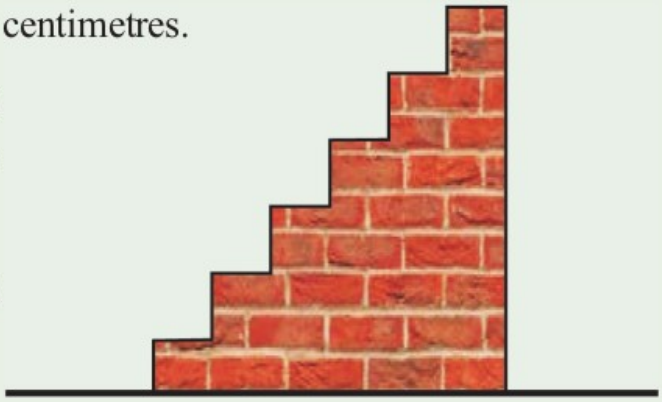
Here the sequence start with 2 and adding 3 repeatedly . So it is an arithmetic sequence .

4

In the staircase shown here the height of the first step is 10 centimetres and the height of each step after it is 17.5 centimetres.

i) How high from the ground would be someone climbing up, after each step?

ii) Write these numbers as a sequence.



Answer .

(i) Height from the ground after climbing first step = 10 cm

Height from the ground after climbing second step = 10 + 17.5 = 27.5 cm

Height from the ground after climbing third step =27.5 + 17.5 = 45 cm

Height from the ground after climbing fourth step = 45 + 17.5 = 62.5 cm

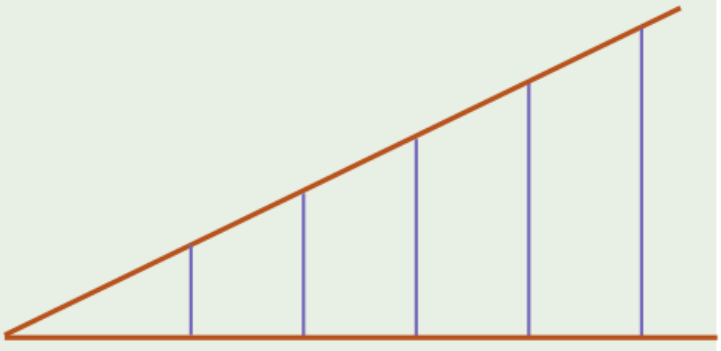
Height from the ground after climbing fifth step = 62.5 + 17.5 = 80 cm

Height from the ground after climbing sixth step = 80 + 17.5 = 97.5 cm

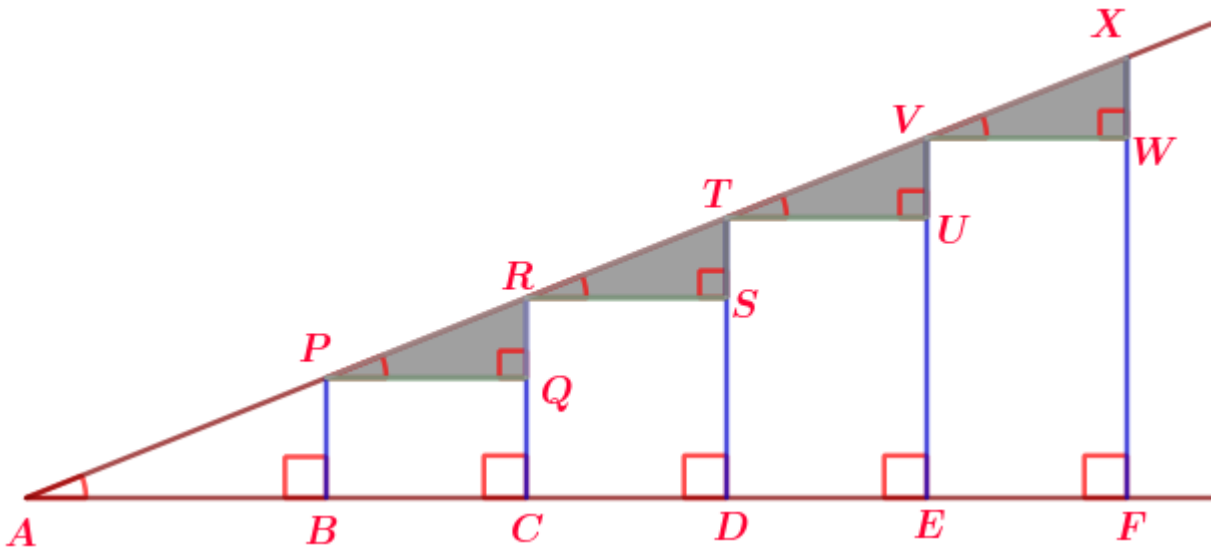
(ii) 10 , 27.5 , 45 , 62.5 , . . .

4

In this picture, the perpendiculars to the bottom line are equally spaced. Prove that, continuing like this, the lengths of perpendiculars form an arithmetic sequence.



Answer



In the figure BC = CD = DE = EF

PQ , RS , TU , VW are the lines drawn parallel to AF .

{ ∠Q = ∠S = ∠U = ∠W = 90^0

∠A = ∠QPR = ∠SRT = ∠UTV = ∠WVX (Corresponding angles)

BCQP , CDSR , DEUT , EFWV are rectangles .

== > PQ = RS = TU = VW }

PQR and RST equal triangles . (If one side and two angles on it of a triangle are equal to one side and two angles on it of another triangle , then the triangles are equal)

That is , PQR , RST, TUV, VWX are equal triangles .

So QR = ST = UV = XW .

That is the shaded right triangles in the figure are equal .The difference of the length of the perpendiculars is the vertical sides of the right triangles . Since the length of the vertical sides are equal , the lengths of the perpendiculars form an arithmetic sequence .

6

The algebraic expression of a sequence is

$$x_n = n^3 - 6n^2 + 13n - 7$$

Is it an arithmetic sequence?

Answer .

$$x_n = n^3 - 6n^2 + 13n - 7$$

$$x_1 = 1^3 - 6 \times 1^2 + 13 \times 1 - 7 = 1 - 6 \times 1 + 13 - 7$$

$$= 1 - 6 + 13 - 7 = 1$$

$$x_2 = 2^3 - 6 \times 2^2 + 13 \times 2 - 7 = 8 - 6 \times 4 + 26 - 7$$

$$= 8 - 24 + 26 - 7 = 3$$

$$x_3 = 3^3 - 6 \times 3^2 + 13 \times 3 - 7 = 27 - 6 \times 9 + 39 - 7$$

$$= 27 - 54 + 39 - 7 = 5$$

$$x_4 = 4^3 - 6 \times 4^2 + 13 \times 4 - 7 = 64 - 6 \times 16 + 52 - 7$$

$$= 64 - 96 + 52 - 7 = 13$$

$$x_3 - x_2 = 5 - 3 = 2$$

$$x_4 - x_3 = 13 - 5 = 8$$

Here the difference of two consecutive terms is not a constant . So it is not an arithmetic sequence .