Mathematics Online Class X On 12-07-2021

ARITHMETIC SEQUENCE

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Question

Find the algebraic form of the sequence 5 , 9 , 13 , . . .

Answer

Given sequence is 5 , 9 , 13 , . . .

common difference = 4

5 , 9 , 13 , 17 , . . .

Multiples of 4 4 , 8 , 12 , 16 , . . .

Here we can see that all the terms of the sequence are 1 more than the multiples of 4 .

∴ Algebraic form of the sequence is $4n + 1$

- **Considering consecutive 3 natural numbers , we have**

  $1 + 2 + 3 = 6 = 3 \times 2$

  $2 + 3 + 4 = 9 = 3 \times 3$

  $3 + 4 + 5 = 12 = 3 \times 4$

We noted that the sum of any three consecutive natural numbers is thrice the middle number.

using algebra: in three consecutive natural numbers,
if we take the middle one as n , then the first number is $n - 1$ and the last one is $n + 1$, so that the sum is $(n - 1) + n + (n + 1) = 3n$

- **Considering consecutive 5 natural numbers , we have**

$1 + 2 + 3 + 4 + 5 = 15 = 5 \times 3$

using algebra: in five consecutive natural numbers,
if we take the middle one as n
Numbers are taken as $(n - 2)$, $(n - 1)$, n , $(n + 1)$, $(n + 2)$

$$\therefore \text{ Sum} = (n - 2) + (n - 1) + n + (n + 1) + (n + 2) = 2n + 2n + n = 5n$$

- Considering consecutive 7 natural numbers , we have

$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28 = 7 \times 4$

using algebra: in seven consecutive natural numbers, if we take the middle one as n . Numbers are taken as $(n - 3)$, $(n - 2)$, $(n - 1)$, n , $(n + 1)$, $(n + 2)$, $(n + 3)$

$\therefore$ Sum $=(n - 3) + (n - 2) + (n - 1) + n + (n + 1) + (n + 2) + (n + 3)$

$= 2n + 2n + 2n + n = 7n$

- If we take 3 consecutive even numbers ,

$2 + 4 + 6 = 12 = 3 \times 4$

$8 + 10 + 12 = 30 = 3 \times 10$

using algebra: in three consecutive even natural numbers , if we take the middle one as x , then the first even number is $x - 2$ and the last one is $x + 2$, so that the sum is $(x - 2) + x + (x + 2) = 3x$

- If we take 3 consecutive odd numbers,

$1 + 3 + 5 = 9 = 3 \times 3$

$7 + 9 + 11 = 27 = 3 \times 9$

using algebra: in three consecutive odd natural numbers , If we take the middle one as x , then the first odd number is $x - 2$ and the last one is $x + 2$, so that the sum is $(x - 2) + x + (x + 2) = 3x$

- If we take 3 consecutive multiples of 3 .That is 3 , 6 , 9

Sum $= 3 + 6 + 9 = 18 = 3 \times 6$

using algebra: in three consecutive multiples of 3 , If we take the middle one as x , then the first multiples of 3 is $x - 3$ and the last one is $x + 3$, so that the sum is $(x - 3) + x + (x + 3) = 3x$

- If we consider the sequence obtained by adding 1 to the multiples of 4 That is 5 , 9 , 13 , . . .

If we take 3 consecutive terms 5 , 9 , 13

$$\text{Sum} = 5 + 9 + 13 = 27 = 3 \times 9$$

using algebra: if we take the middle term as x , then the first term is $x - 4$ and the last term is $x + 4$, so that the sum is $(x - 4) + x + (x + 4) = 3x$

- If we consider three consecutive terms of an arithmetic sequence and let the common difference be y and the the middle term as x

  the first number is $x - y$ and the third one is $x + y$.

  $$\text{Sum} = x - y + x + x + y = 3x$$

- If we consider five consecutive terms of an arithmetic sequence and let the common difference be y and the the middle term as x

  Then terms are $x - 2y$, $x - y$, x , $x + y$, $x + 2y$

  $$\text{Sum} = x - 2y + x - y + x + x + y + x + 2y = 5x$$

  From above we get ,

  If we consider some odd number of consecutive terms of an arithmetic sequence , Sum = Number of terms × Middle term

- If we take 11 consecutive terms of an arithmetic sequence

  1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 , 10 , 11

  Here the middle term is the 6^{th} term .

  Now take the sum of terms equidistant from the middle terms we have the sum as

  $1^{st} + 11^{th} = 2^{nd} + 10^{th} = 3^{rd} + 9^{th} = 4^{th} + 8^{th} = 5^{th} + 7^{th}$

  From above we get ,

  If we consider some odd number of consecutive terms of an arithmetic sequence , there is a middle term and the sum of terms equidistant from the middle term will have equal sum.

