ONLINE MATHS CLASS - X - 10 (12 / 07 /2021)

1. ARITHMETIC SEQUENCE - CLASS 8

What did we study in the last class ?

- **If the first term of an arithmetic sequence is f and its common difference is d , then its n^{th} term is $dn + f - d$.**
- **The algebraic form of any arithmetic sequence is of the form $an + b$, where a and b are fixed numbers . a is the common difference .**
- $a = d$, $b = f - d$

Activity 1

Write the algebraic form of the arithmetic sequence 5 , 9 , 13 , . . .

Answer

$x_n = dn + f - d$

$= 4 \times n + 5 - 4$

$= 4n + 1$

$f = 5$

$d = 9 - 5 = 4$

(The terms of this sequence are got by adding 1 to the multiples of 4)

Activity 2 **(Sum of three consecutive natural numbers)**

	Middle number	Relation between the middle number and the sum
1 + 2 + 3 = 6	2	3 x 2 = 6
2 + 3 + 4 = 9	3	3 x 3 = 9
3 + 4 + 5 = 12	4	3 x 4 = 12
4 + 5 + 6 = 15	5	3 x 5 = 15
5 + 6 + 7 = 18	6	3 x 6 = 18

Activity 3

Prove that the sum of three consecutive natural numbers is three times the middle number

Answer

Take the middle number among three consecutive natural numbers as n .

Numbers = $n - 1$, n , $n + 1$

Sum of the numbers = $(n - 1) + n + (n + 1) = 3n$

= 3 x Middle number .

Activity 3 (Sum of five consecutive natural numbers)

	Middle term	Relation between the middle number and the sum
1 + 2 + 3 + 4 + 5 = 15	3	5 x 3 = 15
2 + 3 + 4 + 5 + 6 = 20	4	5 x 4 = 20
3 + 4 + 5 + 6 + 7 = 25	5	5 x 5 = 25
4 + 5 + 6 + 7 + 8 = 30	6	5 x 6 = 30
5 + 6 + 7 + 8 + 9 = 35	7	5 x 7 = 35

Activity 4

Prove that the sum of five consecutive natural numbers is five times the middle number

Answer

Take the middle number among five consecutive natural numbers as n .

Numbers = $n - 2$, $n - 1$, n , $n + 1$, $n + 2$

Sum of the numbers = $(n - 2) + (n - 1) + n + (n + 1) + (n + 2) = 5n$

= 5 x Middle number

Activity 5 (Sum of seven consecutive natural numbers)

Take the middle number among seven consecutive natural numbers as n .

Numbers = $n-3$, $n-2$, $n-1$, n , $n+1$, $n+2$, $n+3$

Sum of the numbers = $(n-3)+(n-2)+(n-1)+n+(n+1)+(n+2)+(n+3)$

= $7n$ = 7 x Middle number

Activity 6 (Sum of three consecutive even numbers)

	Middle term	Relation between the middle number and the sum
2 + 4 + 6 = 12	4	3 x 4 = 12
4 + 6 + 8 = 18	6	3 x 6 = 18
6 + 8 + 10 = 24	8	3 x 8 = 24
8 + 10 + 12 = 30	10	3 x 10 = 30
10 + 12 + 14 = 36	12	3 x 12 = 36

Take the middle number among three consecutive natural numbers as x .

Numbers = $x-2$, x , $x+2$

Sum of the numbers = $(x-2)+x+(x+2)$ = $3x$ = 3 x Middle term

Activity 7 (Sum of three consecutive odd numbers)

	Middle term	Relation between the middle number and the sum
1 + 3 + 5 = 9	3	3 x 3 = 9
3 + 5 + 7 = 15	5	3 x 5 = 15

5 + 7 + 9 = 21	7	3 x 7 = 21
7 + 9 + 11 = 27	9	3 x 9 = 27
9 + 11 + 13 = 33	11	3 x 11 = 33

Take the middle number among three consecutive natural numbers as x .

Numbers = $x - 2$, x , $x + 2$

Sum of the numbers = $(x - 2) + x + (x + 2) = 3x$ = 3 x Middle term

Activity 8 **(Sum of three consecutive multiples of three)**

Take the middle number among three consecutive multiples of three as x .

