Mathematics Online Class X On 08-07-2021

ARITHMETIC SEQUENCE

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In the previous class we studied
The algebraic form of an arithmetic sequence is of the form $x_n = an + b$, where a and b are fixed numbers and a is the common difference ;
conversely, any sequence of this form is an arithmetic sequence .
The algebraic form of an arithmetic sequence can also be written in the form $x_n = dn + (f-d)$
where f is the first term and d is the common difference .

Question

The algebraic form an arithmetic sequence is 5n + 3 .
Find the fist term and common difference .

Answer

Algebraic form = $x_n = 5n + 3$

First term = $x_1 = 5 \times 1 + 3 = 8$

Second term = $x_2 = 5 \times 2 + 3 = 13$

Third term = $x_3 = 5 \times 3 + 3 = 18$

∴ Arithmetic sequence is 8 , 13 , 18 , . . .

$$\text{Common difference } d = \left.\begin{matrix} 13 - 8 \\ 18 - 13 \\ \dots \end{matrix}\right\} = 5$$

Question

In the arithmetic sequence $\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{9}{6}, \dots$

i) Find the algebraic form of the sequence.

ii)Prove that this sequence contains no natural numbers.

Answer

Common difference d = $x_3 - x_2 = \frac{7}{6} - \frac{5}{6} = \frac{2}{6} = \frac{1}{3}$

Algebraic form = x_n = an + b Where a = d and b = f – d

$$a = d = \frac{1}{3}$$

$$b = f - d = \frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$

Algebraic form = $x_n = \frac{1}{3}n + \frac{1}{6} = \frac{2}{6}n + \frac{1}{6} = \frac{(2n+1)}{6}$

In this sequence each term contains numerator as an odd number and denominator as an even number .

Since odd numbers cannot have 2 as a factor ,

$\frac{Numerator}{Denominator} = \frac{Odd\ number}{Even\ number}$ cannot be a natural number .

∴ we get , the sequence contains no natural numbers .

Question

In the arithmetic sequence $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \ldots$

i) Find the algebraic form of the sequence.

ii)Prove that this sequence contains all natural numbers.

Answer

Common difference d = $x_2 - x_1 = \frac{2}{7} - \frac{1}{7} = \frac{1}{7}$

Algebraic form = x_n = an + b Where a = d and b = f – d

$$a = d = \frac{1}{7}$$

$$b = f - d = \frac{1}{7} - \frac{1}{7} = 0$$

Algebraic form = $x_n = \frac{1}{7}n + 0 = \frac{1}{7}n$

When n = 7 , 14 , 21 , 28 , 35 , . . . we get all natural numbers as terms of this sequence .

Question

The 8th term of an arithmetic sequence is 12 and 12th term is 8 .What is the algebraic expression for this sequence ? Find the 20th term ?

Answer

8th term = $x_8 = 12$

12th term = $x_{12} = 8$

We have Common difference = $\frac{\textit{Term difference}}{\textit{Position difference}}$

$$d = \frac{x_{12} - x_8}{12-8} = \frac{8-12}{12-8} = \frac{-4}{4} = -1$$

$$f = x_1 = x_8 - 7d = 12 - 7 \times -1 = 12 + 7 = 19$$

Algebraic form = $x_n = an + b$ Where $a = d$ and $b = f - d$

$a = d = -1$

$b = f - d = 19 - (-1) = 19 + 1 = 20$

Algebraic form = $x_n = -1n + 20 = 20 - n$

20th term = $20 - 20 = 0$

NOTE

If m th term of an arithmetic sequence is n and n th term is m . Then

i) Common difference $d = -1$

ii) $(m + n)^{th}$ term $= x_{(m+n)} = 0$

Question

Prove that the squares of all the terms of the arithmetic sequence 4 , 7 , 10 , . . .belong to the sequence .

Answer

Given arithmetic sequence is 4 , 7 , 10 , . . .

Common difference $d = x_2 - x_1 = 7 - 4 = 3$

$$\begin{array}{r|l} & 1 \\ 3 & 4 \\ & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} & 2 \\ 3 & 7 \\ & 6 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} & 3 \\ 3 & 10 \\ & 9 \\ \hline & 1 \end{array}$$

Here d = 3 and each term divided by 3 gives remainder 1 .

Now squares of the terms are $4^2 = 16$, $7^2 = 49$, $10^2 = 100$, . . .

$$\begin{array}{r} 5 \\ 3\,\overline{)\,16} \\ 15 \\ \hline 1 \end{array} \qquad \begin{array}{r} 16 \\ 3\,\overline{)\,49} \\ 48 \\ \hline 1 \end{array} \qquad \begin{array}{r} 33 \\ 3\,\overline{)100} \\ 99 \\ \hline 1 \end{array}$$

Here the squares of the terms of the sequence will also give remainder 1 when divided by 3.

From this we get the squares of all the terms of the arithmetic sequence 4 , 7 , 10 , . . .belongs to the sequence .

OR

Algebraic form = x_n = an + b Where a = d and b = f – d

$a = d = 3$

$b = f - d = 4 - 3 = 1$

Algebraic form = $x_n = 3n + 1$

That is each term of the sequence is 1 added to a multiple of 3 .

Now $x_n^2 = (3n + 1)^2 = 9n^2 + 6n + 1 = 3(3n^2 + 2n) + 1$ is also 1 added to a multiple of 3 .

From this we get the squares of all the terms of the arithmetic sequence 4 , 7 , 10 , . . .belongs to the sequence .