Question

The sum of five consecutive terms of an arithmetic sequence is 500 . Can you write the sequence ?

Answer

Given sum = 500

Number of terms = 5 (odd number of terms)

We have Sum = Number of terms × Middle term

500 = 5 × Middle term

∴ Middle term = $\frac{500}{5}$ = 100

If d=0 sequence is 100 , 100 , 100 , 100 , 100

If d=1 sequence is 98 , 99 , 100 , 101 , 102

If d=2 sequence is 96 , 98 , 100 , 102 , 104

If d=5 sequence is 90 , 95 , 100 , 105 , 110

Question

The sum of five consecutive terms of an arithmetic sequence is 500 and first term is 10 . Can you write the sequence ?

Answer

Given First term = 10 , sum = 500 and Number of terms = 5 (odd)

We have Sum = Number of terms × Middle term

500 = 5 × Middle term

∴ Middle term = $\frac{500}{5}$ = 100

10, _ , 100 , _ , _

Common difference = $\frac{\textit{Term difference}}{\textit{Position difference}} = \frac{100-10}{3-1} = \frac{90}{2} = 45$

Sequence is 10 , 55 , 100 , 145 , 190

Now consider even number of terms

Consider First four terms of the arithmetic sequence 1 , 3 , 5 , 7 , . . .

Sum of first four terms = $\underbrace{1 + \overbrace{3 + 5}^{8} + 7}_{8}$ = 8 + 8 = 16

Consider ,

Six consecutive terms of the arithmetic sequence 1 , 3 , 5 , 7 , . . .

Sum of six consecutive terms = $\underbrace{1 + \overbrace{3 + \underbrace{5 + 7}_{12} + 9}^{12} + 11}_{12}$

$= 12 + 12 + 12 = 36$

If the first term of an arithmetic sequence is x and common difference is y , then sequence is x , $x+y$, $x+2y$, $x+3y$, . . .

Here 1^{st} term + 4^{th} term = $x + x+3y = 2x + 3y$

2^{nd} term + 3^{rd} term = $x+y + x+2y = 2x + 3y$

$\therefore$ 1^{st} term + 4^{th} term = 2^{nd} term + 3^{rd} term

We have the algebraic form of an arithmetic sequence is of the form $x_n = an + b$

1^{st} term + 10^{th} term = $a+b + 10a+b = 11a+2b$

2^{nd} term + 9^{th} term = $2a+b + 9a+b = 11a+2b$

3^{rd} term + 8^{th} term = $3a+b + 8a+b = 11a+2b$

4^{th} term + 7^{th} term = $4a+b + 7a+b = 11a+2b$

5^{th} term + 6^{th} term = $5a+b + 6a+b = 11a+2b$

From this we get ,

In an arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of pairs of the terms are also equal .

Question

The sum of fifth term and sixteenth term of an arithmetic sequence is 50 . Find the sum of first 20 terms ?

Answer

Given 5^{th} term + 16^{th} term = 50

We have to find the sum of first 20 terms . By pairing the terms we get 10 pairs with equal sums with equal sums of positions .

1^{st} term + 20^{th} term = 2^{nd} term + 19^{th} term = 3^{rd} term + 18^{th} term

= 4^{th} term + 17^{th} term = 5th term + 16th term = . . .

Sum = No.of pairs × One pair sum = $10 \times 50 = 500$

Question

The sum of first 5 terms of an arithmetic sequence is 100 . The sum of first 10 terms of an arithmetic sequence is 250. Find the sequence

Answer

Given sum of first 5 terms = 100

We have Sum = Number of terms × Middle term

$100 = 5 \times$ Third term

$\therefore$ Third term $= \frac{100}{5} = 20$

Sum of first ten terms = 250

10 terms can be paired as 5 pairs with equal sums with equal sums of positions . They are 1^{st} term + 10^{th} term , 2^{nd} term + 9^{th} term , 3^{rd} term + 8^{th} term , 4^{th} term + 7^{th} term , 5^{th} term + 6^{th} term

We have Sum = No.of pairs × One pair sum

$250 = 5 \times$ (3^{rd} term + 8^{th} term)

3^{rd} term + 8^{th} term $= \frac{250}{5} = 50$

$\therefore$ 8^{th} term $= 50 - 20 = 30$

Common difference $= \frac{\text{Term difference}}{\text{Position difference}} = \frac{30-20}{8-3} = \frac{10}{5} = 2$

1^{st} term = 3^{rd} term - 2 Common difference $= 20 - 2\times 2 = 20 - 4 = 16$

$\therefore$ Required sequence is 16 ,18 , 20 , 22 , 24 , 26 , 28 , 30 , 32 , 34