Mathematics Online Class X On 05-07-2021

ARITHMETIC SEQUENCE

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Question on the previous class

How many two digit numbers are there which leave a remainder 2 on division by 3 ?

Answer

Two digit numbers which leave a remainder 2 on division by 3 are 11 , 14 , 17 , . . . , 98 .

This is an arithmetic sequence with common difference 3

First term 11 and Last term 98

$$TOTAL\ NUMBER\ OF\ TERMS = \frac{LAST\ TERM - FIRST\ TERM}{COMMON\ DIFFERENCE} + 1$$

$$= \frac{98-11}{3} + 1$$

$$= \frac{87}{3} + 1$$

$$= 29 + 1$$

$$= 30$$

Algebra of arithmetic sequences

1) Sequence of natural numbers

1 , 2 , 3 , 4 , . . .

What is the 10^{th} term of the sequence ? 10

What is the 100^{th} term of the sequence ? 100

What is the 199^{th} term of the sequence ? 199

What is the n^{th} term of the sequence ? n

Algebraic form of the sequence of natural numbers is n

2) Sequence of even numbers

2 , 4 , 6 , 8 , . . .

What is the 10^{th} term of the sequence ? $2 \times 10 = 20$

What is the 100^{th} term of the sequence ? $2 \times 100 = 200$

What is the 199^{th} term of the sequence ? $2 \times 199 = 398$

What is the n^{th} term of the sequence ? $2 \times n = 2n$

Algebraic form of the sequence of even numbers is 2n

3) Sequence of odd numbers

$1 , 3 , 5 , 7 , \ldots$

$1 = 2 - 1 = 2 \times 1 - 1$,

$3 = 4 - 1 = 2 \times 2 - 1$,

$5 = 6 - 1 = 2 \times 3 - 1$,

$7 = 8 - 1 = 2 \times 4 - 1 , \ldots$

What is the 10^{th} term of the sequence ? $2 \times 10 - 1 = 19$

What is the 100^{th} term of the sequence ? $2 \times 100 - 1 = 199$

What is the n^{th} term of the sequence ? $2 \times n - 1 = 2n-1$

Algebraic form of the sequence of odd numbers is $2n-1$

4) Sequence of multiples of 5

$5 , 10 , 15 , 20 , \ldots$

What is the 10^{th} term of the sequence ? $5 \times 10 = 50$

What is the 100^{th} term of the sequence ? $5 \times 100 = 500$

What is the n^{th} term of the sequence ? $5 \times n = 5n$

Algebraic form of sequence of multiples of 5 is $5n$

5) Sequence of multiples of 3

$3 , 6 , 9 , 12 , \ldots$

What is the 10^{th} term of the sequence ? $3 \times 10 = 30$

What is the 100^{th} term of the sequence ? $3 \times 100 = 300$

What is the n^{th} term of the sequence ? $3 \times n = 3n$

Algebraic form of the sequence of multiples of 3 is $3n$

Similarly

	Sequence	Algebraic form
Multiples of 4	$4 , 8 , 12 , 16 , \ldots$	$4n$
Multiples of 6	$6 , 12 , 18 , 24 , \ldots$	$6n$
Multiples of 7	$7 , 14 , 21 , 28 , \ldots$	$7n$
Multiples of 8	$8 , 16 , 24 , 32 , \ldots$	$8n$

We know that we can generate another arithmetic sequence by adding or subtracting a constant to the multiples of a number

1) Sequence of multiples of 5 5 ,10 , 15 , 20 , . . .

Here common difference is 5 and algebraic form is 5n

Adding 1 to each terms we get another sequence 6 ,11 , 16 , 21, . . .

This is also an arithmetic sequence with common difference 5

10^{th} term of the sequence ? $5 \times 10 + 1 = 51$

100^{th} term of the sequence ? $5 \times 100 + 1 = 501$

n^{th} term of the sequence ? $5 \times n + 1 = 5n+1$

Algebraic form of sequence is 5n+1

2) Sequence of multiples of 3 3 ,6 , 9 , 12 , . . .

