ONLINE MATHS CLASS - X - 09 (08 / 07 /2021)

1. ARITHMETIC SEQUENCE - CLASS 7

What did we study in the last class ?

- If the first term of an arithmetic sequence is f and its common difference is d , then its n^{th} term is $dn + f - d$.
- The algebraic form of any arithmetic sequence is of the form $an + b$, where a and b are fixed numbers . a is the common difference .
- $a = d$, $b = f - d$

Activity 1

Each term of an arithmetic sequence got by multiplying the position number by the common difference and adding a fixed number . That is algebraic form of any arithmetic sequence is of the form $x_n = an + b$.

(NB : The n^{th} term of a sequence is also called its *algebraic form* .)

On the other hand , is any sequence $x_n = an + b$, an arithmetic sequence ?

Any two consecutive terms of this sequence are of the form $an + b$ and $a(n+1) + b$.

Their difference is ,

$$a(n+1) + b - (an + b) = an + a + b - (an + b)$$
$$= an + a + b - an - b$$
$$= a$$

That is , the difference between any two consecutive terms is the same number a and so it is an arithmetic sequence .

Finding

Any sequence with the algebraic form $x_n = an + b$, *is an arithmetic sequence*

Activity 2

The algebraic form of a sequence is $5n + 3$. Check whether it is an arithmetic sequence .

Answer

$$x_n = 5n + 3$$

$$x_1 = 5 \times 1 + 3 = 5 + 3 = 8$$

$$x_2 = 5 \times 2 + 3 = 10 + 3 = 13$$

$$x_3 = 5 \times 3 + 3 = 15 + 3 = 18$$

$$x_4 = 5 \times 4 + 3 = 20 + 3 = 23$$

$$x_5 = 5 \times 5 + 3 = 25 + 3 = 28$$

Sequence = 8 , 13 , 18 , 23 , 28 , . . .

Here the sequence start with 8 and adding 5 repeatedly . So it is an arithmetic sequence

Activity 3

Write the algebraic form of the arithmetic sequence $\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{9}{6}, \ldots$

Answer

$$d = \frac{7}{6} - \frac{5}{6} = \frac{2}{6}$$

$$x_n = dn + f - d$$

$$= \frac{2}{6} \times n + \frac{1}{2} - \frac{2}{6}$$

$$= \frac{2}{6}n + \frac{3}{6} - \frac{2}{6} \qquad \left[\frac{1}{2} = \frac{3}{6} \right]$$

$$= \frac{2}{6}n + \frac{1}{6} = \frac{2n + 1}{6}$$

Activity 4

Prove that the arithmetic sequence $\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{9}{6}, \ldots$ **contains no natural numbers .**

Answer

$$d = \frac{7}{6} - \frac{5}{6} = \frac{2}{6}$$

Algebraic form of the sequence $= dn + f - d$

$$= \frac{2}{6} \times n + \frac{1}{2} - \frac{2}{6}$$

$$= \frac{2}{6} n + \frac{3}{6} - \frac{2}{6} \qquad \left[\frac{1}{2} = \frac{3}{6} \right]$$

$$= \frac{2}{6} n + \frac{1}{6} = \frac{2n + 1}{6}$$

Here the numerator of the terms of this sequence are odd numbers . (odd numbers are got by adding 1 to the multiples of 2) and the denominator 6 is an even number .

Since all the multiples of 6 are even , this sequence does not contain any natural number .

Activity 5

Prove that the arithmetic sequence $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \ldots$ **contains all natural numbers .**

Answer

$$d = \frac{2}{7} - \frac{1}{7} = \frac{1}{7}$$

Algebraic form of the sequence $= dn + f - d$

$$= \frac{1}{7} \times n + \frac{1}{7} - \frac{1}{7} = \frac{1}{7} n + 0 = \frac{n}{7}$$

Here the numerator of the terms of this sequence are consecutive natural numbers . So the multiples of the denominator 7 come as the numerator of the terms . So this sequence contains all natural numbers .

Activity 6

Eighth term of an arithmetic sequence is 12 and its 12th term is 8 . Write the algebraic form of this sequence ?

Answer

$$Common\ \ difference = \frac{Term\ \ difference}{Position\ \ difference} = \frac{x_{12} - x_8}{12 - 8} = \frac{8 - 12}{4} = \frac{-4}{4} = -1$$

First term $= x_8 - 7d = 12 - 7 \times -1 = 12 + 7 = 19$

$$x_n = d\ n + f - d$$

$$= -1 \times n + 19 - (-1)$$

$$= -n + 19 + 1$$

$$= -n + 20$$

NOTE :

What is the 20th term of the sequence taken in the above activity ?

$$x_n = -n + 20$$

$$x_{20} = -20 + 20 = 0$$

(We may write $x_n = -n + 20$ **as** $x_n = 20 - n$ **)**

Activity 7

Prove that the squares of all the terms of the arithmetic sequence 4 , 7 , 10 , 13 , . . . belong to the sequence .

Answer

$$d = 7 - 4 = 3$$

$$x_n = d\,n + f - d$$

$$= 3 \times n + 4 - 3$$

$$= 3\,n + 1$$

$$(x_n)^2 = (3\,n + 1)^2 \qquad [\ (a+b)^2 = a^2 + b^2 + 2ab\ ]$$

$$= (3\,n)^2 + 1^2 + 2 \times 3n \times 1$$

$$= 9\,n^2 + 1 + 6\,n$$

$$(x_n)^2 - 4 = 9\,n^2 + 1 + 6\,n - 4$$

$$= 9\,n^2 + 6\,n + 1 - 4$$

$$= 9\,n^2 + 6\,n - 3$$

$$= 3 \times 3\,n^2 + 3 \times 2\,n - 3 \times 1$$

$$= 3\,(3\,n^2 + 2\,n - 1\,)$$

Here the difference between $(x_n)^2$ and 4 is divisible by the common difference .

(Difference is the multiple of the common difference) . So $(x_n)^2$ is a term of this sequence

That is , the squares of all the terms of this sequence belong to it .

Findings

- **If the algebraic form an arithmetic sequence is given , we can form the sequence or find any term .**
- **If an arithmetic sequence or any two terms of this sequence are given , we can find its algebraic form .**