Mathematics Online Class X On 15-07-2021

ARITHMETIC SEQUENCE

Click 

Question on the previous class with answer

The common difference of an arithmetic sequence is 6 . The sum of first 20 terms is 1200 .Write the sequence?

Answer

Given common difference = 6 $\quad\therefore$ Algebraic form $x_n = 6n + b$

sum of first n terms $= 6 \times \frac{n(n+1)}{2} + b \times n$

sum of first 20 terms $= 6 \times \frac{20(20+1)}{2} + b \times 20 = 1200$

$6 \times 210 + 20b = 1200$

$1260 + 20b = 1200$

$20b = 1200 - 1260 = -60$

$b = \frac{-60}{20} = -3$

$\therefore$ Algebraic form of the sequence is $6n - 3$

Sequence is 3 , 9 , 15 , . . .

Question

Find the algebraic form of the sum of first n even natural numbers?

Answer

We have $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

$2 + 4 + 6 + \ldots + 2n = 2\,(1 + 2 + 3 + \ldots + n) = 2 \times \frac{n(n+1)}{2}$

$= n\,(n + 1)$

$= n^2 + n$

Question

Find the sum of first 10 even numbers and sum of first 25 even numbers?

Answer

Sum of first 10 even numbers $= 10\,(10 + 1) = 100 + 10 = 110$

Sum of first 25 even numbers $= 25\,(25 + 1) = 625 + 25 = 650$

Question

Find the sum of first 10 odd numbers ?

Answer

Here number of terms = 10 [even]

∴ Sum = No.of pairs × One pair sum

10^{th} odd number = 2 × 10-1 = 20-1 = 19

Sum of first 10 odd numbers

$$= \frac{10}{2} \times (1 + 19) = 5 \times 20 = 100$$

Question

Find the sum of first 25 odd numbers ?

Answer

Here number of terms = 25 [odd]

Sum = Number of terms × Middle term

25^{th} odd number = 2 × 25-1 = 50-1 = 49

Middle odd number = $\frac{(49+1)}{2} = 25$

Sum of first 25 odd numbers

$$= 25 \times 25 = 625$$

Question

Find the algebraic form of the sum of first n odd natural numbers?

Answer

Here number of terms = n

n^{th} odd number = (2n – 1)

$$1 + 3 + 5 + \ldots + (2n - 1) = \frac{n}{2} \times [1 + (2n - 1)] = \frac{n}{2} \times 2n = n^2$$

Question

Find the sum of n consecutive terms of the arithmetic sequence 3 , 6 , 9 , 12 , . . .

Answer

$$3 + 6 + 9 + \ldots + 3n = 3 (1 + 2 + 3 + \ldots + n)$$

$$= 3 \frac{n(n+1)}{2}$$

$$= \frac{3}{2} n^2 + \frac{3}{2} n$$

Question

Find the sum of n consecutive terms of the arithmetic sequence
4 , 8 , 12 , 16 , . . .

Answer

$$4 + 8 + 12 + \ldots + 4n = 4\,(1 + 2 + 3 + \ldots + n)$$
$$= 4\,\frac{n(n+1)}{2}$$
$$= 2n^2 + 2n$$

Question

Find the sum of n consecutive terms of the arithmetic sequence obtained by adding 1 to the multiples of 4 ?

Answer

Sequence 5 , 9 , 13 , . . . , (4n + 1)

$$\text{Sum} = 5 + 9 + 13 + \ldots + (4n + 1)$$
$$= 4\times1+1 + 4\times2+1 + 4\times3+1 + \ldots + 4\times n+1$$
$$= 4(1 + 2 + 3 + \ldots + n) + \underbrace{(1+1+1+\ldots+1)}_{n \text{ terms}}$$
$$= 4\,\frac{n(n+1)}{2} + 1\times n$$
$$= 2n^2 + 2n + n$$
$$= 2n + 3n$$

Question

The algebraic form of an arithmetic sequence is an + b .
find the sum of first n terms ?