Numbers = $x - 3$, x , $x + 3$

Sum of the numbers = $(x - 3) + x + (x + 3) = 3x$ = 3 x Middle term

Activity 9 **(Sum of three consecutive terms of the arithmetic sequence 5 , 9 , 13 , . . .)**

Take the middle number among three consecutive terms of this sequence as x .

Terms = $x - 4$, x , $x + 4$

Sum of the terms = $(x - 4) + x + (x + 4) = 3x$

= 3 x Middle term

Activity 10 **(Sum of five consecutive terms of the arithmetic sequence 5 , 9 , 13 , . . .)**

Take the middle number among five consecutive terms of this sequence as x .

Terms = $x - 8$, $x - 4$, x , $x + 4$, $x + 8$

Sum of the terms = $(x - 8) + (x - 4) + x + (x + 4) + (x + 8) = 5x$

= 5 x Middle term

Activity 11

Prove that the sum of any three consecutive terms of an arithmetic sequence is three times the middle term .

Answer

Take the middle term of three consecutive terms of an arithmetic sequence with common difference y as x .

Terms = $x - y$, x , $x + y$

Sum of the terms = $(x - y) + x + (x + y) = 3x$ = 3 x Middle term

Activity 12

Prove that the sum of any five consecutive terms of an arithmetic sequence is five times the middle term .

Answer

Take the middle term of five consecutive terms of an arithmetic sequence with common difference y as x .

Terms = $x - 2y$, $x - y$, x , $x + y$, $x + 2y$

Sum of the terms = $(x - 2y) + (x - y) + x + (x + y) + (x + 2y) = 5x$

= 5 x Middle term

Activity 13

Prove that the sum of any seven consecutive terms of an arithmetic sequence is seven times he middle term .

Answer

Take the middle term of seven consecutive terms of an arithmetic sequence with common difference y as x .

Terms = $x - 3y$, $x - 2y$, x – y , x , $x + y$, $x + 2y$, $x + 3y$

Sum of the terms = $(x – 3y) + (x – 2y) + (x – y) + x + (x + y) + (x + 2y) + (x + 3y)$

= $7x$ = 7 x Middle term

Findings

- The sum of any three consecutive terms of an arithmetic sequence is three times the middle term .
- The sum of any five consecutive terms of an arithmetic sequence is five times the middle term .
- The sum of any seven consecutive terms of an arithmetic sequence is seven times the middle term .

Conclusion

If n is an odd number , then the sum of n consecutive terms of an arithmetic sequence is n times the middle term .

Activity 14

The sum of first three consecutive terms of an arithmetic sequence is 12 . Calculate its second term .

Answer

Sum of first three consecutive terms = 3 x Middle term = 3 x Second term

3 x Second term = 12

Second term = $\frac{12}{3} = 4$

Activity 15

Fifth term of an arithmetic sequence is 10 . Calculate the sum of first 9 terms of this sequence .

Answer

Sum of first 9 consecutive terms = 9 x Middle term

= 9 x Ffth term

= 9 x 10 = 90

Activity 16

First term of an arithmetic sequence is 10 and the sum of first five terms of this sequence is 500 . Write down the sequence .

Answer

Sum of first 5 consecutive terms = 5 x Middle term = 5 x Third term

5 x Third term = 500

$$\text{Third term} = \frac{500}{5} = 100$$

$$Common\ \ difference = \frac{Term\ \ difference}{Position\ \ difference} = \frac{x_3 - x_1}{3 - 1} = \frac{100 - 10}{2} = \frac{90}{2} = 45$$

Sequence = 10 , 55 , 100 , 145 , . . .

Activity 17 (Three consecutive terms)

	Middle term	First term + Last term
1 , 2 , 3	2	1 + 3 = 4
2 , 4 , 6	4	2 + 6 = 8
1 , 3 , 5	3	1 + 5 = 6
7 , 11 , 15	11	7 + 15 = 22
9 , 14 , 19	14	9 + 19 = 28

The sum of three consecutive terms of an arithmetic sequence is three times the middle term . So the sum of the first and the last must be twice the middle .