Here common difference is 3 and algebraic form is 3n

Adding 2 to each terms we get another sequence 5 , 8 , 11 , 14 , . . .

This is also an arithmetic sequence with common difference 3.

10^{th} term of the sequence ? $3 \times 10 + 2 = 32$

100^{th} term of the sequence ? $3 \times 100 + 2 = 302$

n^{th} term of the sequence ? $3 \times n + 2 = 3n+2$

Algebraic form of sequence is 3n+2

3) Sequence of multiples of 3 3 ,6 , 9 , 12 , . . .

Here common difference is 3 and algebraic form is 3n

Subtracting 1 from each terms we get another sequence

2 , 5 , 8 , 11 , . . .

This is also an arithmetic sequence with common difference 3.

10^{th} term of the sequence ? $3 \times 10 - 1 = 29$

100^{th} term of the sequence ? $3 \times 100 - 1 = 299$

n^{th} term of the sequence ? $3 \times n - 1 = 3n-1$

Algebraic form of sequence is 3n-1

From above we see that we can make the algebraic form of an arithmetic sequence by first writing the multiples of common difference and see that which constant is added or subtracted to make the first sequence.

Thus algebraic form of an arithmetic sequence is written as common difference × n + constant OR common difference × n – constant according as constant is added or subtracted, to make the first sequence.

The algebraic form of an arithmetic sequence is of the form $x_n = an + b$, where a and b are fixed numbers and a is the common difference ; conversely, any sequence of this form is an arithmetic sequence .

We know n^{th} term , $x_n = f + (n-1)\,d$ where f is the first term and d is the common difference

$x_n = f + (n-1)\,d = f + dn - d = dn + (f-d)$

That is $x_n = dn + (f-d)$

Question

Write the algebraic form of the sequence 2 , 5 , 8 , 11 , . . .

Answer

This is an arithmetic sequence with common difference 3 .

Now ,

2 = 3 × 1 – 1 ,

5 = 3 × 2 – 1 ,

8 = 3 × 3 – 1 , . . .

Algebraic form of the sequence is $x_n = 3n - 1$

Question

Write down the arithmetic sequence of multiples of 4 and write its algebraic form?

Answer

Sequence 4 , 8 , 12 , 16 , . . .

Algebraic form $x_n = 4n$

Here common difference is 4

Multiples of 4 are 4 , 8 ,12 ,16 , . . .

4 = 4 × 1

8 = 4 × 2

12 = 4 × 3 , . . .

Question

Write down the arithmetic sequence which leave a remainder 1 on division by 4 and write its algebraic form?

Answer

Sequence 1 , 5 , 9 , 13 , . . .

Algebraic form $x_n = 4n - 3$

Here common difference is 4

Multiples of 4 are 4 , 8 ,12 ,16 , . . .

Now 1 , 5 , 9 , 13 , . . .

$1 = 4 \times 1 - 3$

$5 = 4 \times 2 - 3$

$9 = 4 \times 3 - 3$, . . .

Question

Write down the arithmetic sequence which leave a remainder 2 on division by 4 and write its algebraic form?

Answer

Sequence 2 , 6 , 10 , 14 , . . .

Algebraic form $x_n = 4n - 2$

Here common difference is 4

Multiples of 4 are 4 , 8 ,12 ,16 , . . .

Now 2 , 6 , 10 , 14 , . . .

$2 = 4 \times 1 - 2$

$6 = 4 \times 2 - 2$

$10 = 4 \times 3 - 2$, . . .

Question

Write down the arithmetic sequence which leave a remainder 3 on division by 4 and write its algebraic form?

Answer

Sequence 3 , 7 , 11 , 15 , . . .

Algebraic form $x_n = 4n - 1$

Here common difference is 4

Multiples of 4 are 4 , 8 ,12 ,16 , . . .

Now 3 , 7 , 11 , 15 , . . .

$3 = 4 \times 1 - 1$

$7 = 4 \times 2 - 1$

$11 = 4 \times 3 - 1$, . . .