Mathematics Online Class X On 28-06-2021

ARITHMETIC SEQUENCE

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Answers of questions on previous class

(1) Write the algebraic expression for each of the sequences below:

- i) Sequence of odd numbers
- ii) Sequence of natural numbers which leave remainder 1 on division by 3.
- iii) The sequence of natural numbers ending in 1.
- iv) The sequence of natural numbers ending in 1 or 6.

i) Sequence of odd numbers

1 , 3 , 5 , 7 , 9 , 11 , . . .

First term $x_1 = 1 = 2 - 1 = 2 \times 1 - 1$

Second term $x_2 = 3 = 4 - 1 = 2 \times 2 - 1$

Third term $x_3 = 5 = 6 - 1 = 2 \times 3 - 1$

Fourth term $x_4 = 7 = 8 - 1 = 2 \times 4 - 1$

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n th term $x_n = 2 \times n - 1$

Algebraic expression of the sequence = **2 n – 1**

ii) Sequence of natural numbers which leave remainder 1 on division by 3

1 , 4 , 7 , 10 , 13 , . . .

First term $x_1 = 1 = 3 - 2 = 3 \times 1 - 2$

Second term $x_2 = 4 = 6 - 2 = 3 \times 2 - 2$

Third term $x_3 = 7 = 9 - 2 = 3 \times 3 - 2$

Fourth term $x_4 = 10 = 12 - 2 = 3 \times 4 - 2$

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n th term $x_n = 3 \times n - 2$

Algebraic expression of the sequence = **3 n – 2**

iii) The sequence of natural numbers ending in 1

1 , 11 , 21 , 31 , 41 , . . .

First term $x_1 = 1 = 10 - 9 = 10 \times 1 - 9$

Second term $x_2 = 11 = 20 - 9 = 10 \times 2 - 9$

Third term $x_3 = 21 = 30 - 9 = 10 \times 3 - 9$

Fourth term $x_4 = 31 = 40 - 9 = 10 \times 4 - 9$

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n th term $x_n = 10 \times n - 9$

Algebraic expression of the sequence = **10 n – 9**

iv) The sequence of natural numbers ending with 1 or 6

1 , 6 , 11 , 16 , 21 , 26 , . . .

First term $x_1 = 1 = 5 - 4 = 5 \times 1 - 4$

Second term $x_2 = 6 = 10 - 4 = 5 \times 2 - 4$

Third term $x_3 = 11 = 15 - 4 = 5 \times 3 - 4$

Fourth term $x_4 = 16 = 20 - 4 = 5 \times 4 - 4$

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n th term $x_n = 5 \times n - 4$

Algebraic expression of the sequence = **5 n – 4**

(2) For the sequence of regular polygons starting with an equilateral triangle, write the algebraic expressions for the sequence of the sums of inner angles, the sums of the outer angles, the measures of an inner angle, and the measures of an outer angle.

i) Sequence of sums of inner angles

$180^0 , 360^0 , 540^0 , 720^0 , 900^0 , . . .$

$$180 \times 1 , 180 \times 2 , 180 \times 3 , 180 \times 4 , \ldots , 180 \times n , \ldots$$

Algebraic expression of the sequence = 180n

ii) Sequence of sums of outer angles

$$360^0 , 360^0 , 360^0 , \ldots$$

Algebraic expression of the sequence = 360

iii) Sequence of the measures of an inner angle

$$60^0 , 90^0 , 108^0 , 120^0 , \ldots$$

$$\frac{180}{3} , \frac{360}{4} , \frac{540}{5} , \frac{720}{6} , \ldots$$

$$\frac{180\times1}{(1+2)} , \frac{180\times2}{(2+2)} , \frac{180\times3}{(3+2)} , \frac{180\times4}{(4+2)} , \ldots , \frac{180\times n}{(n+2)} , \ldots$$

Algebraic expression of the sequence = $\frac{180\times n}{(n+2)}$

iv) Sequence of the measures of an outer angle

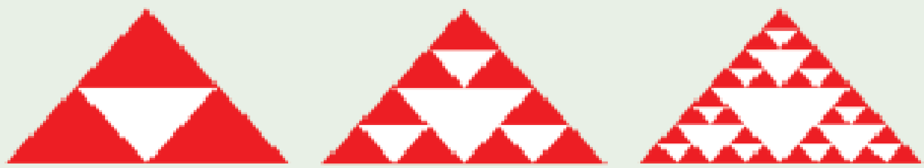
$$120^0 , 90^0 , 72^0 , 60^0 , \ldots$$

$$\frac{360}{3} , \frac{360}{4} , \frac{360}{5} , \frac{360}{6} , \ldots$$

$$\frac{360}{(1+2)} , \frac{360}{(2+2)} , \frac{360}{(3+2)} , \frac{360}{(4+2)} , \ldots , \frac{360}{(n+2)} , \ldots$$

Algebraic expression of the sequence = $\frac{360}{(n+2)}$

(3) Look at these pictures:



The first picture is got by removing the small triangle formed by joining the midpoints of an equilateral triangle. The second picture is got by removing such a middle triangle from each of the red triangles of the first picture. The third picture shows the same thing done on the second.

i) How many red triangles are there in each picture?

ii) Taking the area of the original uncut triangle as 1, compute the area of a small triangle in each picture.

iii) What is the total area of all the red triangles in each picture?

iv) Write the algebraic expressions for these three sequences obtained by continuing this process.