Answer

Here $x_n = an + b$ where a = common difference & a+b = first term

$$\text{Sum of first n terms} = (a\times1+b)+(a\times2+b)+(a\times3+b) + \ldots+(a\times n+b)$$
$$= a\,(1 + 2 + 3 + \ldots + n) + \underbrace{(b + b + b + \ldots + b)}_{n \text{ terms}}$$
$$= a \times \frac{n(n+1)}{2} + bn$$
$$= \frac{an^2}{2} + \frac{an}{2} + bn$$
$$= \frac{a}{2}n^2 + \left(\frac{a}{2} + b\right)n$$

From this we get ,

Algebraic form of the sum of an arithmetic sequence is $pn^2 + qn$

where $p = \frac{a}{2}$ = half of common difference and

$p + q = \frac{a}{2} + \frac{a}{2} + b = a + b$ = first term .

Question

The algebraic form of the sum of an arithmetic sequence is $3n^2 + 4n$

Find the algebraic form of the sequence ?

Answer

Given sum of first n terms = $3n^2 + 4n$

when n = 1 , sum of first 1 term = first term = $3(1)^2 + 4(1) = 3+4 = 7$

when n = 2 , sum of first 2 terms = $3(2)^2 + 4(2) = 12 + 8 = 20$

first term + second term = 20

7 + second term = 20

second term = 20 – 7 = 13

∴ Common difference = 13 – 7 = 6

Sequence is 7 , 13 , 19 , . . .

Algebraic form of the sequence is 6n + 1

Question

1

2 3

4 5 6

7 8 9 10

.....

.....

(i) write the next two lines of the pattern above

(ii) write the first and last numbers of the tenth line

(iii) find the sum of all the numbers in the tenth line

Answer

(i) Next two lines of the above pattern are

11 , 12 , 13 , 14 , 15

16 , 17 , 18 , 19 , 20 , 21

(ii) First line contains 1 number

Second line contains 2 numbers

Third line contains 3 numbers

Continue like this tenth line contains 10 numbers

1

2 3 = 1 + 2

4 5 6 = 1+ 2 + 3

7 8 9 10 = 1 + 2 + 3 + 4

11 12 13 14 15 = 1 + 2 + 3 + 4 + 5

16 17 18 19 20 21 = 1 + 2 + 3 + 4 + 5 + 6

From this ,

we get last number in the 10^{th} line =1 + 2 + 3 + . . . + 10 = 55

first number in the 10^{th} line = 55 – 9 = 46

(iii) Numbers in tenth line are 46 , 47 , 48 , . . . , 55

Sum of all numbers in the 10^{th} line = No.of pairs × One pair sum

$= \frac{10}{2}(46 + 55)$

$= 5 \times 101$

$= 505$

Question

3

7 11

15 19 23

27 31 35 39

.....

.....

(i) How many numbers will be there in the tenth line

(ii) Last number in the tenth line

(iii) First number in the tenth line

(iv) Sum of all numbers in the tenth line

Answer

(i) Tenth line contains 10 numbers

(ii)

Terms triangle	Position triangle
3 = 4×1- 1	1
4×2- 1 = 7 11 = 4×3- 1	2 3
15 19 23	4 5 6
27 31 35 39	7 8 9 10

From above we can see that each number of the given pyramid is obtained by subtracting 1 from the multiples of 4 .

using position triangle last number in the tenth line is the 55^{th} term .

Last number in the tenth line $= 4 \times 55 - 1 = 220 - 1 = 219$

(iii) using position triangle first number in the tenth line is the 46^{th} term .

First number in the tenth line $= 4 \times 46 - 1 = 184 - 1 = 183$

(iv) Sum of all numbers in the 10^{th} line = No.of pairs × One pair sum

$$= \frac{10}{2} (183 + 219)$$

$$= 5 \times 402$$

$$= 2010$$