

Mathematics Online Class X On 01-07-2021

ARITHMETIC SEQUENCE



Answers of questions on previous class

1. In each of the arithmetic sequences below, some terms are missing and their positions are marked with \bigcirc .

Find them.

i) $24, 42, \bigcirc, \bigcirc, \dots$

$$x_2 - x_1 = (2-1)d \quad d = 42 - 24 = 18$$

$$x_3 = 42 + 18 = 60$$

$$x_4 = 60 + 18 = 78$$

ii) $\bigcirc, 24, 42, \bigcirc, \dots$

$$x_3 - x_2 = (3-2)d \quad d = 42 - 24 = 18$$

$$x_1 = 24 - 18 = 6$$

$$x_4 = 42 + 18 = 60$$

iii) $\bigcirc, \bigcirc, 24, 42, \dots$

$$x_4 - x_3 = (3-2)d \quad d = 42 - 24 = 18$$

$$x_2 = 24 - 18 = 6$$

$$x_1 = 6 - 18 = -12$$

iv) $24, \bigcirc, 42, \bigcirc, \dots$

$$x_3 - x_1 = (3-1)d \quad 2d = 42 - 24 = 18 \quad \therefore d = \frac{18}{2} = 9$$

$$x_2 = 24 + 9 = 33$$

$$x_4 = 42 + 9 = 51$$

v) $\bigcirc, 24, \bigcirc, 42, \dots$

$$x_4 - x_2 = (4-2)d \quad 2d = 42 - 24 = 18 \quad \therefore d = \frac{18}{2} = 9$$

$$x_1 = 24 - 9 = 15$$

$$x_3 = 24 + 9 = 33$$

vi) $24, \bigcirc, \bigcirc, 42, \dots$

$$x_4 - x_1 = (4-1)d \quad 3d = 42 - 24 = 18 \quad \therefore d = \frac{18}{3} = 6$$

$$x_2 = 24 + 6 = 30$$

$$x_3 = 30 + 6 = 36$$

2. The terms in two positions of some arithmetic sequences are given below .Write the first five terms of each .

i) 3rd term 34

6th term 67

$$x_6 - x_3 = (6-3)d \quad 3d = 67 - 34 = 33 \quad \therefore d = \frac{33}{3} = 11$$

$$x_1 = x_3 - 2d = 34 - 2 \times 11 = 34 - 22 = 12$$

$$x_2 = 12 + 11 = 23$$

$$x_3 = 23 + 11 = 34$$

$$x_4 = 34 + 11 = 45$$

$$x_5 = 45 + 11 = 56$$

ii) 3rd term 43

6th term 76

$$x_6 - x_3 = (6-3)d \quad 3d = 76 - 43 = 33 \quad \therefore d = \frac{33}{3} = 11$$

$$x_1 = x_3 - 2d = 43 - 2 \times 11 = 43 - 22 = 21$$

$$x_2 = 21 + 11 = 32$$

$$x_3 = 32 + 11 = 43$$

$$x_4 = 43 + 11 = 54$$

$$x_5 = 54 + 11 = 65$$

iii) 3rd term 2

5th term 3

$$x_5 - x_3 = (5-3)d \quad 2d = 3 - 2 = 1 \quad \therefore d = \frac{1}{2}$$

$$x_1 = x_3 - 2d = 2 - 2 \times \frac{1}{2} = 2 - 1 = 1$$

$$x_2 = 1 + \frac{1}{2} = 1 \frac{1}{2}$$

$$x_3 = 1 \frac{1}{2} + \frac{1}{2} = 2$$

$$x_4 = 2 + \frac{1}{2} = 2 \frac{1}{2}$$

$$x_5 = 2 \frac{1}{2} + \frac{1}{2} = 3$$

iv) 4th term 2

7th term 3

$$x_7 - x_4 = (7-4)d \quad 3d = 3 - 2 = 1 \quad \therefore d = \frac{1}{3}$$

$$x_1 = x_4 - 3d = 2 - 3 \times \frac{1}{3} = 2 - 1 = 1$$

$$x_2 = 1 + \frac{1}{3} = 1 \frac{1}{3}$$

$$x_3 = 1 \frac{1}{3} + \frac{1}{3} = 1 \frac{2}{3}$$

$$x_4 = 1 \frac{2}{3} + \frac{1}{3} = 2$$

$$x_5 = 2 + \frac{1}{3} = 2 \frac{1}{3}$$

v) 2nd term 5

5th term 2

$$x_5 - x_2 = (5-2)d \quad 3d = 2 - 5 = -3 \quad \therefore d = \frac{-3}{3} = -1$$