Answer

i)

Number of red triangles in the first figure = 3

Number of red triangles in the second figure = 9

Number of red triangles in the third figure = 27

ii)

The area of a small triangle in the first figure $= \frac{1}{4}$

The area of a small triangle in the second figure $= \frac{1}{16}$

The area of a small triangle in the third figure $= \frac{1}{64}$

iii)

Total area of all the red triangles in the first figure $= 3 \times \frac{1}{4} = \frac{3}{4}$

Total area of all the red triangles in the second figure $= 9 \times \frac{1}{16} = \frac{9}{16}$

Total area of all the red triangles in the third figure $= 27 \times \frac{1}{64} = \frac{27}{64}$

iv)

a) **Sequence of the number of red triangles in the each figure**

$3, 9, 27, \ldots$

$3^1, 3^2, 3^3, \ldots, 3^n, \ldots$

Algebraic expression of the sequence = 3^n

b) **Sequence of the area of a small triangle in the each figure**

$\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots$

$\left(\frac{1}{4}\right), \left(\frac{1}{4}\right)^2, \left(\frac{1}{4}\right)^3, \ldots, \left(\frac{1}{4}\right)^n, \ldots$

Algebraic expression of the sequence = $\left(\frac{1}{4}\right)^n$

c) **Sequence of the total area of all the red triangles in the each figure**

$\frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \ldots$

$\left(\frac{3}{4}\right), \left(\frac{3}{4}\right)^2, \left(\frac{3}{4}\right)^3, \ldots, \left(\frac{3}{4}\right)^n, \ldots$

Algebraic expression of the sequence = $\left(\frac{3}{4}\right)^n$

Sequence of natural numbers $1, 2, 3, 4, \ldots$

Sequence of even natural numbers $2, 4, 6, 8, \ldots$

Sequence of multiples of 5 $5, 10, 15, 20, \ldots$

Sequence of natural numbers which leave remainder 2 on division by 3 — 2 , 5 , 8 , 11 , . . .

Sequence of perimeters of squares having sides 1unit , 2unit , 3unit , 4unit , . . . — 4 , 8 , 12 , 16 , . . .

Sequence of perimeters of squares having sides 1unit , $1\frac{1}{2}$ unit , 2unit , $2\frac{1}{2}$ unit , . . . — 4 , 6 , 8 , 10 , . . .

A sequence got by starting with any number and adding a fixed number repeatedly is called an arithmetic sequence.

Look some arithmetic sequence given below :

(a) Sequence of sums of outer angles of a polygons
360^0 , 360^0 , 360^0 , . . .

(b) Sequence of sides of squares having sides 1unit , $1\frac{1}{2}$ unit , 2unit , $2\frac{1}{2}$ unit , . . . — 1 , $1\frac{1}{2}$, 2 , $2\frac{1}{2}$, . . .

(c) Sequence of length of diagonals of squares having sides 1unit , 2 unit , 3 unit , 4unit , . . . — $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$, . . .

(d) Sequence of speed of particle with speed 10 m/s moves in a straight line when a force is applied opposite ,it reduces a speed of 2m/s — 10 , 8 , 6 , 4 , . . .

The constant difference got by subtracting from any term the just previous term is called the common difference (d) of an arithmetic sequence .

Common difference of the above arithmetic sequences are

(a) 360^0 , 360^0 , 360^0 , . . . Common difference d = 0

(b) 1 , $1\frac{1}{2}$, 2 , $2\frac{1}{2}$, . . . Common difference d = $\frac{1}{2}$

(c) $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$, . . . Common difference d = $\sqrt{2}$

(d) 10 , 8 , 6 , 4 , . . . Common difference d = -2

Consider

Natural numbers **1 , 2 , 3 , 4 , . . .**

This is an arithmetic sequence with Common difference d = 1

Multiplying each term by 6 **6 , 12 , 18 , 24 , . . .**

This is an arithmetic sequence with Common difference d = 6

Adding each term by 1 **7 , 13 , 19 , 25 , . . .**

This is an arithmetic sequence with Common difference d = 6

Subtracting each term by 2 **5 , 11 , 17 , 23 , . . .**

This is an arithmetic sequence with Common difference d = 6

From this we get if we multiply a constant to the sequence of natural numbers and adding or subtracting a constant to it we get an arithmetic sequence .