Mathematics Online Class X On 01-07-2021

ARITHMETIC SEQUENCE

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Answers of questions on previous class

1. In each of the arithmetic sequences below , some terms are missing and their positions are marked with ◌. Find them .

i) 24 , 42 , ◌ , ◌ , . . .

$x_2 - x_1 = (2-1)d$ $\quad d = 42 - 24 = 18$

$x_3 = 42 + 18 = 60$

$x_4 = 60 + 18 = 78$

ii) ◌ , 24 , 42 , ◌ , . . .

$x_3 - x_2 = (3-2)d$ $\quad d = 42 - 24 = 18$

$x_1 = 24 - 18 = 6$

$x_4 = 42 + 18 = 60$

iii) ◌ , ◌ , 24 , 42 , . . .

$x_4 - x_3 = (3-2)d$ $\quad d = 42 - 24 = 18$

$x_2 = 24 - 18 = 6$

$x_1 = 6 - 18 = -12$

iv) 24 , ◌ , 42 , ◌ , . . .

$x_3 - x_1 = (3-1)d$ $\quad 2d = 42 - 24 = 18$ $\quad \therefore d = \frac{18}{2} = 9$

$x_2 = 24 + 9 = 33$

$x_4 = 42 + 9 = 51$

v) ◌ , 24 , ◌ , 42 , . . .

$x_4 - x_2 = (4-2)d$ $\quad 2d = 42 - 24 = 18$ $\quad \therefore d = \frac{18}{2} = 9$

$x_1 = 24 - 9 = 15$

$x_3 = 24 + 9 = 33$

vi) 24 , ◌ , ◌ , 42 , . . .

$x_4 - x_1 = (4-1)d$ $\quad 3d = 42 - 24 = 18$ $\quad \therefore d = \frac{18}{3} = 6$

$x_2 = 24 + 6 = 30$

$x_3 = 30 + 6 = 36$

2. The terms in two positions of some arithmetic sequences are given below .Write the first five terms of each .

i) 3^{rd} term 34

6^{th} term 67

$x_6 - x_3 = (6-3)d \qquad 3d = 67 - 34 = 33 \qquad \therefore d = \frac{33}{3} = 11$

$x_1 = x_3 - 2d = 34 - 2 \times 11 = 34 - 22 = 12$

$x_2 = 12 + 11 = 23$

$x_3 = 23 + 11 = 34$

$x_4 = 34 + 11 = 45$

$x_5 = 45 + 11 = 56$

ii) 3^{rd} term 43

6^{th} term 76

$x_6 - x_3 = (6-3)d \qquad 3d = 76 - 43 = 33 \qquad \therefore d = \frac{33}{3} = 11$

$x_1 = x_3 - 2d = 43 - 2 \times 11 = 43 - 22 = 21$

$x_2 = 21 + 11 = 32$

$x_3 = 32 + 11 = 43$

$x_4 = 43 + 11 = 54$

$x_5 = 54 + 11 = 65$

iii) 3^{rd} term 2

5^{th} term 3

$x_5 - x_3 = (5-3)d \qquad 2d = 3 - 2 = 1 \qquad \therefore d = \frac{1}{2}$

$x_1 = x_3 - 2d = 2 - 2 \times \frac{1}{2} = 2 - 1 = 1$

$x_2 = 1 + \frac{1}{2} = 1\ \frac{1}{2}$

$x_3 = 1\ \frac{1}{2} + \frac{1}{2} = 2$

$x_4 = 2 + \frac{1}{2} = 2\ \frac{1}{2}$

$x_5 = 2\ \frac{1}{2} + \frac{1}{2} = 3$

iv) 4^{th} term 2

7^{th} term 3

$x_7 - x_4 = (7-4)d \qquad 3d = 3 - 2 = 1 \qquad \therefore d = \frac{1}{3}$

$x_1 = x_4 - 3d = 2 - 3 \times \frac{1}{3} = 2 - 1 = 1$

$x_2 = 1 + \frac{1}{3} = 1\ \frac{1}{3}$

$x_3 = 1\ \frac{1}{3} + \frac{1}{3} = 1\ \frac{2}{3}$

$x_4 = 1\ \frac{2}{3} + \frac{1}{3} = 2$

$x_5 = 2 + \frac{1}{3} = 2\ \frac{1}{3}$

v) 2^{nd} term 5

5^{th} term 2

$x_5 - x_2 = (5-2)d \qquad 3d = 2 - 5 = -3 \qquad \therefore d = \frac{-3}{3} = -1$