In three consecutive terms of any arithmetic sequence , the middle term is half the sum of the first and the last .

Activity 18 (**Five consecutive terms**)

	Middle term	$x_1 + x_5$	$x_2 + x_4$
1 , 2 , 3 , 4 , 5	3	1 + 5 = 6	2 + 4 = 6
2 , 4 , 6 , 8 , 10	6	2 + 10 = 12	4 + 8 = 12
1 , 3 , 5 , 7 , 9	5	1 + 9 = 10	3 + 7 = 10
5 , 8 , 11 , 14 , 17	11	5 + 17 = 22	8 + 14 = 22
3 , 10 , 17 , 24 , 31	17	3 + 31 = 34	10 + 24 = 34

Findings

- **In five consecutive terms of any arithmetic sequence ,**

$$x_1 + x_5 = x_2 + x_4$$

- **In five consecutive terms of any arithmetic sequence , the sum of the pairs of terms equidistant from the centre are equal .**
- **In five consecutive terms of any arithmetic sequence , the sums of the pairs of terms equidistant from the centre are twice the middle term .**
- **In five consecutive terms of any arithmetic sequence , the middle term is half the sum of the pairs of terms equidistant from the centre .**
- **In five consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .**

NOTE :

In five consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .

We can prove this result using algebra .

Take the middle term of five consecutive terms of an arithmetic sequence with common difference d as c .

Terms = $c - 2d$, $c - d$, c , $c + d$, $c + 2d$

$x_1 + x_5 = (c - 2d) + (c + 2d) = 2c$

$x_2 + x_4 = (c - d) + (c + d) = 2c$

Activity 19 (**Seven consecutive terms**)

	Middle term	$x_1 + x_7$	$x_2 + x_6$	$x_3 + x_5$
1 , 2 , 3 , 4 , 5 , 6 , 7	4	1 + 7 = 8	2 + 6 = 8	3 + 5 = 8
2 , 4 , 6 , 8 , 10 , 12 ,14	8	2 + 14 =16	4 + 12 = 16	6 + 10 = 16
1 , 3 , 5 , 7 , 9 , 11 , 13	7	1 +13 = 14	3 + 11 = 14	5 + 9 = 14
5 , 8 , 11 , 14 , 17 ,20, 23	14	5 + 23 = 28	8 + 20 = 28	11 + 17 = 28
3 , 10 , 17, 24 , 31 ,38 , 45	24	3 + 45 = 48	10 +38 = 48	17 + 31 =48

Findings

- **In seven consecutive terms of any arithmetic sequence ,**

$$x_1 + x_7 = x_2 + x_6 = x_3 + x_5$$

- **In seven consecutive terms of any arithmetic sequence , the sum of the pairs of terms equidistant from the centre are equal .**
- **In seven consecutive terms of any arithmetic sequence , the sums of the pairs of terms equidistant from the centre are twice the middle term .**
- **In seven consecutive terms of any arithmetic sequence , the middle term is half the sum of the pairs of terms equidistant from the centre .**
- **In seven consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .**

NOTE :

In seven consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .

We can prove this result using algebra .

Take the middle term of seven consecutive terms of an arithmetic sequence with common difference d as c .

Terms $= c-3d \;,\; c-2d \;,\; c-d \;,\; c \;,\; c+d \;,\; c+3d$

$x_1 + x_7 = (c-3d) + (c+3d) = 2c$

$x_2 + x_6 = (c-2d) + (c+2d) = 2c$

$x_3 + x_5 = (c-d) + (c+d) = 2c$

Activity 20 (Nine consecutive terms)

Take the middle term of nine consecutive terms of an arithmetic sequence with common difference d as c .

