Mathematics Online Class X On 14-07-2021

ARITHMETIC SEQUENCE

Click 

When we consider some consecutive terms of an arithmetic sequence ,if the number of terms is odd ,

Sum of all terms = Number of terms × Middle term

Also the sum of terms equidistant from the middle term will have equal sum.

SUMS

1 + 2 = 3

1 + 2 + 3 = 3 × 2 = 6

1 + 2 + 3 + 4 = 2 × (1+4) =2 × 5 = 10

1 + 2 + 3 + 4 + 5 = 5 × 3 = 15

1 + 2 + 3 + 4 + 5 + 6 = 3 × (1+6) = 3 × 7 = 21

1 + 2 + 3 + 4 + 5 + 6 + 7 = 7 × 4 = 28

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 4 × (1+8) = 4 × 9 = 36

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 9 × 5 = 45

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 5 × (1+10) = 5 × 11 = 55

If number of terms is odd ,

Sum = Number of terms × Middle term

If number of terms is even ,

Sum = No.of pairs × One pair sum

Question

Find the sum of first 20 natural numbers

Answer

1 + 2 + 3 + 4 + . . . + 19 + 20

Here the number of terms is even ,

Sum = No.of pairs × One pair sum

= 10 × (1+ 20) = 10 × 21

= 210

Question

Find the sum of first 25 natural numbers

Answer

1 + 2 + 3 + 4 + . . . + 24 + 25

Here the number of terms is odd ,

Sum = Number of terms × Middle term

= 25 × 13

= 325

Question

Find the sum of first 50 natural numbers

Answer

1 + 2 + 3 + 4 + . . . + 49 + 50

Here the number of terms is even ,

Sum = No.of pairs × One pair sum

= 25 × (1 + 50) = 25 × 51

= 1275

When we consider first n natural numbers ,

- If n is even , there are $\frac{n}{2}$ pairs

$$\therefore \text{sum} = \frac{n}{2} \times (n+1)$$

- If n is odd , there is a middle term and middle term = $\frac{n+1}{2}$

$$\therefore \text{sum} = n \times \frac{(n+1)}{2}$$

That is ,

The sum of any number of consecutive natural numbers , starting with one , is half the product of the last number and the next natural number .

Question

Find the sum of first 100 natural numbers

Answer

1 + 2 + 3 + 4 + . . . + 99 + 100

$$= 100 \times \frac{(100+1)}{2} = 50 \times 101 = 5050$$

Question

Find the sum of first 100 even natural numbers

Answer

$2 + 4 + 6 + 8 + \ldots + 198 + 200$

$= 2 (1 + 2 + 3 + 4 + \ldots + 99 + 100)$

$= 2 \times 5050$

$= 10100$

Question

Find the sum of first 100 multiples of 3

Answer

$3 + 6 + 9 + 12 + \ldots + 300$

$= 3 (1 + 2 + 3 + 4 + \ldots + 99 + 100)$

$= 3 \times 5050$

$= 15150$

Question

Find the sum of first 100 multiples of 5

Answer

$5 + 10 + 15 + 20 + \ldots + 500$

$= 5 (1 + 2 + 3 + 4 + \ldots + 99 + 100)$

$= 5 \times 5050$

$= 25250$

Question

Find the sum of first 100 terms of the arithmetic sequence

$6 + 11 + 16 + \ldots$

Answer

Common difference = 5

Algebraic form = $5n + 1$

We have $5 + 10 + 15 + 20 + \ldots + 500 = 25250$

$6 + 11 + 16 + 21 + \ldots + 501 = (5 + 1) + (10 + 1) + (15 + 1) + \ldots + (500 + 1)$

$= (5 + 10 + 15 + 20 + \ldots + 500) + (1 + 1 + 1 + \ldots + 1)$

$= 25250 + 100 = 25350$

(100 terms)

Question

Find the sum of first 100 terms of the arithmetic sequence

4 + 9 + 14 + . . .

Answer

Common difference = 5

Algebraic form = 5 n – 1

We have 5 + 10 + 15 + 20 + . . . + 500 = 25250

4 + 9 + 14 + . . . + 499

$= (5 -1) + (10 - 1) + (15 - 1) + \ldots + (500 -1)$

$= (5 + 10 + 15 + 20 + \ldots + 500) - \underbrace{(1 + 1 + 1 + \ldots + 1)}_{100 \text{ terms}}$

$= 5 \times \frac{100(100+1)}{2} - 100$

$= 5 \times 5050 - 100$

$= 25250 - 100 = 25150$

Question

The algebraic form of an arithmetic sequence is 10n – 4 .

find the sum of first 20 terms ?

Answer

Here $x_n = 10n - 4$

Sum of first 20 terms $= (10\times1-4)+(10\times2-4)+(10\times3-4) + \ldots+(10\times20-4)$

$= 10 (1 + 2 +3 + \ldots + 20) - \underbrace{(4 + 4 + 4 + \ldots + 4)}_{20 \text{ terms}}$

$= 10 \times \frac{20(20+1)}{2} - 4 \times 20$

$= 10 \times 210 - 80 = 2100 - 80 = 2020$

Question

The algebraic form of an arithmetic sequence is an + b .

find the sum of first n terms ?

Answer

Here $x_n = an + b$

Sum of first n terms $=(a\times1+b)+(a\times2+b)+(a\times3+b) + \ldots+(a\times n+b)$

$= a (1 + 2 +3 + \ldots + n) + \underbrace{(b + b + b + \ldots + b)}_{n \text{ terms}}$

$= a \times \frac{n(n+1)}{2} + bn$

Question

Calculate the difference between the sums of the first 20 terms of the arithmetic sequences 2 , 9 , 16 , . . . and 5 , 12 , 19 , . . .

Answer

Difference between the sums of first 20 terms

$$\begin{array}{l} 5 + 12 + 19 + \ldots \text{ 20 terms} \quad - \\ 2 + 9 + 16 + \ldots \text{ 20 terms} \\ \hline 3 + 3 + 3 + \ldots \text{ 20 terms} \quad = 3 \times 20 = 60 \end{array}$$

Question

What is the difference between the sum of the first 10 terms and the next 10 terms of the arithmetic sequence 7 , 11 , 15 , . . . ?

Answer

Given sequence is 7 , 11 , 15 , . . .

common difference = d = 4

Difference between the sums of the first 10 terms and the next 10 terms

$$\begin{array}{l} 11^{th} + 12^{th} + 13^{th} + \ldots + 20^{th} \quad - \\ 1^{st} + 2^{nd} + 3^{rd} + \ldots + 10^{th} \\ \hline 10d + 10d + 10d + \ldots + 10d \quad = 10 \times 10d = 10^2 d = 100 \times 4 = 400 \end{array}$$

Question

The common difference of an arithmetic sequence is 6 . The sum of first 20 terms is 1300 .Write the sequence?

Answer

Given common difference = 6

∴ Algebraic form $x_n = 6n + b$

sum of first n terms = $6 \times \frac{n(n+1)}{2} + b \times n$

sum of first 20 terms = $6 \times \frac{20(20+1)}{2} + b \times 20 = 1300$

$$6 \times 210 + 20b = 1300$$
$$1260 + 20b = 1300$$
$$20b = 1300 - 1260 = 40$$
$$b = \frac{40}{20} = 2$$

∴ Algebraic form of the sequence is 6n + 2

Sequence is 8 , 14 , 20 , . . .

ASSIGNMENT

The common difference of an arithmetic sequence is 6 . The sum of first 20 terms is 1200 .Write the sequence?