$$x_1 = x_2 - 1d = 5 - 1 \times -1 = 5 + 1 = 6$$

$$x_2 = 6 + (-1) = 5$$

$$x_3 = 5 + (-1) = 4$$

$$x_4 = 4 + (-1) = 3$$

$$x_5 = 3 + (-1) = 2$$

3. The 5th term of an arithmetic sequence is 38 and 9th term is 66 .What is its 25th term?

$$x_5 = 38 \quad x_9 = 66$$

$$x_9 - x_5 = (9-5)d \quad 4d = 66 - 38 = 28 \quad \therefore d = \frac{28}{4} = 7$$

$$x_{25} = x_9 + 16d = 66 + 16 \times 7 = 66 + 112 = 178$$

4. Is 101 a term of the arithmetic sequence 13 , 24 , 35 , ...
What about 1001?

$$x_1 = 13 \quad d = 24 - 13 = 11$$

$$101 - 13 = 88 \text{ Which is a multiple of common difference } 11$$

\therefore 101 is a term of this sequence.

Now

$$1001 - 13 = 988 \text{ Which is not a multiple of common difference } 11$$

\therefore 1001 is not a term of this sequence.

Fill each box with numbers such that each row and column must be an arithmetic sequence .

Answer

Simply we can fill each box with natural numbers from 1 to 16

1	2	3	4	$d = 1$
5	6	7	8	$d = 1$
9	10	11	12	$d = 1$
13	14	15	16	$d = 1$
$d = 4$	$d = 4$	$d = 4$	$d = 4$	

Here $\frac{16-1}{1} = 15$

Total number of boxes = $15 + 1 = 16$

Here each row is an arithmetic sequence with common difference 1 and each column is an arithmetic sequence with common difference 4 .

Question

Fill each box with numbers such that each row and column must be an arithmetic sequence .

2			
			32

Answer

Here two numbers are fixed .

We can simply we write continuous even natural numbers , we get

2	4	6	8	$d = 2$
10	12	14	16	$d = 2$
18	20	22	24	$d = 2$
26	28	30	32	$d = 2$
$d = 8$	$d = 8$	$d = 8$	$d = 8$	

Here $\frac{32-2}{2} = 15$

Total number of boxes = $15 + 1 = 16$

Here each row is an arithmetic sequence with common difference 2

and

each column is an arithmetic sequence with common difference 8 .

Question

Fill each box with numbers such that each row and column must be an arithmetic sequence .

1			
			31

Answer

Here two numbers are fixed .

We can simply we write continuous odd natural numbers , we get,

1	3	5	7	$d = 2$
9	11	13	15	$d = 2$
17	19	21	23	$d = 2$
25	27	29	31	$d = 2$
∞	∞	∞	∞	

Here $\frac{31 - 1}{2} = 15$

Total number of boxes = $15 + 1 = 16$

Also

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

$d = 6$

$d = 10$

Here we can see that the numbers in both the diagonals is also an arithmetic sequence .

From the above questions , we can see that

$$\text{TOTAL NUMBER OF TERMS} = \frac{\text{LAST TERM} - \text{FIRST TERM}}{\text{COMMON DIFFERENCE}} + 1$$

Question

Fill up the empty cells of the given square such that the numbers in each row and column form an arithmetic sequences

1			4
7			28

Answer

We have

Term difference = position difference \times common difference

$$\therefore \text{Common difference} = \frac{\text{Term difference}}{\text{Position difference}}$$

1			4
7			28

$$d = \frac{4 - 1}{3} = 1$$

$$d = \frac{28 - 7}{3} = 7$$

$$d = \frac{7 - 1}{3} = 2$$

$$d = \frac{28 - 4}{3} = 8$$

Like this we can fill the empty cells

1	2	3	4
3	6	9	12
5	10	15	20
7	14	21	28

Question

How many three digit numbers are there , which leave remainder 3 on division by 7 ?

OR

101 , 108 , 115 , . . . , 997 . How many terms are there in this arithmetic sequence ?

Answer

Three digit numbers which leave remainder 3 on division by 7 are 101 , 108 , 115 , . . . , 997 .

This is an arithmetic sequence with first term 101 and last term 997 with common difference 7.

$$\begin{aligned} \text{TOTAL NUMBER OF TERMS} &= \frac{\text{LAST TERM} - \text{FIRST TERM}}{\text{COMMON DIFFERENCE}} + 1 \\ &= \frac{997 - 101}{7} + 1 \\ &= \frac{896}{7} + 1 \\ &= 128 + 1 \\ &= 129 \end{aligned}$$

ASSIGNMENT

How many two digit numbers are there which leave a remainder 2 on division by 3 ?