$x_1 = x_2 - 1d = 5 - 1 \times -1 = 5 + 1 = 6$

$x_2 = 6 + (-1) = 5$

$x_3 = 5 + (-1) = 4$

$x_4 = 4 + (-1) = 3$

$x_5 = 3 + (-1) = 2$

3. The 5^{th} term of an arithmetic sequence is 38 and 9^{th} term is 66 .What is its 25^{th} term?

$x_5 = 38 \qquad x_9 = 66$

$x_9 - x_5 = (9-5)d \qquad 4d = 66 - 38 = 28 \qquad \therefore d = \frac{28}{4} = 7$

$x_{25} = x_9 + 16d = 66 + 16 \times 7 = 66 + 112 = 178$

4. Is 101 a term of the arithmetic sequence 13 , 24 , 35 , . . . What about 1001 ?

$x_1 = 13 \qquad d = 24 - 13 = 11$

101 – 13 = 88 Which is a multiple of common difference 11

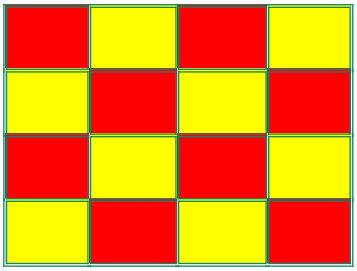
∴ 101 is a term of this sequence.

Now

1001 – 13 = 988 Which is not a multiple of common difference 11

∴ 1001 is not a term of this sequence.

Fill each box with numbers such that each row and column must be an arithmetic sequence .



Answer

Simply we can fill each box with natural numbers from 1 to 16

1	2	3	4	d = 1
5	6	7	8	d = 1
9	10	11	12	d = 1
13	14	15	16	d = 1
d = 4	d = 4	d = 4	d = 4	

Here $\frac{16-1}{1} = 15$

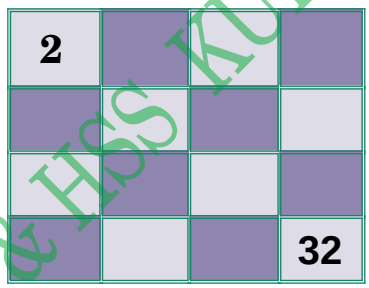
Total number of boxes= 15 + 1 = 16

Here each row is an arithmetic sequence with common difference 1 and

each column is an arithmetic sequence with common difference 4 .

Question

Fill each box with numbers such that each row and column must be an arithmetic sequence .



Answer

Here two numbers are fixed .

We can simply we write continuous even natural numbers , we get

2	4	6	8	d = 2
10	12	14	16	d = 2
18	20	22	24	d = 2
26	28	30	32	d = 2
d = 8	d = 8	d = 8	d = 8	

Here $\frac{32-2}{2} = 15$

Total number of boxes= 15 + 1 = 16

Here each row is an arithmetic sequence with common difference 2

and

each column is an arithmetic sequence with common difference 8 .

Question

Fill each box with numbers such that each row and column must be an arithmetic sequence .

1			
			31

Answer

Here two numbers are fixed .

We can simply we write continuous odd natural numbers , we get,

1	3	5	7	d = 2
9	11	13	15	d = 2
17	19	21	23	d = 2
25	27	29	31	d = 2
d = 8	d = 8	d = 8	d = 8	

Here $\frac{31 - 1}{2} = 15$

Total number of boxes= 15 + 1 = 16

Also

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

d = 6 d =10

Here we can see that the numbers in both the diagonals is also an arithmetic sequence .

From the above questions , we can see that

$$TOTAL\ NUMBER\ OF\ TERMS = \frac{LAST\ TERM - FIRST\ TERM}{COMMON\ DIFFERENCE} + 1$$

Question

Fill up the empty cells of the given square such that the numbers in each row and column form an arithmetic sequences

1			4
7			28

Answer

We have

Term difference = position difference × common difference

∴ Common difference = $\frac{Term\ difference}{Position\ difference}$

1			4
7			28

$d = \frac{4 - 1}{3} = 1$

$d = \frac{28 - 7}{3} = 7$

$d = \frac{7 - 1}{3} = 2$

$d = \frac{28 - 4}{3} = 8$

Like this we can fill the empty cells

1	2	3	4
3	6	9	12
5	10	15	20
7	14	21	28

Question

How many three digit numbers are there , which leave remainder 3 on division by 7 ?

OR

101 , 108 , 115 , . . . , 997 . How many terms are there in this arithmetic sequence ?

Answer

Three digit numbers which leave remainder 3 on division by 7 are 101 , 108 , 115 , . . . , 997 .

This is an arithmetic sequence with first term 101 and last term 997 with common difference 7.

$$TOTAL\ NUMBER\ OF\ TERMS = \frac{LAST\ TERM - FIRST\ TERM}{COMMON\ DIFFERENCE} + 1$$

$$= \frac{997-101}{7} + 1$$

$$= \frac{896}{7} + 1$$

$$= 128 + 1$$

$$= 129$$

ASSIGNMENT

How many two digit numbers are there which leave a remainder 2 on division by 3 ?