Terms $= c-4d \;,\; c-3d \;,\; c-2d \;,\; c-d \;,\; c \;,\; c+d \;,\; c+3d \;,\; c+4d$

$$x_1 + x_9 = (c - 4d) + (c + 4d) = 2c$$

$$x_2 + x_8 = (c - 3d) + (c + 3d) = 2c$$

$$x_3 + x_7 = (c - 2d) + (c + 2d) = 2c$$

$$x_4 + x_6 = (c - d) + (c + d) = 2c$$

Findings

- **In nine consecutive terms of any arithmetic sequence ,**

  $$x_1 + x_9 = x_2 + x_8 = x_3 + x_7 = x_4 + x_6$$

- **In nine consecutive terms of any arithmetic sequence , the sum of the pairs of terms equidistant from the centre are equal .**
- **In nine consecutive terms of any arithmetic sequence , the sums of the pairs of terms equidistant from the centre are twice the middle term .**
- **In nine consecutive terms of any arithmetic sequence , the middle term is half the sum of the pairs of terms equidistant from the centre .**
- **In nine consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .**

Conclusion

In an odd number of consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .

Activity 21 (Four consecutive terms)

	$x_1 + x_4$	$x_2 + x_3$
1 , 2 , 3 , 4	1 + 4 = 5	2 + 3 = 5
2 , 4 , 6 , 8	2 + 8 = 10	4 + 6 = 10
1 , 3 , 5 , 7	1 + 7 = 8	3 + 5 = 8
5 , 8 , 11 , 14	5 + 14 = 19	8 + 11 = 19
3 , 10 , 17 , 24	3 + 24 = 27	10 + 17 = 27

Findings

- In four consecutive terms of any arithmetic sequence ,

$$x_1 + x_4 = x_2 + x_3$$

- In four consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .

NOTE :

In four consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .

We can prove this result using algebra .

Take the first term of four consecutive terms of an arithmetic sequence with common difference d as c .

Terms = c , $c + d$, $c + 2d$, $c + 3d$

$$x_1 + x_4 = c + (c + 3d) = 2c + 3d$$

$$x_2 + x_3 = (c + d) + (c + 2d) = 2c + 3d$$

Activity 22 (Six consecutive terms)

	$x_1 + x_6$	$x_2 + x_5$	$x_3 + x_4$
1 , 2 , 3 , 4 , 5 , 6	1 + 6 = 7	2 + 5 = 7	3 + 4 = 7
2 , 4 , 6 , 8 , 10 , 12	2 + 12 = 14	4 + 10 = 14	6 + 8 = 14
1 , 3 , 5 , 7 , 9 , 11	1 + 11 = 12	3 + 9 = 12	5 + 7 = 12
5 , 8 , 11 , 14 , 17 ,20	5 + 20 = 25	8 + 17 = 25	11 + 14 = 25
3 , 10 , 17, 24 , 31 ,38	3 + 38 = 41	10 + 31 = 41	17 + 24 = 41

Findings

- In six consecutive terms of any arithmetic sequence ,

$$x_1 + x_6 = x_2 + x_5 = x_3 + x_4$$

- In six consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .

NOTE :

In six consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of termsare equal , then the sums of the pairs of the terms are equal .

We can prove this result using algebra .

Take the first term of six consecutive terms of an arithmetic sequence with common difference d as c .

Terms = c , $c + d$, $c + 2d$, $c + 3d$, $c + 4d$, $c + 5d$

$x_1 + x_6 = c + (c + 5d) = 2c + 5d$

$x_2 + x_5 = (c + d) + (c + 4d) = 2c + 5d$

$x_3 + x_4 = (c + 2d) + (c + 3d) = 2c + 5d$

Activity 23 (Eight consecutive terms)

Take the first term of four consecutive terms of an arithmetic sequence with common difference d as c .

Terms = c , $c + d$, $c + 2d$, $c + 3d$, $c + 4d$, $c + 5d$, $c + 6d$, $c + 7d$

$x_1 + x_8 = c + (c + 7d) = 2c + 7d$

$x_2 + x_7 = (c + d) + (c + 6d) = 2c + 7d$

$x_3 + x_6 = (c + 2d) + (c + 5d) = 2c + 7d$

$x_4 + x_5 = (c + 3d) + (c + 4d) = 2c + 7d$

Findings

- In eight consecutive terms of any arithmetic sequence ,

$$x_1 + x_8 = x_2 + x_7 = x_3 + x_6 = x_4 + x_5$$

- In eight consecutive terms of any arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .

Conclusion

In an arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .