ONLINE MATHS CLASS - X - 11 (14 / 07 /2021)

1. ARITHMETIC SEQUENCE - CLASS 9

What did we study in the last class ?

- **If n is an odd number , then the sum of n consecutive terms of an arithmetic sequence is n times the middle term .**
- **In an odd number of consecutive terms of any arithmetic sequence , the sums of the pairs of terms equidistant from the centre are twice the middle term .**
- **In an arithmetic sequence , if the sums of positions of two pairs of terms are equal , then the sums of the pairs of the terms are equal .**

Activity1

7^{th} term of an arithmetic sequence is 15 . Calculate the sum of first 13 terms of this sequence

Answer

Sum of first 13 terms = 13 x Middle term

= 13 x 7^{th} term

= 13 x 15 = 195

Activity 2

The sum of 4^{th} and 14^{th} terms of an arithmetic sequence is 56 .

a) Find the sum of first and 17^{th} terms of this sequence .

b) What is its 9^{th} term ?

c) Calculate the sum of 17 terms of this sequence .

Answer

$x_4 + x_{14} = 56$

a) $x_1 + x_{17} = 56$

b) $x_9 = \frac{x_1 + x_{17}}{2} = \frac{56}{2} = 28$

c) Sum of first 17 terms = 17 x Middle term

$= 17 \times x_9 = 17 \times 28 = 476$

Activity 3

The sum of first and 12th terms of an arithmetic sequence is 69 .

a) Find the sum of second and 11th terms of this sequence .

b) Find the sum of third and 10th terms of this sequence .

c) Find the sum of 4th and 9th terms of this sequence .

d) Find the sum of 5th and 8th terms of this sequence .

e) Find the sum of 6th and 7th terms of this sequence .

f) Calculate the sum of 12 terms of this sequence .

Answer

$x_1 + x_{12} = 69$

a) $x_2 + x_{11} = 69$

b) $x_3 + x_{10} = 69$

c) $x_4 + x_9 = 69$

d) $x_5 + x_8 = 69$

e) $x_6 + x_7 = 69$

f) Sum of first 12 terms = $6 \times 69 = 414$ (Total 6 pairs)

Activity 4 (Sum of a fixed number of natural numbers starting from 1)

Case 1 (Number is odd)

	Sum	
1 + 2 + 3	3 x Middle number	3 x 2 = 6
1 + 2 + 3 + 4 + 5	5 x Middle number	5 x 3 = 15
1 + 2 + 3 + 4 + 5 + 6 + 7	7 x Middle number	7 x 4 = 28
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	9 x Middle number	9 x 5 = 45
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11	11 x Middle number	11 x 6 = 66

NOTE :

Number of terms (starting from 1)	Middle number
3	2
5	3
11	6
25	13
n , an odd number	$\frac{n+1}{2}$

$$1 + 2 + 3 + \ldots + 15 = 15 \times \text{Middle term} = 15 \times \left(\frac{15+1}{2}\right) = 15 \times \frac{16}{2} = 15 \times 8 = 120$$

$$1 + 2 + 3 + \ldots + 25 = 25 \times \text{Middle term} = 25 \times \left(\frac{25+1}{2}\right) = 15 \times \frac{26}{2} = 25 \times 13 = 325$$

$$1 + 2 + 3 + \ldots + 31 = 31 \times \text{Middle term} = 31 \times \left(\frac{31+1}{2}\right) = 31 \times \frac{32}{2} = 31 \times 16 = 496$$

$$1 + 2 + 3 + \ldots + 49 = 49 \times \text{Middle term} = 49 \times \left(\frac{49+1}{2}\right) = 49 \times \frac{50}{2} = 49 \times 25 = 1225$$

If n is an odd number ,

$$1 + 2 + 3 + \ldots + n = n \times \text{Middle term} = n \times \left(\frac{n+1}{2}\right) = \frac{n\,(n+1)}{2}$$

Finding

If n is an odd number , $1 + 2 + 3 + \ldots + n = \dfrac{n\,(n+1)}{2}$

Case 2 (Number is even)

	Number of pairs of terms with equal sum of positions	Pair sum	Sum of the terms
1 + 2 + 3 + 4	2	5	2 x 5 = 10
1 + 2 + 3 + 4 + 5 + 6	3	7	3 x 7 = 21
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	4	9	4 x 9 = 36
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10	5	11	5 x 11 = 55
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12	6	13	6 x 13 = 78

NOTE :

1. If the first 12 consecutive terms of an arithmetic sequence are given ,

$$x_1 + x_{12} = x_2 + x_{11} = x_3 + x_{10} = x_4 + x_9 = x_5 + x_8 = x_6 + x_7$$

2.

Number of terms (starting from 1)	Number of pairs of terms with equal sum of positions
4	2
6	3
10	5
50	25
n , an even number	$\frac{n}{2}$

$$1 + 2 + 3 + \ldots + 16 = \frac{16}{2} \times Sum\ of\ a\ pair\ of\ terms = \frac{16}{2} \times (1+16) = 8 \times 17 = 136$$

$$1 + 2 + 3 + \ldots + 20 = \frac{20}{2} \times Sum\ of\ a\ pair\ of\ terms = \frac{20}{2} \times (1+20) = 10 \times 21 = 210$$

$$1 + 2 + 3 + \ldots + 30 = \frac{30}{2} \times Sum\ of\ a\ pair\ of\ terms = \frac{30}{2} \times (1+30) = 15 \times 30 = 450$$

$$1 + 2 + 3 + \ldots + 50 = \frac{50}{2} \times Sum\ of\ a\ pair\ of\ terms = \frac{50}{2} \times (1+50) = 25 \times 51 = 1275$$

If n is an even number ,

$$1 + 2 + 3 + \ldots + n = \frac{n}{2} \times Sum\ of\ a\ pair\ of\ terms = \frac{n}{2} \times (1+n) = \frac{n\ (1+n)}{2}$$

$$= \frac{n\ (n+1)}{2}$$

Finding

If n is an even number , $1 + 2 + 3 + \ldots + n = \frac{n\,(n+1)}{2}$

Conclusion

The sum of any number of consecutive natural numbers , starting with one , is half the product of the last number and the next natural number .

That is , $1 + 2 + 3 + \ldots + n = \frac{n\,(n+1)}{2}$

Activity 5

Compute the following sums .

a) $1 + 2 + 3 + \ldots + 100$

b) $2 + 4 + 6 + \ldots + 200$

c) $3 + 6 + 9 + \ldots + 300$

d) $4 + 8 + 12 + \ldots + 400$

Answer

a) $1 + 2 + 3 + \ldots + 100 = \frac{100 \times 101}{2} = 5050$

b) $2 + 4 + 6 + \ldots + 200 = 2(1 + 2 + 3 + \ldots + 100) = 2 \times 5050 = 10100$

c) $3 + 6 + 9 + \ldots + 300 = 3(1 + 2 + 3 + \ldots + 100) = 3 \times 5050 = 15150$

d) $4 + 8 + 12 + \ldots + 400 = 4(1 + 2 + 3 + \ldots + 100) = 4 \times 5050 = 20200$

Activity 6

Compute the following sums .

a) $1 + 2 + 3 + \ldots + 100$

b) $5 + 10 + 15 + \ldots + 500$

c) $6 + 11 + 16 + \ldots + 501$

Answer

a) $1 + 2 + 3 + \ldots + 100 = \frac{100 \times 101}{2} = 5050$

b) $5 + 10 + 15 + \ldots + 500 = 5(1 + 2 + 3 + \ldots + 100) = 5 \times 5050 = 25250$

c) $6 + 11 + 16 + \ldots + 501 = 25250 + (100 \times 1) = 25250 + 100 = 25350$

(Here the terms of the arithmetic sequence $6, 11, 16, \ldots, 501$ **are got by adding 1 to the terms of the arithmetic sequence** $5, 10, 15, \ldots, 500$ **)**

Activity 7

Compute the following sums .

a) $1 + 2 + 3 + \ldots + 100$

b) $7 + 14 + 21 + \ldots + 700$

c) $4 + 11 + 18 + \ldots + 697$

Answer

a) $1 + 2 + 3 + \ldots + 100 = \frac{100 \times 101}{2} = 5050$

b) $7 + 14 + 21 + \ldots + 700 = 7(1 + 2 + 3 + \ldots + 100) = 7 \times 5050 = 35350$

c) $4 + 11 + 18 + \ldots + 697 = 25250 - (100 \times 3) = 25250 - 300 = 24950$

(Here the terms of the arithmetic sequence $4, 11, 18, \ldots, 697$ **are got by subtracting 3 from the terms of the arithmetic sequence** $7, 14, 21, \ldots, 700$ **)**

Activity 8

Compute the following sums .

a) $1 + 2 + 3 + \ldots + 20$

b) $10 + 20 + 30 + \ldots + 200$

c) $6 + 16 + 26 + \ldots + 196$

Answer

a) $1 + 2 + 3 + \ldots + 20 = \dfrac{20 \times 21}{2} = 210$

b) $10 + 20 + 30 + \ldots + 200 = 10(1 + 2 + 3 + \ldots + 20) = 10 \times 210 = 2100$

c) $6 + 16 + 26 + \ldots + 196 = 2100 - (20 \times 4) = 2100 - 80 = 2020$

(Here the terms of the arithmetic sequence $6, 16, 26, \ldots, 196$ **are got by subtracting 4 from the terms of the arithmetic sequence** $10, 20, 30, \ldots, 200$ **)**

Activity 9 (The sum of first n terms of an arithmetic sequence)

An arithmetic sequence is of the form ,

$$x_n = an + b$$

To calculate the sum its first n **terms , we put** $n = 1, 2, 3, \ldots$ **in this and add .**

First term $= a \times 1 + b = a + b$

Second term $= a \times 2 + b = 2a + b$

Third term $= a \times 3 + b = 3a + b$

Fourth term $= a \times 4 + b = 4a + b$

Fifth term $= a \times 5 + b = 5a + b$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

n^{th} term $= an + b$

$$x_1 + x_2 + x_3 + \ldots + x_n = (a + 2a + 3a + \ldots + an) + \underbrace{(b + b + b + \ldots + b)}_{n \text{ times}}$$

$$= a(1 + 2 + 3 + \ldots + n) + b \times n$$

$$= a\frac{n(n+1)}{2} + bn$$

Finding

For the arithmetic sequence ,

$$x_n = an + b$$

the sum of the first n terms is

$$x_1 + x_2 + x_3 + \ldots + x_n = a\frac{n(n+1)}{2} + bn$$

NOTE :

Arithmetic sequence	Algebraic form	Sum of the first n terms
5 , 8 , 11 , . . .	$3n + 2$	$3 \times \frac{n(n+1)}{2} + 2n$
7 , 11 , 15 , . . .	$4n + 3$	$4 \times \frac{n(n+1)}{2} + 3n$
11 , 21 , 31 , . . .	$10n + 1$	$10 \times \frac{n(n+1)}{2} + n$
1 , 6 , 11 , . . .	$5n - 4$	$5 \times \frac{n(n+1)}{2} - 4n$
7 , 15 , 23 , . . .	$8n - 1$	$8 \times \frac{n(n+1)}{2} - n$

Activity 10

Calculate the difference between the sums of the first 20 terms of the arithmetic sequences

2 , 9 , 16 , . . . and 5 , 12 , 19 , ...

Answer

$$5 + 12 + 19 + \ldots + x_{20} \quad -$$

$$2 + 9 + 16 + \ldots + y_{20}$$

$$3 + 3 + 3 + \ldots + 3 = 3 \times 20 = 60$$

Activity 11

What is the difference between the sum of the first 10 terms and the next 10 terms of the arithmetic sequence 7 , 11 , 15 , . . .

Answer

$d = 11 - 7 = 4$

$$\begin{array}{r} x_{11} + x_{12} + x_{13} + \ldots + x_{20} \\ x_1 + x_2 + x_3 + \ldots + x_{10} \\ \hline 10d + 10d + 10d + \ldots + 10d = 10 \times 10d \end{array}$$

$$= 10 \times 10 \times 4 = 400$$

Activity 12

Common difference of an arithmetic sequence is 6 and the sum of the first 20 terms is 1300 . Write down the sequence .

Answer

$$x_1 + x_{20} = \frac{1300}{10} = 130$$ **(20 terms = total 10 pairs)**

$$x_1 + (x_1 + 19d) = 130$$

$$2x_1 + 19d = 130$$

$$2x_1 + 19 \times 6 = 130$$

$$2x_1 + 114 = 130$$

$$2x_1 = 130 - 114 = 16$$

$$x_1 = \frac{16}{2} = 8$$

Sequence = 8 , 14 , 20 , . . .

NOTE : (Another method)

The algebraic form any arithmetic sequence of common difference 6 can be taken as

$6n + b$.

Sum of first 20 terms = 1300

$$\Rightarrow \quad 6 \times \frac{20 \times 21}{2} + b \times 20 = 1300$$

$$6 \times 210 + 20b = 1300$$

$$1260 + 20b = 1300$$

$$20b = 1300 - 1260 = 40$$

$$b = \frac{40}{20} = 2$$

$x_n = 6n + b = 6n + 2$

$x_1 = 6 \times 1 + 2 = 6 + 2 = 8$

Sequence = 8 , 14 , 20 , . . .